# Color Science

M074 / GV14 2013
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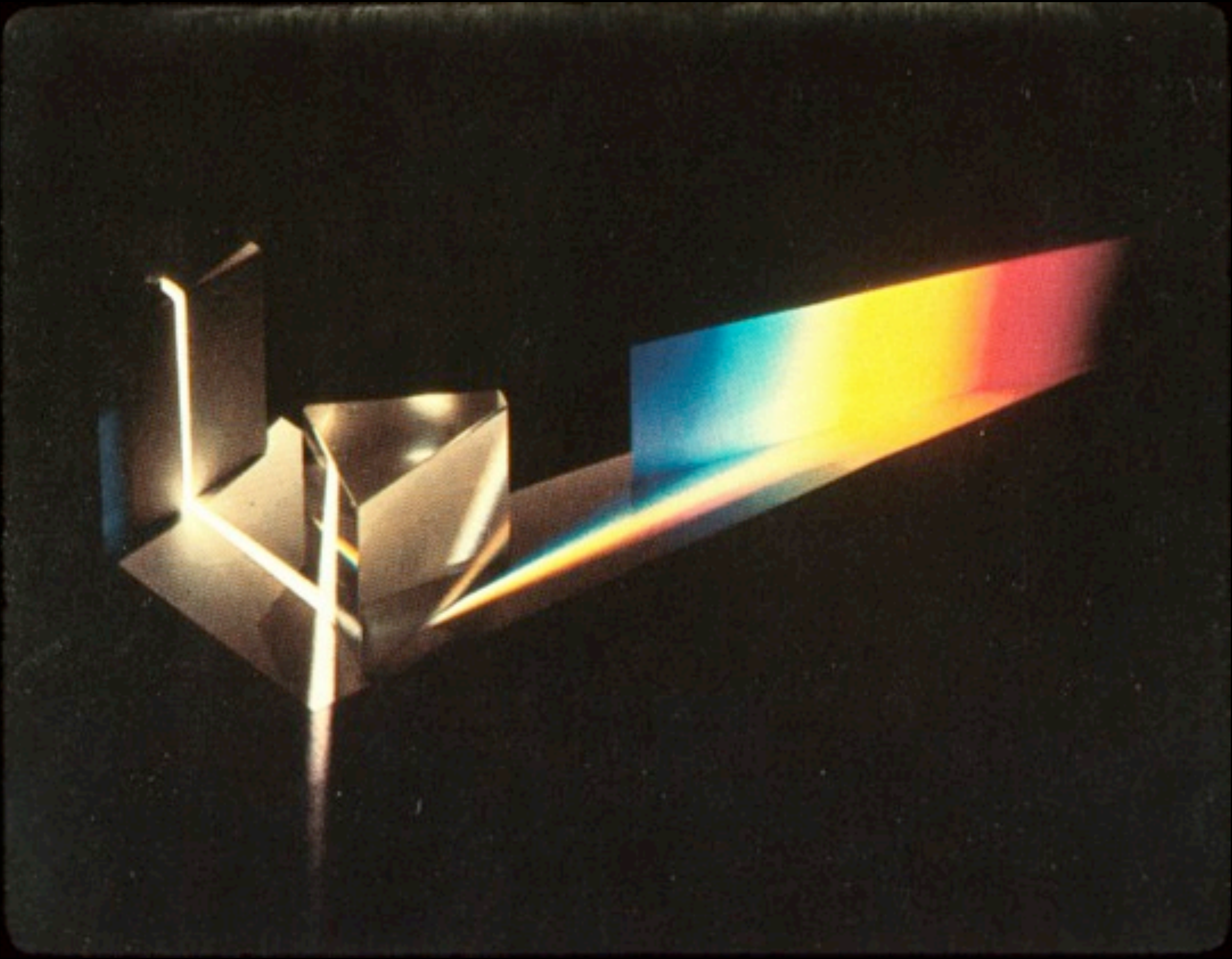
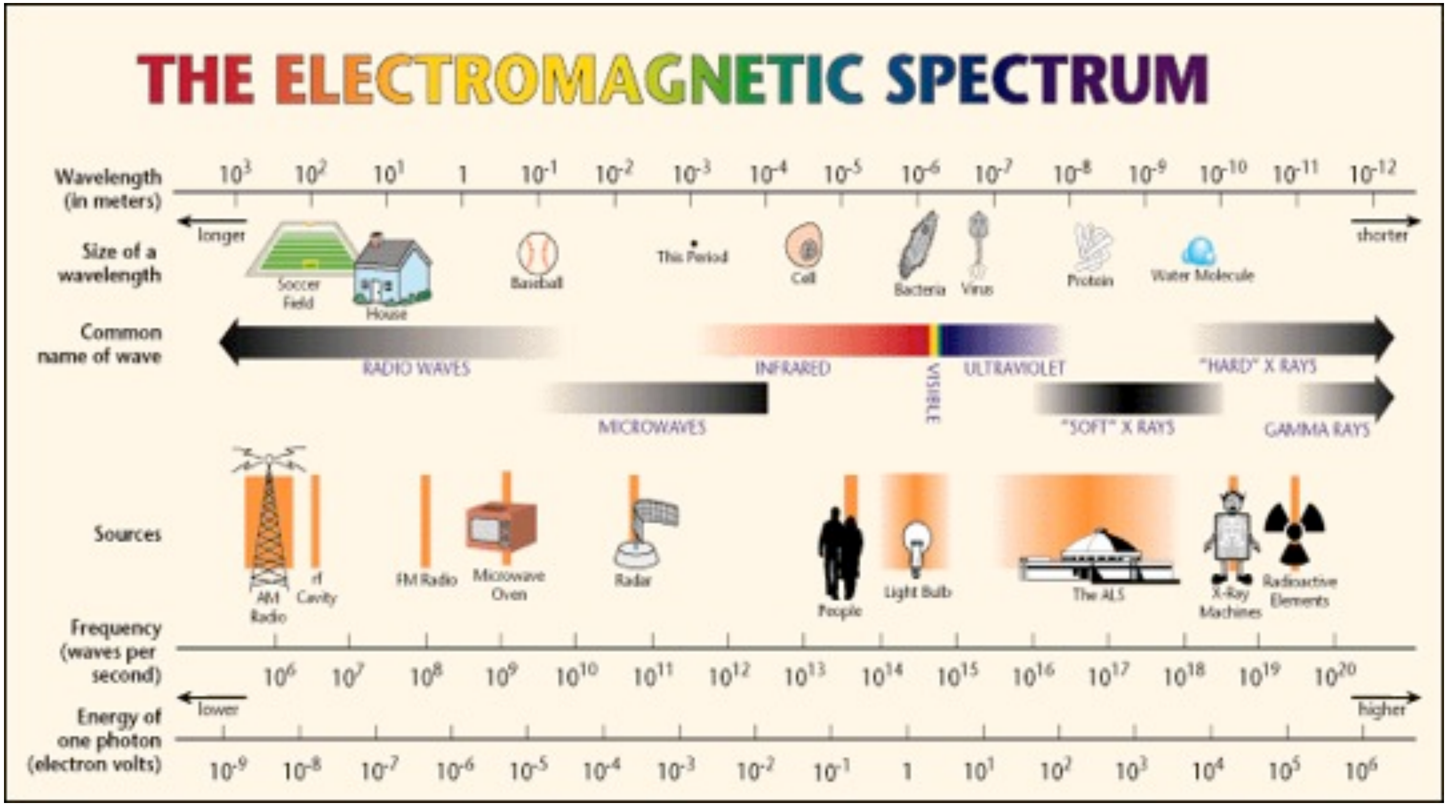
presenting slides by Steve Marschner



[source unknown]

# What light is

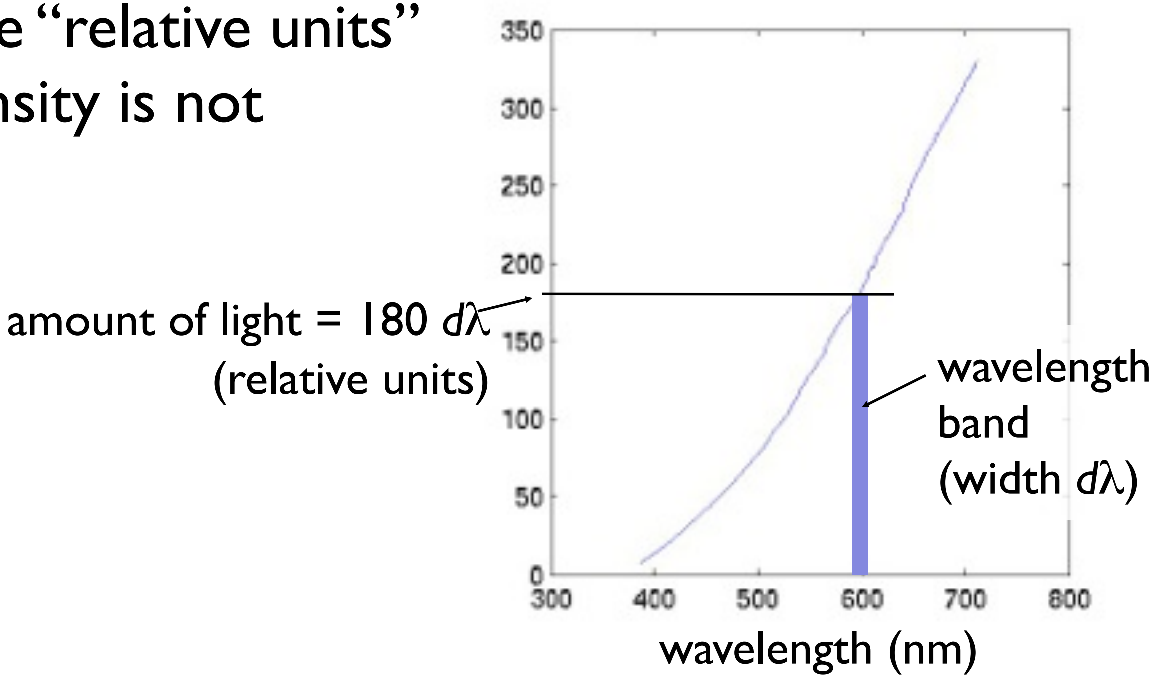
- Light is electromagnetic radiation
  - exists as oscillations of different frequency (or, wavelength)



[Lawrence Berkeley Lab / MicroWorlds]

# Measuring light

- Salient property is the *spectral power distribution (SPD)*
  - the amount of light present at each wavelength
  - units: Watts per nanometer (tells you how much power you'll find in a narrow range of wavelengths)
  - for color, often use "relative units" when overall intensity is not important

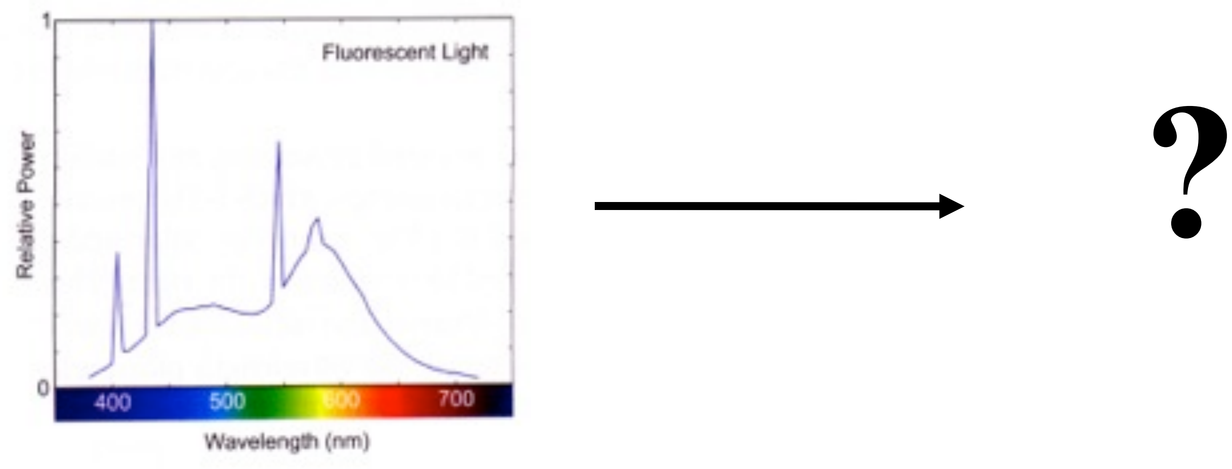


# What color is

- Colors are the sensations that arise from light energy of different wavelengths
  - we are sensitive from about 380 to 760 nm—one "octave"
- Color is a phenomenon of human perception; it is **not** a universal property of light
- Roughly speaking, things appear "colored" when they depend on wavelength and "gray" when they do not.

# The problem of color science

- Build a model for human color perception
- That is, map a *Physical light description* to a *Perceptual color sensation*

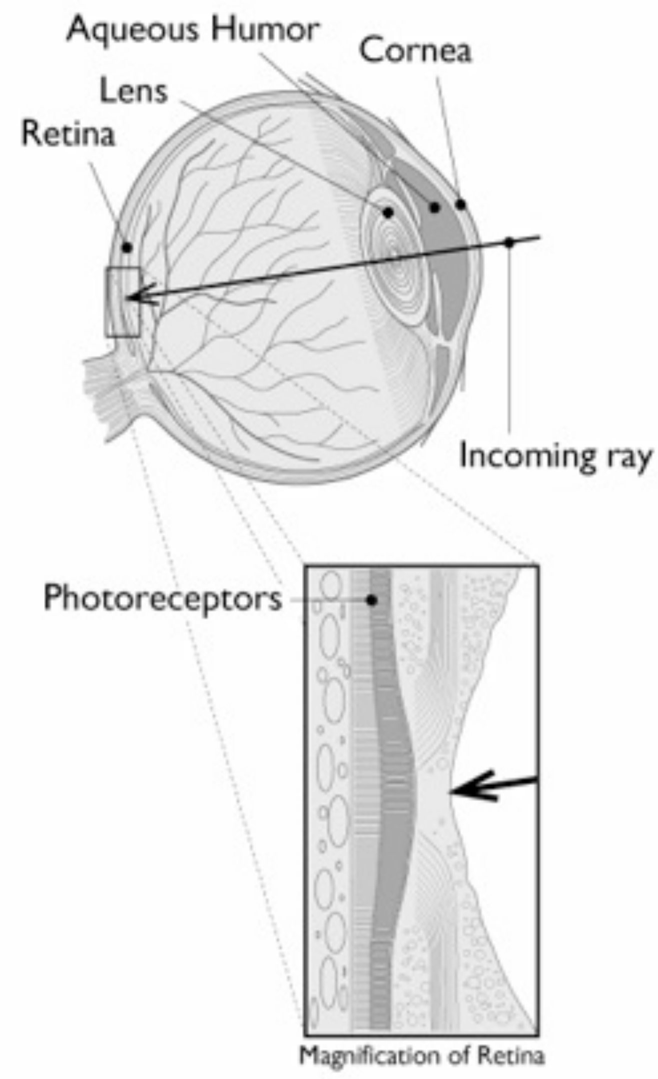


[Stone 2003]

*Physical*

*Perceptual*

# The eye as a measurement device



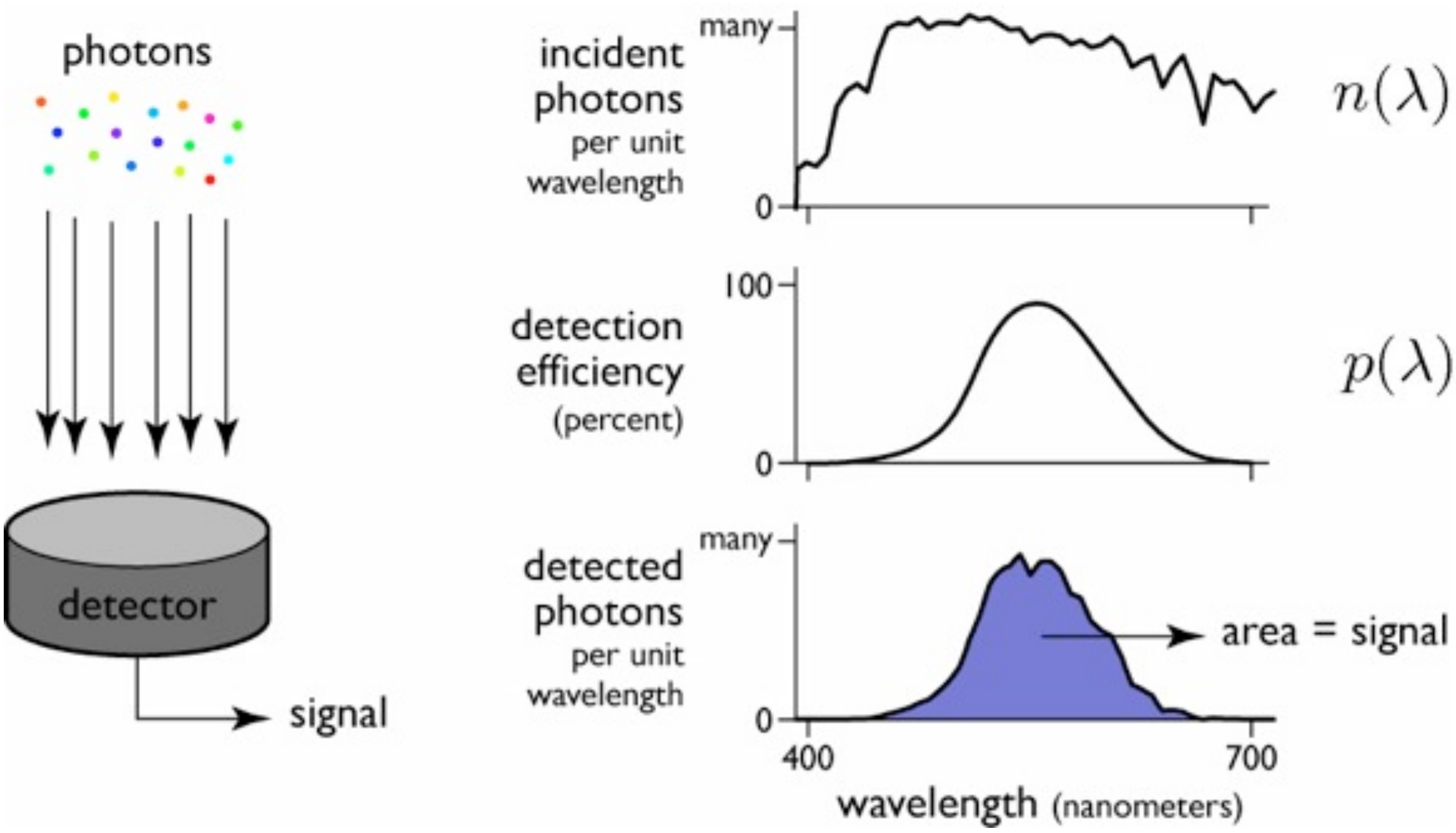
[Greger et al. 1995]

- We can model the low-level behavior of the eye by thinking of it as a light-measuring machine
  - its optics are much like a camera
  - its detection mechanism is also much like a camera
- Light is measured by the *photoreceptors* in the retina
  - they respond to visible light
  - different types respond to different wavelengths

# A simple light detector

- Produces a scalar value (a number) when photons land on it
  - this value depends strictly on the number of photons detected
  - each photon has a probability of being detected that depends on the wavelength
  - there is no way to tell the difference between signals caused by light of different wavelengths: there is just a number
- This model works for many detectors:
  - based on semiconductors (such as in a digital camera)
  - based on visual photopigments (such as in human eyes)

# A simple light detector



$$X = \int n(\lambda) p(\lambda)\, d\lambda$$

# Light detection math

- Same math carries over to power distributions
  - spectum entering the detector has its spectral power distribution (SPD), *s*(λ)
  - detector has its *spectral sensitivity* or *spectral response*, *r*(λ)

$$X = \int s(\lambda) r(\lambda) \, d\lambda$$

measured signal

detector's sensitivity

input spectrum

# Light detection math

$$X = \int s(\lambda) r(\lambda)\, d\lambda \qquad \text{or} \qquad X = s \cdot r$$

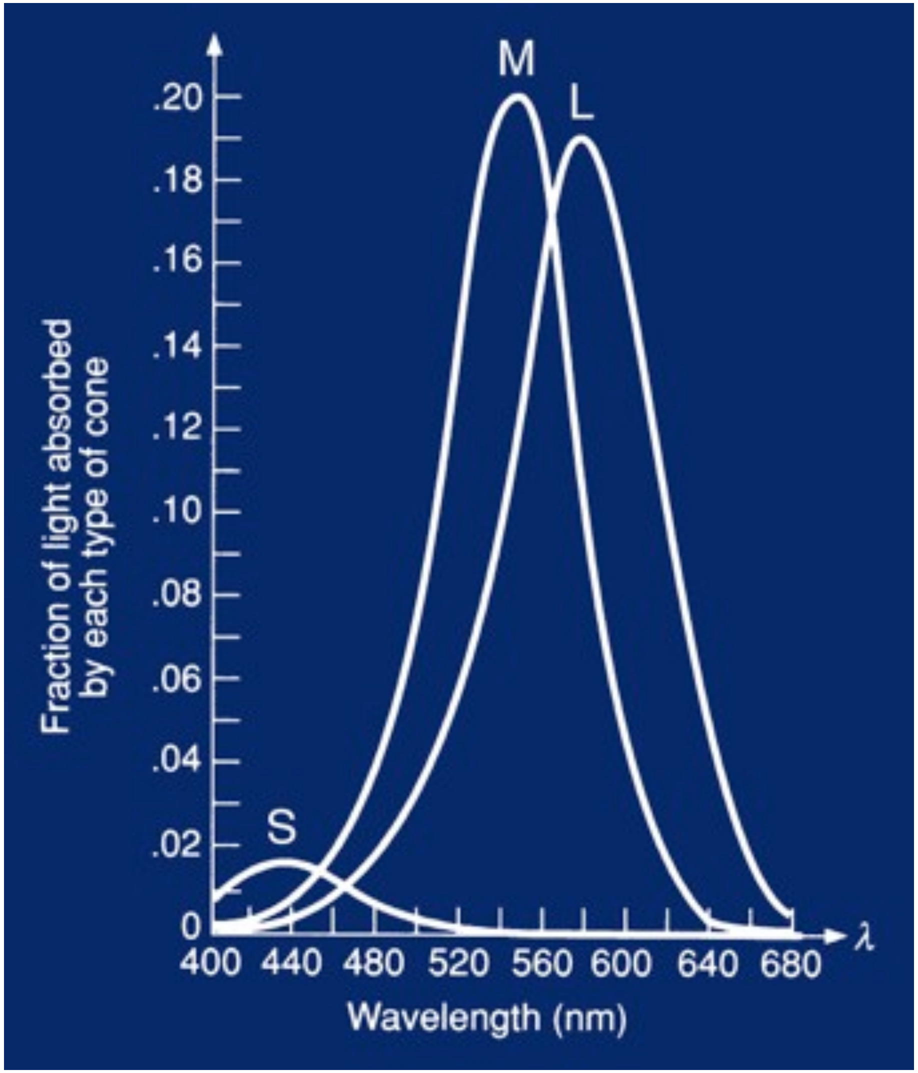
- If we think of *s* and *r* as vectors, this operation is a dot product (aka inner product)
  - in fact, the computation is done exactly this way, using sampled representations of the spectra.
    - let $\lambda_i$ be regularly spaced sample points $\Delta\lambda$ apart; then:

$$\tilde{s}[i] = s(\lambda_i); \tilde{r}[i] = r(\lambda_i)$$

$$\int s(\lambda) r(\lambda)\, d\lambda \approx \sum_i \tilde{s}[i] \tilde{r}[i]\, \Delta\lambda$$

    - this sum is very clearly a dot product

# Cone Responses



- S,M,L cones have broadband spectral sensitivity
- S,M,L neural response is integrated w.r.t. λ
  - we'll call the response functions $r_S, r_M, r_L$
- Results in a trichromatic visual system
- S, M, and L are *tristimulus values*

[source unknown]

# Cone responses to a spectrum *s*

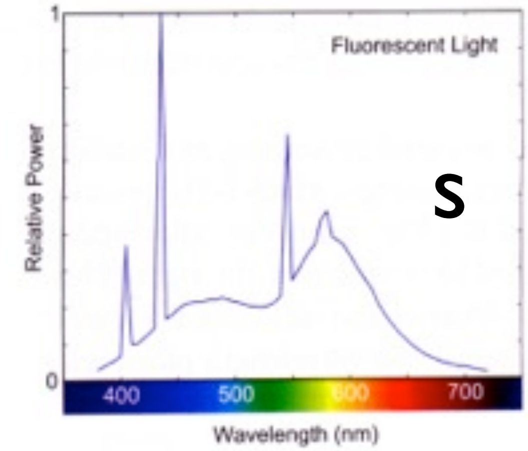
$$S = \int r_S(\lambda) s(\lambda)\, d\lambda = r_S \cdot s$$

$$M = \int r_M(\lambda) s(\lambda)\, d\lambda = r_M \cdot s$$

$$L = \int r_L(\lambda) s(\lambda)\, d\lambda = r_L \cdot s$$

# Colorimetry: an answer to the problem

- Wanted to map a *Physical light description* to a *Perceptual color sensation*
- Basic solution was known and standardized by 1930
  - Though not quite in this form—more on that in a bit



[Stone 2003]

**s** $\longrightarrow$

$$S = r_S \cdot s$$

$$M = r_M \cdot s$$

$$L = r_L \cdot s$$

*Physical*

*Perceptual*

# Basic fact of colorimetry

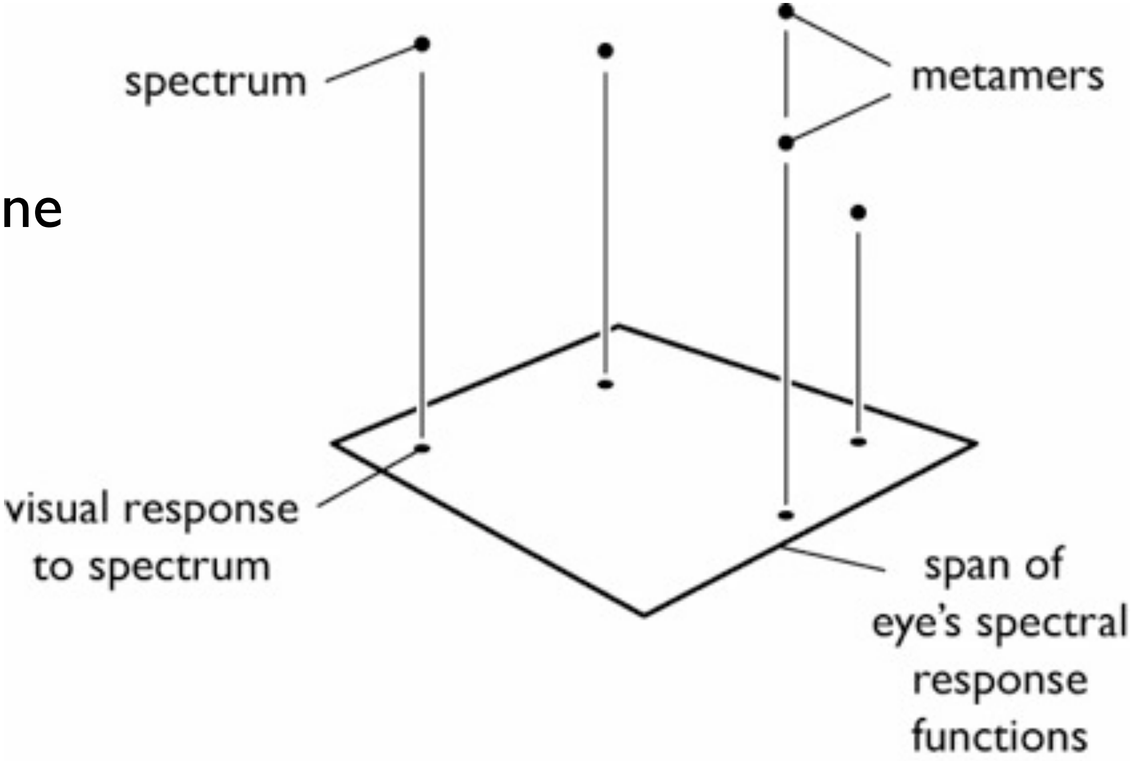
- Take a spectrum (which is a function)
- Eye produces three numbers
- This throws away a lot of information!
  - Quite possible to have two different spectra that have the same S, M, L tristimulus values
  - Two such spectra are *metamers*

# Pseudo-geometric interpretation

- A dot product is a projection
- We are projecting a high dimensional vector (a spectrum) onto three vectors
  - differences that are perpendicular to all 3 vectors are not detectable
- For intuition, we can imagine a 3D analog
  - 3D stands in for high-D vectors
  - 2D stands in for 3D
  - Then vision is just projection onto a plane

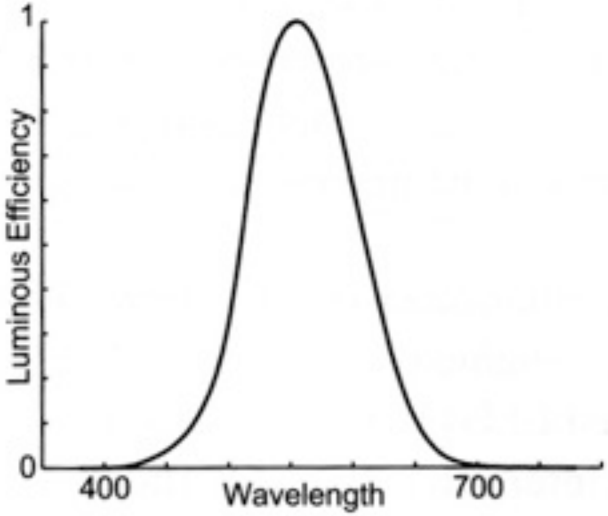
# Pseudo-geometric interpretation

- The information available to the visual system about a spectrum is three values
  - this amounts to a loss of information analogous to projection on a plane
- Two spectra that produce the same response are metamers



# Basic colorimetric concepts

- Luminance
  - the overall magnitude of the the visual response to a spectrum (independent of its color)
    - corresponds to the everyday concept "brightness"
  - determined by product of SPD with the *luminous efficiency function* $V_\lambda$ that describes the eye's overall ability to detect light at each wavelength
  - e.g. lamps are optimized to improve their luminous efficiency (tungsten vs. fluorescent vs. sodium vapor)



[Stone 2003]

# Luminance, mathematically

- Y just has another response curve (like *S*, *M*, and *L*)

$$Y = r_Y \cdot s$$

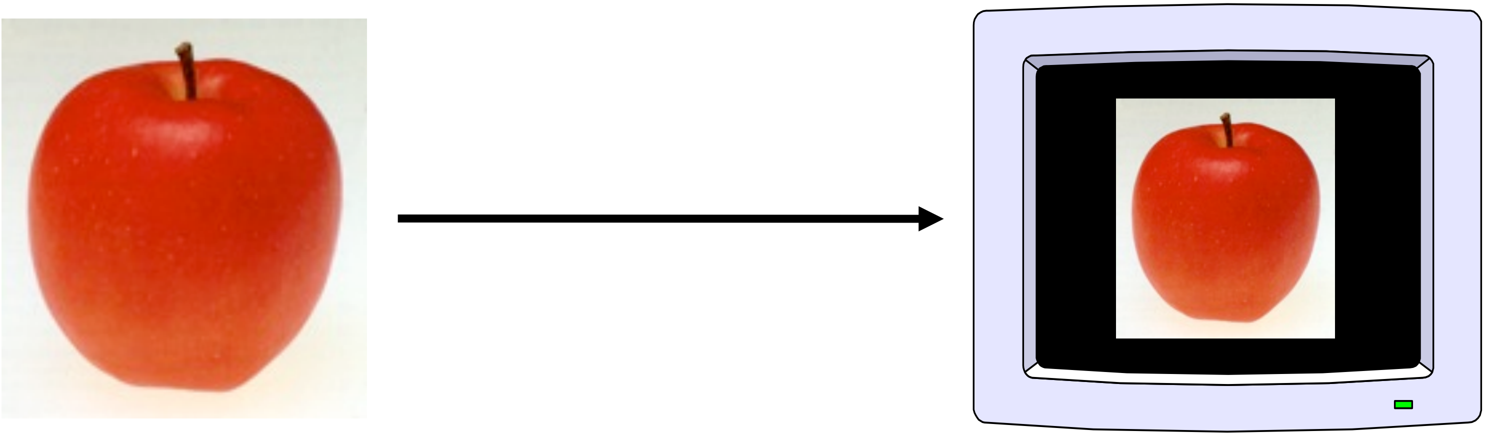
  - $r_Y$ is really called "$V_\lambda$"
- $V_\lambda$ is a linear combination of *S*, *M*, and *L*
  - Has to be, since it's derived from cone outputs

# More basic colorimetric concepts

- Chromaticity
  - what's left after luminance is factored out (the color without regard for overall brightness)
  - scaling a spectrum up or down leaves chromaticity alone
- Dominant wavelength
  - many colors can be matched by white plus a spectral color
  - correlates to everyday concept "hue"
- Purity
  - ratio of pure color to white in matching mixture
  - correlates to everyday concept "colorfulness" or "saturation"

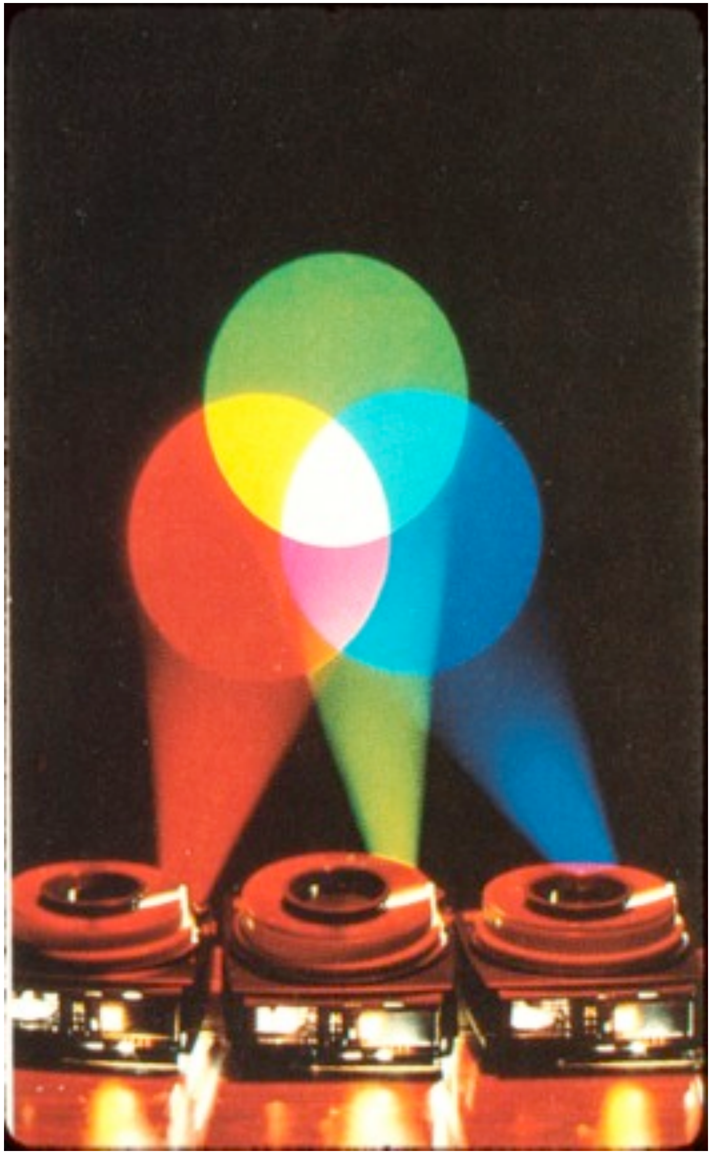
# Color reproduction

- Have a spectrum *s*; want to match on RGB monitor
  - "match" means it looks the same
  - any spectrum that projects to the same point in the visual color space is a good reproduction
- Must find a spectrum that the monitor *can* produce that is a metamer of *s*



[Don Greenberg]

# Additive Color



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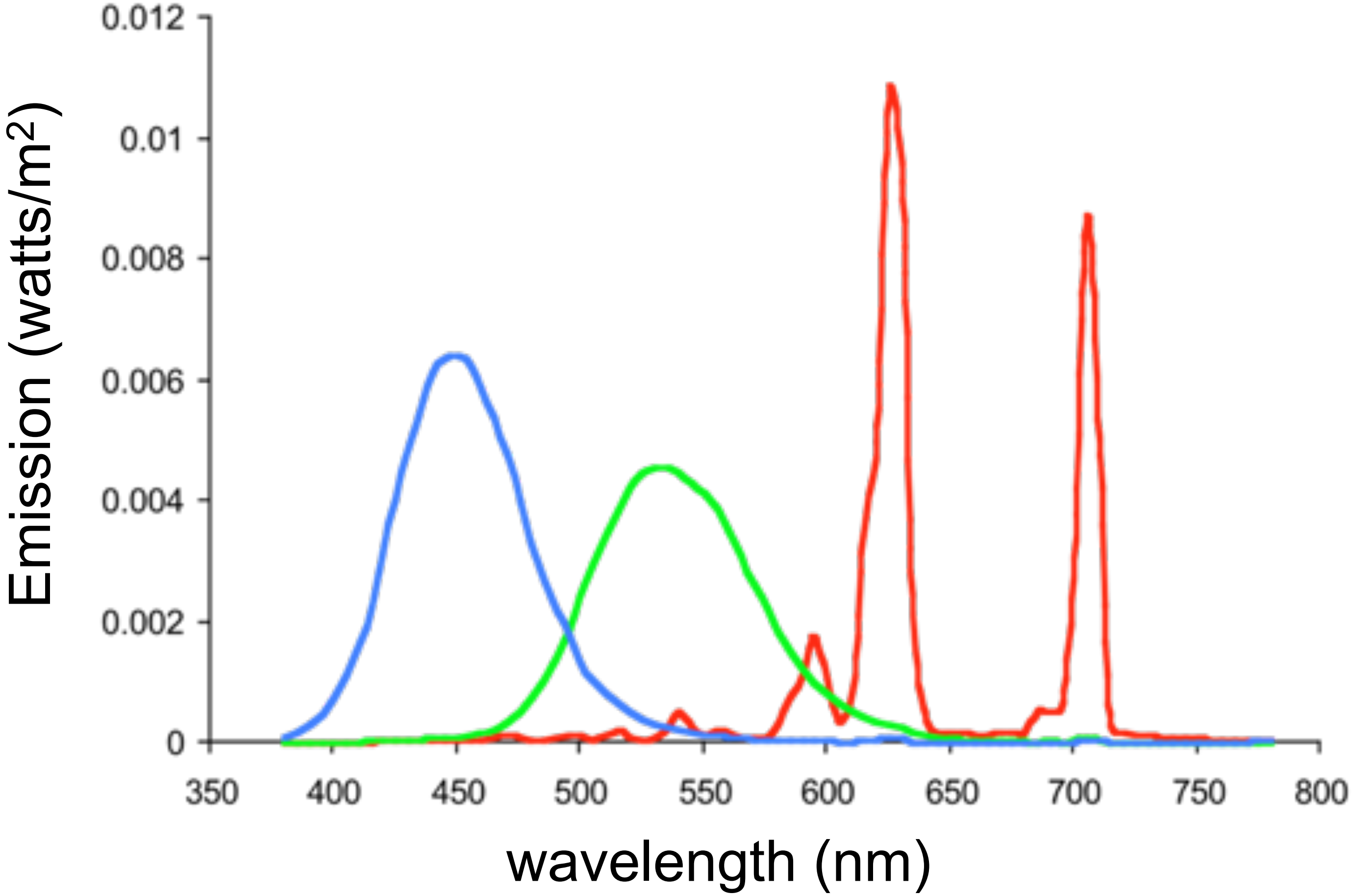
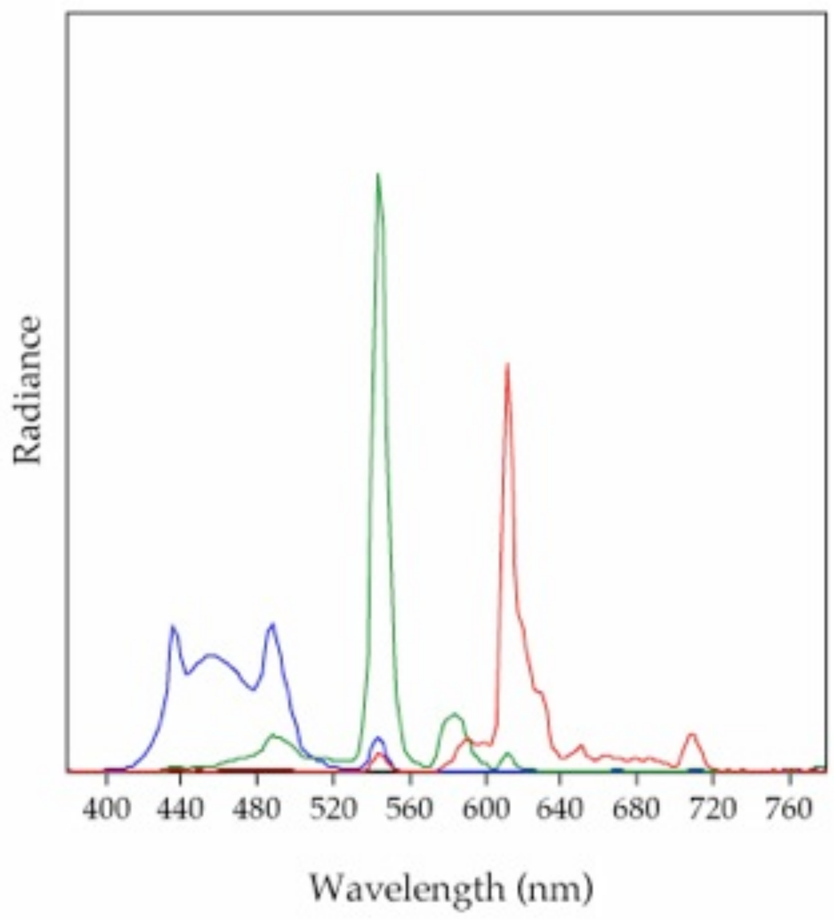
# CRT display primaries



– Curves determined by phosphor emission properties

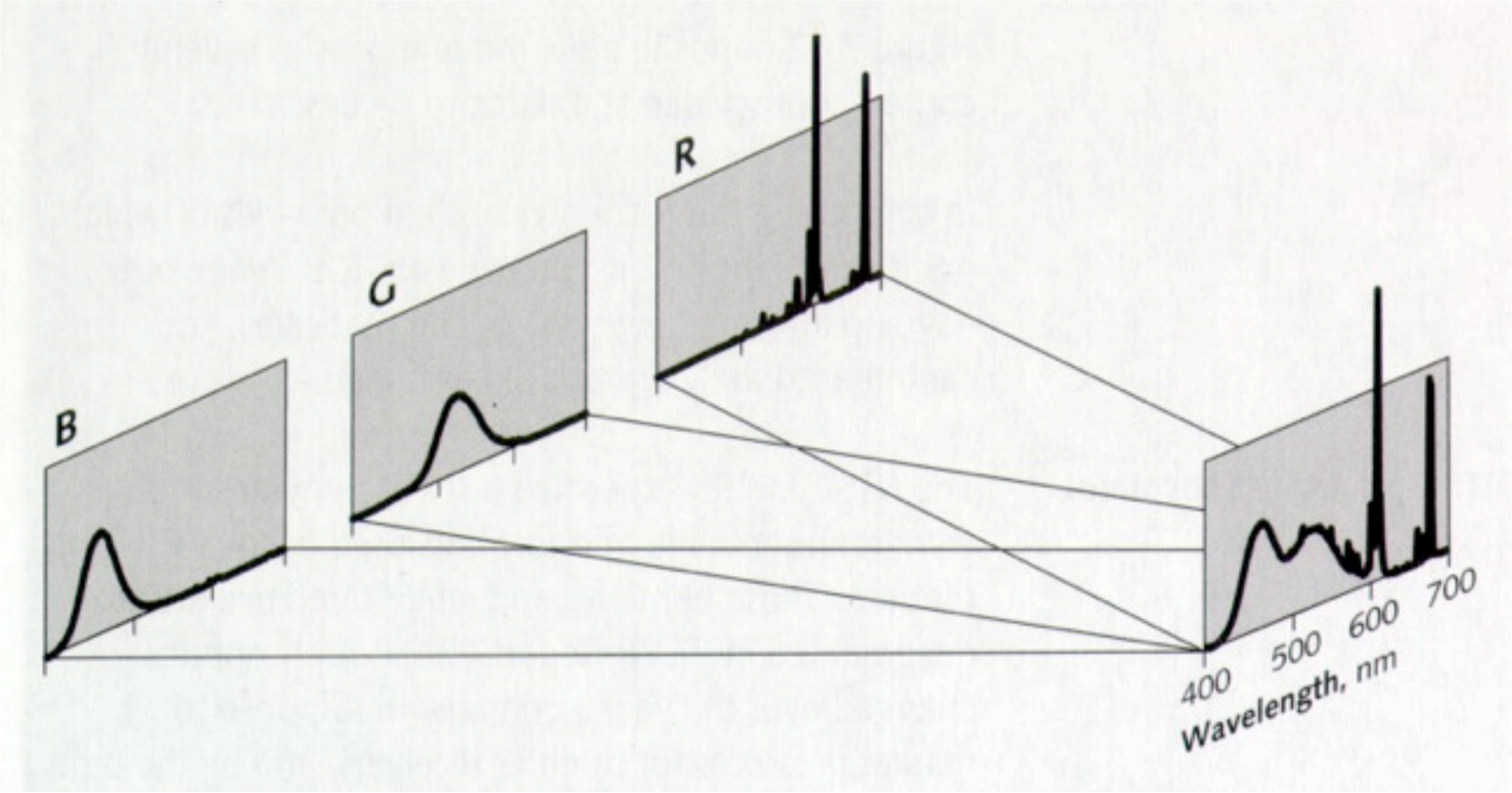
# LCD display primaries



[Fairchild 97]

– Curves determined by (fluorescent) backlight and filters

# Combining Monitor Phosphors with Spatial Integration



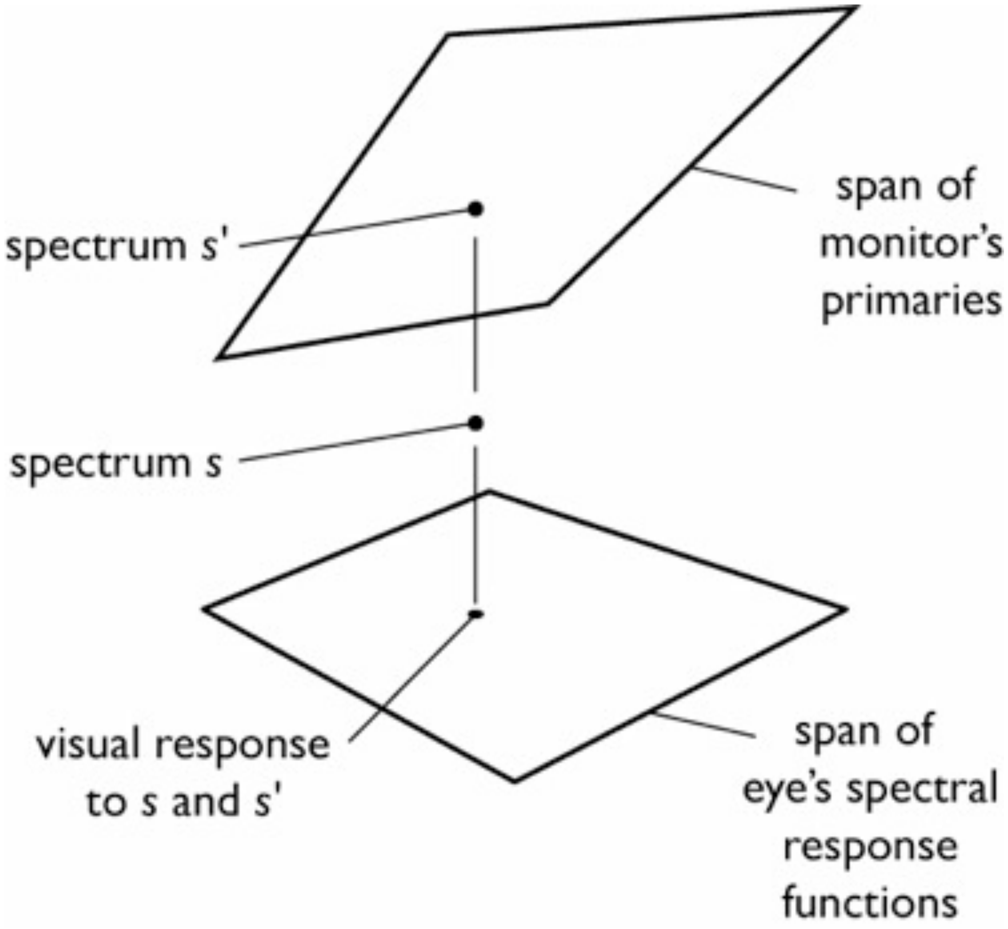
[source unknown]

# Color reproduction

- Say we have a spectrum *s* we want to match on an RGB monitor
  - “match” means it looks the same
  - any spectrum that projects to the same point in the visual color space is a good reproduction
- So, we want to find a spectrum that the monitor can produce that matches *s*
  - that is, we want to display a metamer of *s* on the screen

# Color reproduction

- We want to compute the combination of r, g, b that will project to the same visual response as *s*.



# Color reproduction as linear algebra

- The projection onto the three response functions can be written in matrix form:

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} - & r_S & - \\ - & r_M & - \\ - & r_L & - \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

or,

$$V = M_{SML}\, s.$$

# Color reproduction as linear algebra

- The spectrum that is produced by the monitor for the color signals R, G, and B is:

$$s_a(\lambda) = Rs_r(\lambda) + Gs_g(\lambda) + Bs_b(\lambda).$$

- Again the discrete form can be written as a matrix:

$$\begin{bmatrix} | \\ s_a \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} =$$

or,

$$s_a = M_{RGB}\,C.$$

# Color reproduction as linear algebra

- What color do we see when we look at the display?
  - Feed *C* to display
  - Display produces $s_a$
  - Eye looks at $s_a$ and produces *V*

$$V = M_{SML} M_{RGB} C$$

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

# Color reproduction as linear algebra

- Goal of reproduction: visual response to *s* and $s_a$ is the same:

$$M_{SML}\,\tilde{s} = M_{SML}\,\tilde{s}_a.$$

- Substituting in the expression for $s_a$,

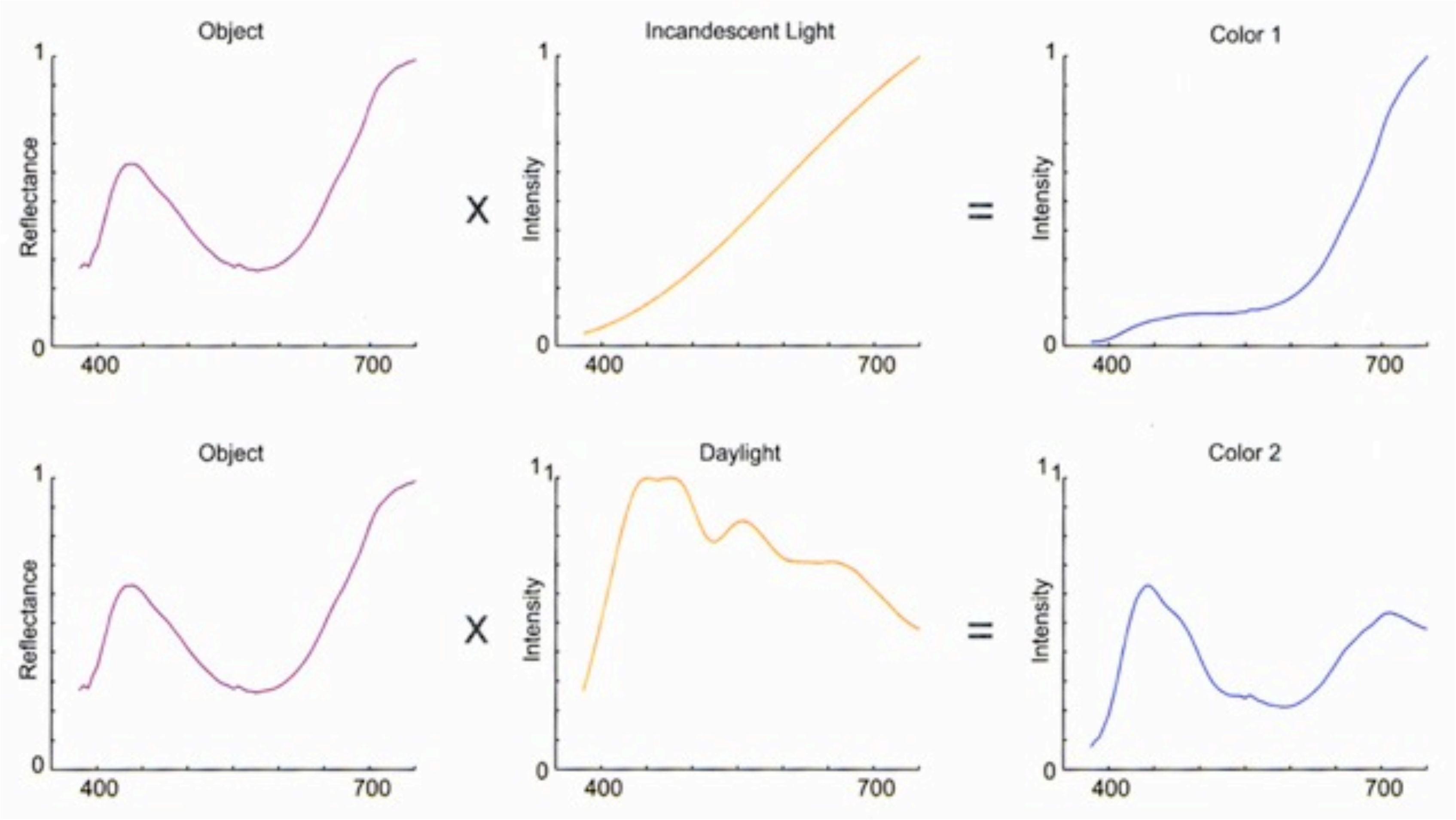
$$M_{SML}\,\tilde{s} = M_{SML}M_{RGB}\,C$$

$$C = \underbrace{(M_{SML}M_{RGB})^{-1}M_{SML}}_{\textit{color matching matrix}\text{ for RGB}}\,\tilde{s}$$

# Subtractive Color

[source unknown]

# Reflection from colored surface



[Stone 2003]

# Subtractive color

- Produce desired spectrum by *subtracting* from white light (usually via absorption by pigments)
- Photographic media (slides, prints) work this way
- Leads to C, M, Y as primaries
- Approximately, 1 – R, 1 – G, 1 – B

# Color spaces

- Need three numbers to specify a color
  - but what three numbers?
  - a *color space* is an answer to this question
- Common example: monitor RGB
  - define colors by what R, G, B signals will produce them on your monitor

    (in math, s = *R***R** + *G***G** + *B***B** for some spectra **R**, **G**, **B**)
  - device dependent (depends on gamma, phosphors, gains, …)
    - therefore if I choose RGB by looking at my monitor and send it to you, you may not see the same color
  - also leaves out some colors (limited *gamut*), e.g. vivid yellow

# Standard color spaces

- Standardized RGB (sRGB)
  - makes a particular monitor RGB standard
  - other color devices simulate that monitor by calibration
  - sRGB is usable as an interchange space; widely adopted today
  - gamut is still limited

# A universal color space: XYZ

- Standardized by CIE (*Commission Internationale de l'Eclairage*, the standards organization for color science)
- Based on three "imaginary" primaries **X**, **Y**, and **Z**

  (in math, $s = X\mathbf{X} + Y\mathbf{Y} + Z\mathbf{Z}$)

  - imaginary = only realizable by spectra that are negative at some wavelengths
  - key properties
    - *any* stimulus can be matched with positive *X*, *Y*, and *Z*
    - separates out luminance: **X**, **Z** have zero luminance, so *Y* tells you the luminance by itself

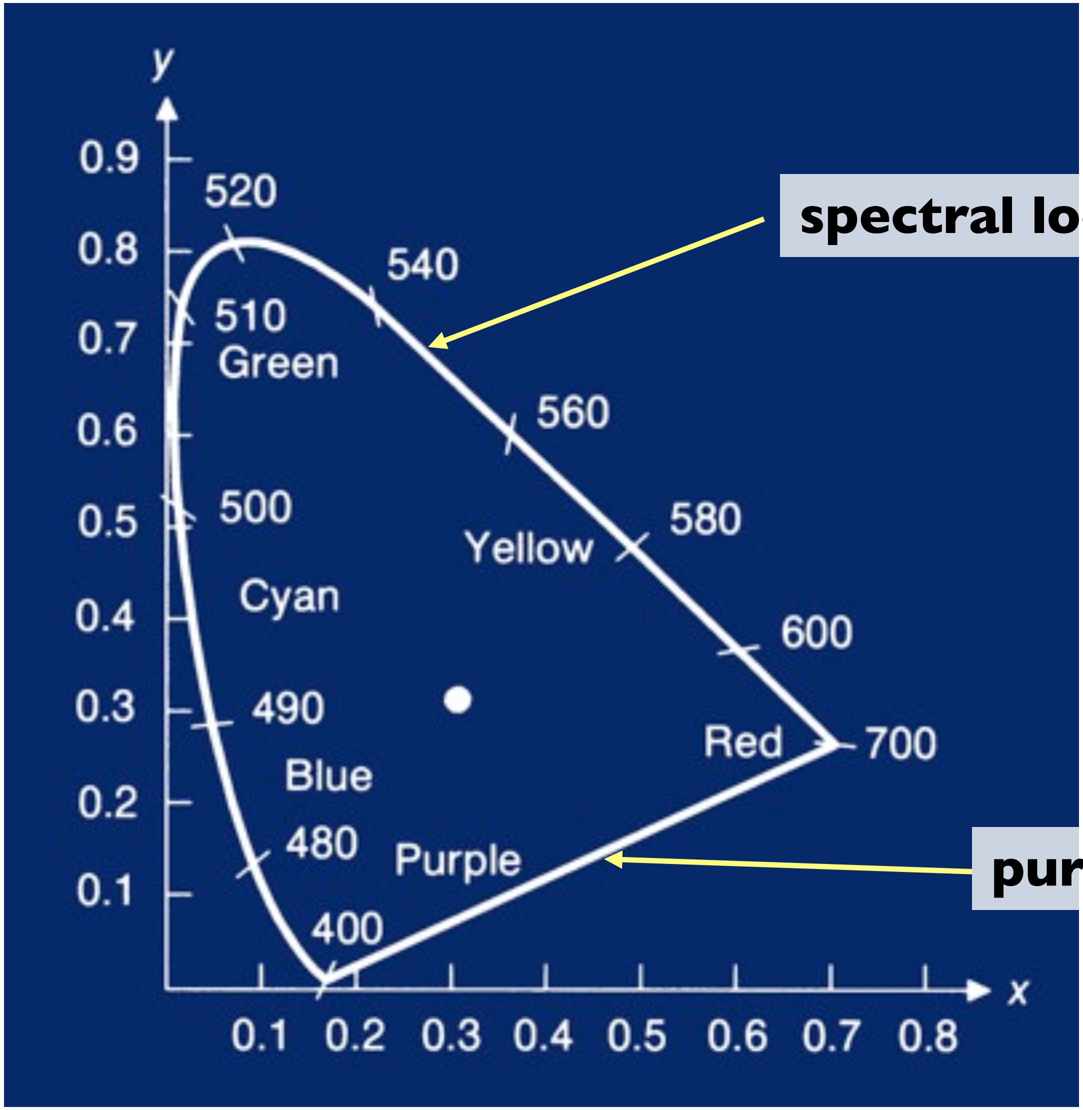
# Separating luminance, chromaticity

- Luminance: *Y*
- Chromaticity: *x*, *y*, *z*, defined as

$$x = \frac{X}{X + Y + Z}$$
$$y = \frac{Y}{X + Y + Z}$$
$$z = \frac{Z}{X + Y + Z}$$

  - since $x + y + z = 1$, we only need to record two of the three
    - usually choose *x* and *y*, leading to (*x*, *y*, *Y*) coords

# Chromaticity Diagram

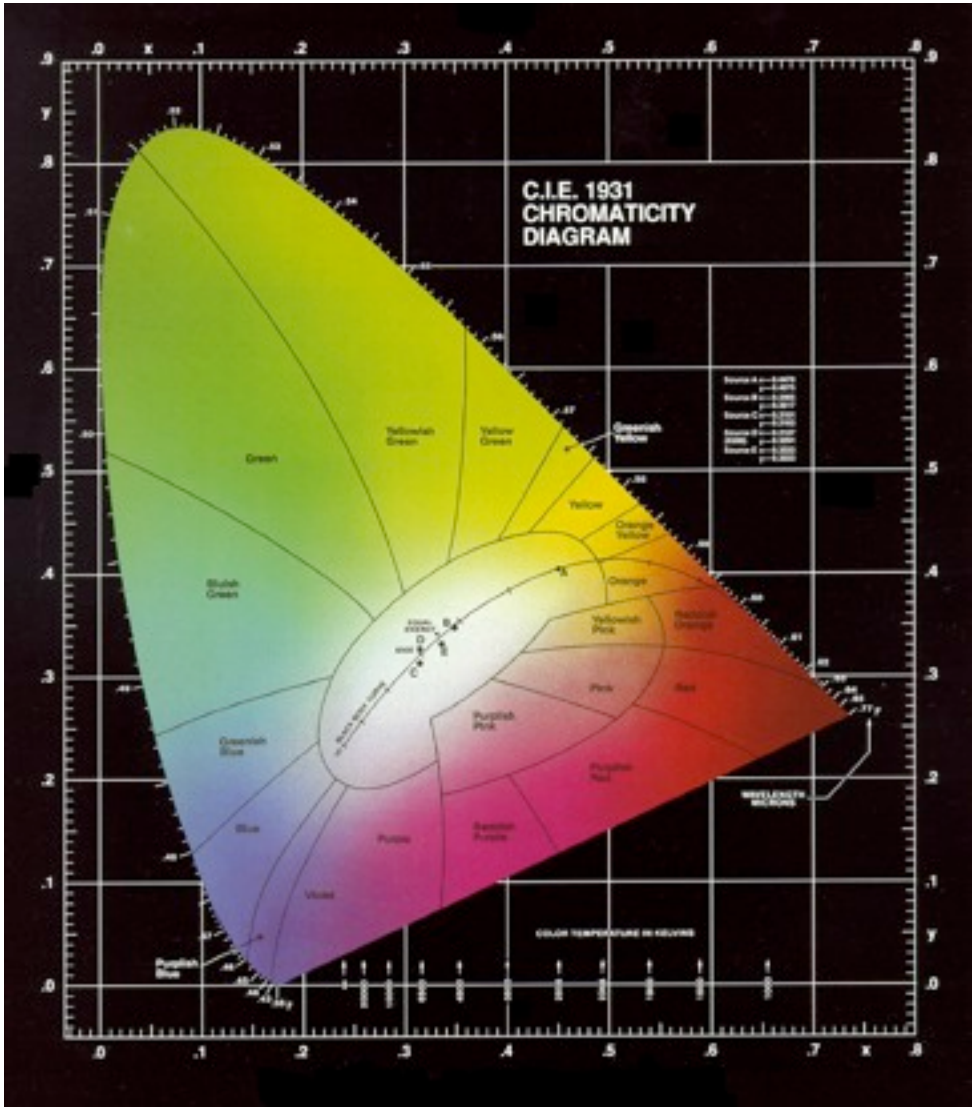


**spectral locus**

**purple line**

[source unknown]

# Chromaticity Diagram



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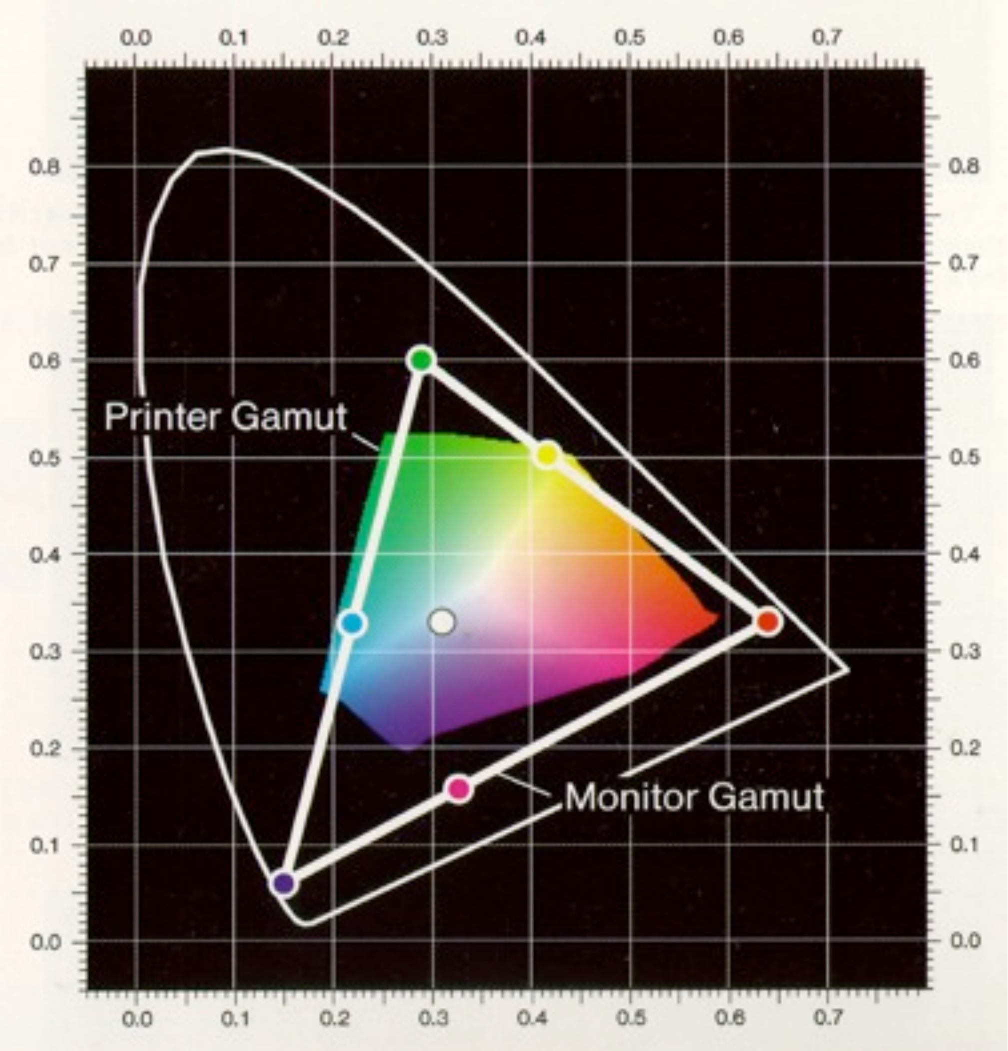
# Color Gamuts



[source unknown]

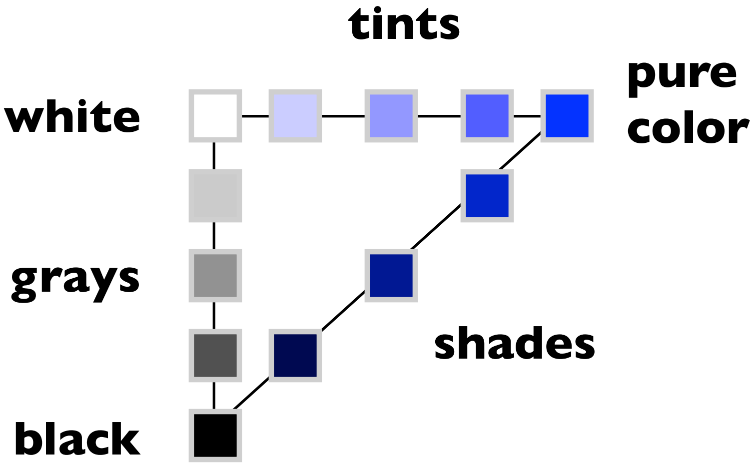
Monitors/printers can't produce all visible colors

Reproduction is limited to a particular domain

For additive color (e.g. monitor) gamut is the triangle defined by the chromaticities of the three primaries.

# Perceptually organized color spaces

- Artists often refer to colors as *tints*, *shades*, and *tones* of pure pigments
  - tint: mixture with white
  - shade: mixture with black
  - tones: mixture with black and white
  - gray: no color at all (aka. neutral)
- This seems intuitive
  - tints and shades are inherently related to the pure color
    - "same" color but lighter, darker, paler, etc.



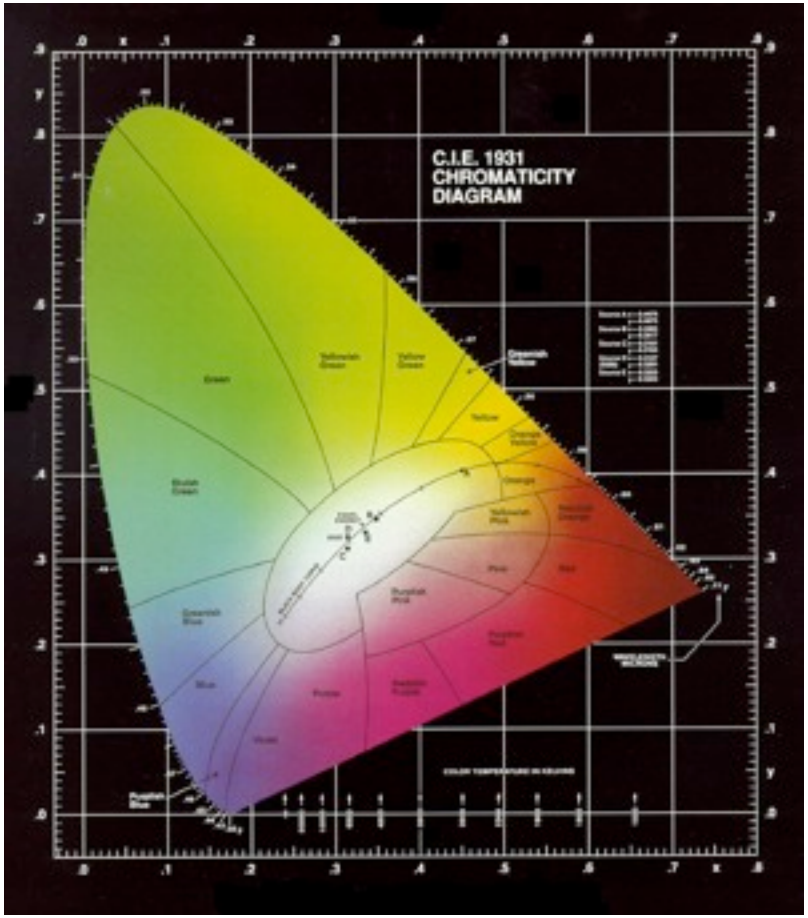
[after Foley et al.]

# Perceptual dimensions of color

- Hue
  - the “kind” of color, regardless of attributes
  - colorimetric correlate: dominant wavelength
  - artist’s correlate: the chosen pigment color
- Saturation
  - the “colorfulness”
  - colorimetric correlate: purity
  - artist’s correlate: fraction of paint from the colored tube
- Lightness (or value)
  - the overall amount of light
  - colorimetric correlate: luminance
  - artist’s correlate: tints are lighter, shades are darker

# Perceptual dimensions: chromaticity

- In x, y, Y (or another luminance/chromaticity space), Y corresponds to lightness
- hue and saturation are then like polar coordinates for chromaticity (starting at white, which way did you go and how far?)

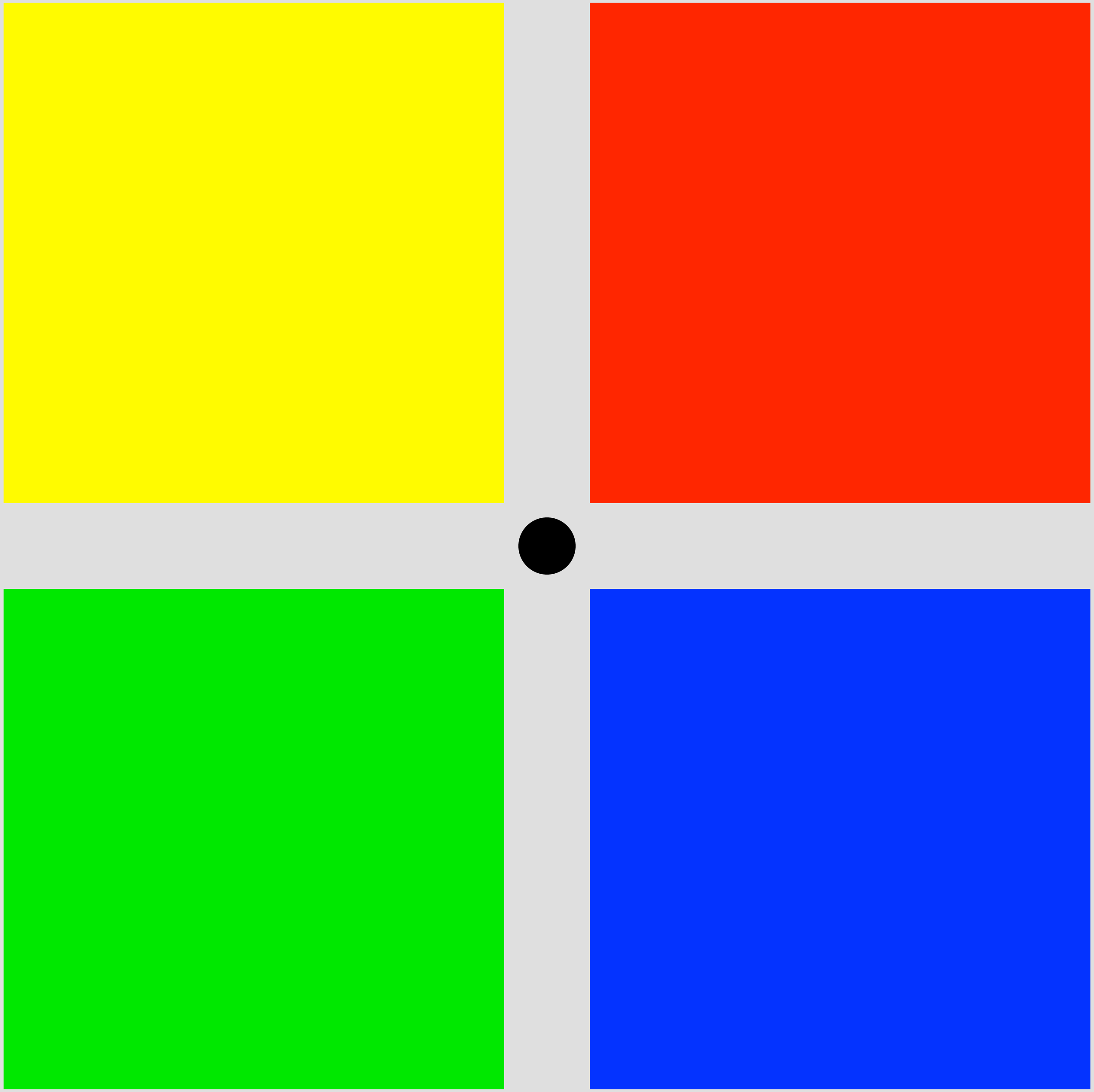


[source unknown]

# Perceptual dimensions of color

- There's good evidence ("opponent color theory") for a neurological basis for these dimensions
  - the brain seems to encode color early on using three axes:

    white — black, red — green, yellow — blue
  - the white—black axis is lightness; the others determine hue and saturation
  - one piece of evidence: you can have a light green, a dark green, a yellow-green, or a blue-green, but you can't have a reddish green (just doesn't make sense)
    - thus red is the *opponent* to green
  - another piece of evidence: afterimages (next slide)



# RGB as a 3D space

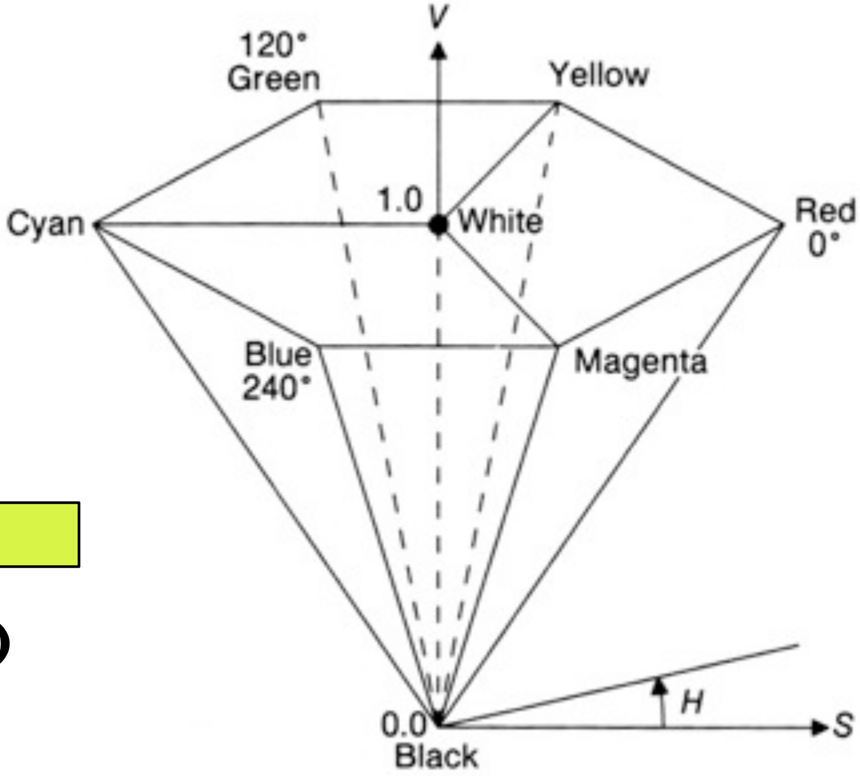
- A cube:

**(demo of RGB cube)**

# Perceptual organization for RGB: HSV

- Uses hue (an angle, 0 to 360), saturation (0 to 1), and value (0 to 1) as the three coordinates for a color
  - the brightest available RGB colors are those with one of R,G,B equal to 1 (top surface)
  - each horizontal slice is the surface of a sub-cube of the RGB cube

**(demo of HSV color pickers)**



[Foley et al.]

# Perceptually uniform spaces

- Two major spaces standardized by CIE
  - designed so that equal differences in coordinates produce equally visible differences in color
  - LUV: earlier, simpler space; *L**, *u**, *v**
  - LAB: more complex but more uniform: *L**, *a**, *b**
  - both separate luminance from chromaticity
  - including a gamma-like nonlinear component is important