Advanced Modelling, Rendering and Animation 2011

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Basic Information

- **Lecturers:** J. Kautz, A. Steed, T. Weyrich
- **Demonstrator:** James Tompkin
- **Lab Time**
  - Fridays, 2–4 PM, in MPEB 1.05 (W20–24)
  - Thursdays, 3–5 PM, in MPEB 4.06 (W26–30)
  - First lab: *TBA*

Basic Information

- **Assessment**
  - Written Examination (2.5 hours, 75%)
  - Coursework Section (2 pieces, 25%)
    - Deadlines: TBA (check web page)

Advanced Modelling, Rendering and Animation 2011
Colour in Computer Graphics

Tim Weyrich

Outline: Today

- Introduction
- Spectral distributions
- Simple Model for the Visual System
- Simple Model for an Emitter System
- Generating Perceivable Colours
- CIE-RGB Colour Matching Functions

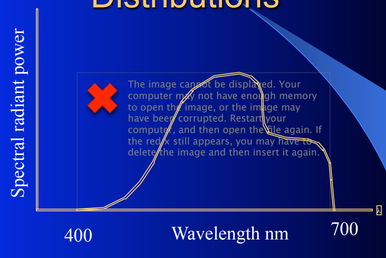
Spectral Distributions

- Radiometry (radiant power, radiance etc)
  - Measurement of light energy
- Photometry (luminance etc)
  - Measurement including response of visual system
- Generally $C(\lambda)$ defines spectral colour distribution $\lambda \varepsilon [\lambda_a, \lambda_b] = \Lambda$
- In computer graphics C is usually radiance.

Monochromatic Light (pure colour)

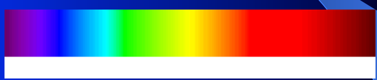
- $\delta(\lambda) = 0, \lambda \neq 0$
- $\int \delta(\lambda)\, d\lambda = 1$
- $\int \delta(t)\, f(x-t) dt = f(x)$
- $C(\lambda) = \delta(\lambda-\lambda_0)$ is spectral distribution for pure colour with wavelength λ_0

Colour as Spectral Distributions



Spectral Energy Distribution

Visible Spectrum



400nm 700nm

Schematic Representation of Colour Spectra

Colour Space

- Space of all visible colours equivalent to set of all functions $C : \Lambda \rightarrow R$
  - $C(\lambda) \geq 0$ all λ
  - $C(\lambda) > 0$ some λ.

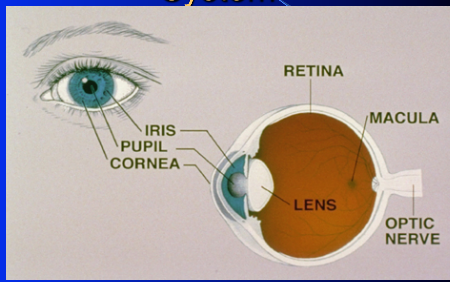
Perception and 'The Sixth Sense' movie

- We do not 'see' $C(\lambda)$ directly but as filtered through visual system.
- Two different people/animals will 'see' $C(\lambda)$ differently.
- Different $C(\lambda)$s can appear exactly the same to one individual (metamer).
- (Ignoring all 'higher level' processing, which basically indicates "we see what we expect to see").

Infinite to Finite

- Colour space is infinite dimensional
- Visual system filters the energy distribution through a finite set of channels
- Constructs a finite signal space (retinal level)
- Through optic nerve to higher order processing (visual cortex ++++).

A Simple Model for the Visual System



Human Eye Schematic

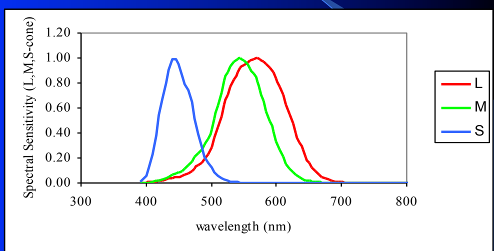
Photosensitive Receptors

- Rods – 130,000,000 night vision + peripheral (scotopic)
- Cones – 5-7,000,000, daylight vision + acuity (one point only)
- Cones
  - L-cones
  - M-cones
  - S-cones

LMS Response Curves

- $l = \int C(\lambda)L(\lambda)d\lambda$
- $m = \int C(\lambda)M(\lambda)d\lambda$
- $s = \int C(\lambda)S(\lambda)d\lambda$
- $C \rightarrow (l,m,s)$ (trichromatic theory)
- $LMS(C) = (l,m,s)$
- $LMS(C_a) = LMS(C_b)$ then C_a, C_b are *metamers*.

2-degree cone normalised response curves



Simple Model for an Emitter System

- Generates chromatic light by mixing streams of energy of light of different spectral distributions
- Finite number (3) and independent of each other

Primaries (Basis) for an Emitter

- $C_E(\lambda) = \alpha_1 E_1(\lambda) + \alpha_2 E_2(\lambda) + \alpha_3 E_3(\lambda)$
- E_i are the primaries (i.e., the display uses them)
- α_i are called the *intensities.*
- CIE-RGB Primaries are:
  - $E_R(\lambda) = \delta(\lambda - \lambda_R)$, $\lambda_R = 700$nm
  - $E_G(\lambda) = \delta(\lambda - \lambda_G)$, $\lambda_G = 546.1$nm
  - $E_B(\lambda) = \delta(\lambda - \lambda_B)$, $\lambda_B = 435.8$nm
    - CIE = Commission Internationale de L'Eclairage

Computing the Intensities

- For a given $C(\lambda)$ problem is to find the intensities α_i such that $C_E(\lambda)$ is metameric to $C(\lambda)$
- First Method to be shown isn't used, but illustrative of the problem.

Computing the Intensities

- For a metamer we require (considering L only):

$$\int C(\lambda)L(\lambda)d\lambda = \int C_E(\lambda)L(\lambda)d\lambda$$

- Expanding:

$$\int_\Lambda C(\lambda)L(\lambda)d\lambda = \int_\Lambda (\alpha_1 E_1(\lambda) + \alpha_2 E_2(\lambda) + \alpha_3 E_3(\lambda))L(\lambda)d\lambda$$

$$= \alpha_1 \int_\Lambda E_1(\lambda)L(\lambda)d\lambda + \alpha_2 \int_\Lambda E_2(\lambda)L(\lambda)d\lambda + \alpha_3 \int_\Lambda E_3(\lambda)L(\lambda)d\lambda$$

Computing the Intensities

- Write

$$\int_\Lambda C(\lambda)L(\lambda)d\lambda = c_L$$

$$\int_\Lambda E_i(\lambda)L(\lambda)d\lambda = e_{iL}$$

In principle C, L and E_i will be known.

- And do the same derivation for M and S:

$$\alpha_1 e_{1L} + \alpha_2 e_{2L} + \alpha_3 e_{3L} = c_L$$

$$\alpha_1 e_{1M} + \alpha_2 e_{2M} + \alpha_3 e_{3M} = c_M$$

$$\alpha_1 e_{1S} + \alpha_2 e_{2S} + e_{3S} = c_S$$

Computing the Intensities

- Or

$$\begin{bmatrix} e_{1L} & e_{2L} & e_{3L} \\ e_{1M} & e_{2M} & e_{3M} \\ e_{1S} & e_{2S} & e_{3S} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} c_L \\ c_M \\ c_S \end{bmatrix}$$

- This gives 3 equations in 3 unknowns which can be solved for the unknown intensities.
- But in practice the L, M and S curves cannot be directly observed, so other techniques are used.

Colour Matching Functions

- Previous method relied on knowing L, M, and S response curves accurately.
- Better method based on colour matching functions.
- Define how to get the colour matching functions $\gamma_i(\lambda)$ relative to a given system of primaries (e.g., RGB).

Colour Matching Functions

- Let λ_0 be a monochromatic colour, and $\gamma_i(\lambda_0)$ (i=1,2,3) be the intensities, then:

$$\delta(\lambda-\lambda_0) \approx \sum_{i=1}^{3} \gamma_i(\lambda_0)E_i(\lambda)$$

- Also, $\int_{\Lambda} \delta(\lambda-\lambda_0)L(\lambda)d\lambda = L(\lambda_0)$
- By substitution: $\int_{\Lambda}\left(\sum_{i=1}^{3} \gamma_i(\lambda_0)E_i(\lambda)\right)L(\lambda)d\lambda = L(\lambda_0)$

Colour Matching Functions

- This further simplifies to:

$$\sum_{i=1}^{3} \gamma_i(\lambda_0)\int_{\Lambda} E_i(\lambda)L(\lambda)d\lambda = \sum_{i=1}^{3} \gamma_i(\lambda_0)e_{iL}$$

- and so:

$$\sum_{i=1}^{3} \gamma_i(\lambda_0)e_{iL} = L(\lambda_0)$$

Colour Matching Functions

- Now replace λ_0 by λ, multiply throughout by colour $C(\lambda)$, and integrate:

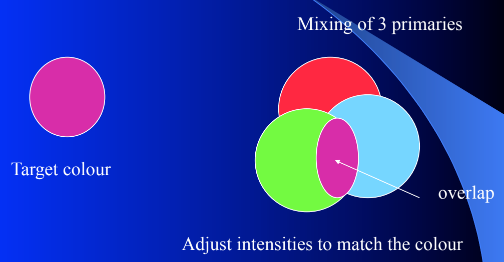
$$\sum_{i=1}^{3} e_{iL}\int_{\Lambda} \gamma_i(\lambda)C(\lambda)d\lambda = \sum_{i=1}^{3} e_{iL}\alpha_i$$

- With further rearrangement, we get the result: $\alpha_i = \int_{\Lambda} \gamma_i(\lambda)C(\lambda)d\lambda$

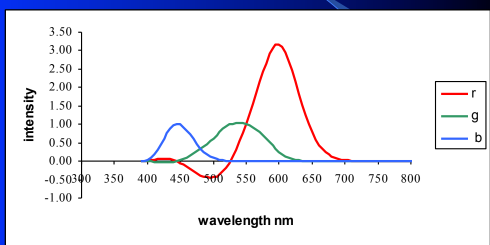
Colour Matching Functions

- Recall that the $\gamma_i(\lambda)$ were the intensities for monochromatic colours.
- The result says that we can find the intensities for a metamer for an arbitrary colour based on these.
- How can we estimate these $\gamma_i(\lambda)$?
- This can be done with a perceptual colour-matching experiment.

Colour Matching Experiment



2-degree RGB Colour Matching Functions



2-degree Colour Matching Functions

- RGB intensities sampled at 5nm intervals between 390nm and 830nm.
- They are '2 degree' color matching functions because the observer only sees a field of view of 2 degrees.
- 2 degree ones are used in computer graphics because of the relatively narrow field of view when looking at a display.

Negative Values?

- Not all monochrome colours can be represented with positive $\gamma_i(\lambda)$.

$$\delta(\lambda-\lambda_0) \approx \gamma_1(\lambda_0)E_R(\lambda)+\gamma_2(\lambda_0)E_G(\lambda)+\gamma_3(\lambda_0)E_B(\lambda)$$

- In that case we add one beam to the target and try to adjust the other two beams to match the new colour:

$$\delta(\lambda-\lambda_0)+\gamma_1(\lambda_0)E_R(\lambda) \approx \gamma_2(\lambda_0)E_G(\lambda)+\gamma_3(\lambda_0)E_B(\lambda)$$

Summary Lecture 1

- Compute the radiance distribution $C(\lambda)$
- Find out the colour matching functions for the display $\gamma_i(\lambda)$
- Perform the 3 integrals $\int \gamma_i(\lambda)C(\lambda)d\lambda$ to get the intensities for the metamer for that colour on the display.
- ….
- Except that's not how it is done …
- ….to be continued….

Outline: Lecture 2

- CIE-RGB Chromaticity Space
- CIE-XYZ Chromaticity Space
- Converting between XYZ and RGB
- Colour Gamuts and Undisplayable Colours
- Summary for Rendering: What to do in practice

CIE-RGB Chromaticity Space

- Consider CIE-RGB primaries:
  - For each $C(\lambda)$ there is a point $(\alpha_R, \alpha_G, \alpha_B)$:
    - $C(\lambda) \approx \alpha_R E_R(\lambda) + \alpha_G E_G(\lambda) + \alpha_B E_B(\lambda)$
  - Considering all such possible points
    - $(\alpha_R, \alpha_G, \alpha_B)$
  - Results in 3D RGB colour space
  - Hard to visualise in 3D
  - so we'll find a 2D representation instead.

CIE-RGB Chromaticity Space

- Consider 1st only monochromatic colours:
  - $C(\lambda) = \delta(\lambda - \lambda_0)$
- Let the CIE-RGB matching functions be
  - $r(\lambda), g(\lambda), b(\lambda)$
- Then, eg,
  - $\alpha_R(\lambda_0) = \int \delta(\lambda - \lambda_0)\, r(\lambda) d\lambda = r(\lambda_0)$
- Generally
  - $(\alpha_R(\lambda_0), \alpha_G(\lambda_0), \alpha_B(\lambda_0)) = (r(\lambda_0), g(\lambda_0), b(\lambda_0))$

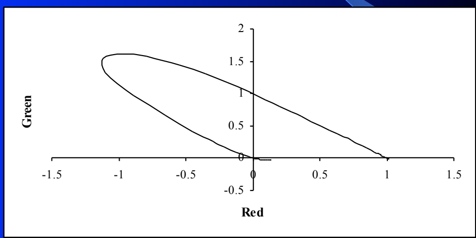
CIE-RGB Chromaticity Space

- As λ_0 varies over all wavelengths
  - $(r(\lambda_0), g(\lambda_0), b(\lambda_0))$ sweeps out a 3D curve.
- This curve gives the metamer intensities for all monochromatic colours.
- To visualise this curve, conventionally project onto the plane
  - $\alpha_R + \alpha_G + \alpha_B = 1$

CIE-RGB Chromaticity Space

- It is easy to show that projection of $(\alpha_R, \alpha_G, \alpha_B)$ onto $\alpha_R + \alpha_G + \alpha_B = 1$ is:
  - $(\alpha_R/D, \alpha_G/D, \alpha_B/D)$,
    - $D = \alpha_R + \alpha_G + \alpha_B$
- Show that interior and boundary of the curve correspond to visible colours.
- CIE-RGB chromaticity space.

CIE-RGB Chromaticity Diagram



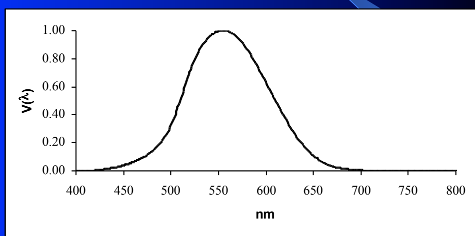
Interpretation of CIE-RGB Chromaticity Diagram

- Suppose α_1 and α_2 are two 3D points corresponding to spectral functions C_1 and C_2.
- Consider the line segment joining them:
  - $(1-t)\,\alpha_1 + t\,\alpha_2$, $t \in [0,1]$
- It is easy to see that for any such t this must correspond to another spectral function.
- When we project the points and line segment to the plane, the line projects to the 2D line joining them.
- All points on the curved boundary and within the curve represent visible colours.

CIE-RGB Chromaticity

- Define:
  - $V(\lambda) = \beta_1 L(\lambda) + \beta_2 M(\lambda) + \beta_3 S(\lambda)$
- For specific constants β_i this is the
  - Spectral Luminous Efficiency curve
- The overall response of visual system to $C(\lambda)$ is
  - $L(C) = K \int C(\lambda)\, V(\lambda) d\lambda$
- For K=680 lumens/watt, and C as radiance, L is called the luminance (candelas per square metre)

Spectral Luminous Efficiency Function



CIE-RGB Chromaticity

- Since
  - $C(\lambda) \approx \alpha_R E_R(\lambda) + \alpha_G E_G(\lambda) + \alpha_B E_B(\lambda)$
- Then
  - $L(C) =$
    $\alpha_R \int E_R(\lambda) V(\lambda) d\lambda$
    $+ \alpha_G \int E_G(\lambda) V(\lambda) d\lambda$
    $+ \alpha_B \int E_B(\lambda) V(\lambda) d\lambda$
- Or
  - $L(C) = \alpha_R l_R + \alpha_G l_G + \alpha_B l_B$

Luminance and Chrominance

- $L(C) = \alpha_R l_R + \alpha_G l_G + \alpha_B l_B$
  - and l_R l_G l_B are constants
- Consider set of all $(\alpha_R, \alpha_G, \alpha_B)$ satisfying this equation…
  - a plane of constant luminance in RGB space
- Only one point on plane corresponds to colour C
  - so what is varying over the plane?
- Chrominance
  - The part of a colour (hue) abstracting away the luminance
- Colour = chrominance + luminance (independent)

Luminance and Chrominance

- Consider plane of constant luminance
  - $\alpha_R l_R + \alpha_G l_G + \alpha_B l_B = L$
- Let $\alpha^* = (\alpha^*_R, \alpha^*_G, \alpha^*_B)$ be a point on this plane.
  - $(t\alpha^*_R, t\alpha^*_G, t\alpha^*_B)$, $t>0$ is a line from 0 through α^*
- Luminance is increasing (tL) but projection on $\alpha_R + \alpha_G + \alpha_B = 1$ is the same.
- Projection on $\alpha_R + \alpha_G + \alpha_B = 1$ is a way of providing 2D coordinate system for chrominance.

(Change of Basis)

- E and F are two different primaries
  - $C(\lambda) \approx \alpha_1 E_1(\lambda) + \alpha_2 E_2(\lambda) + \alpha_3 E_3(\lambda)$
  - $C(\lambda) \approx \beta_1 F_1(\lambda) + \beta_2 F_2(\lambda) + \beta_3 F_3(\lambda)$
- Let **A** be the matrix that expresses F in terms of E
  - $F(\lambda) = \mathbf{A}E(\lambda)$
- Then
  - $\alpha = \beta\mathbf{A}$
  - $\gamma_{Ej}(\lambda) = \sum_i \gamma_{Fi}(\lambda)\, \alpha_{ij}$ (CMFs)

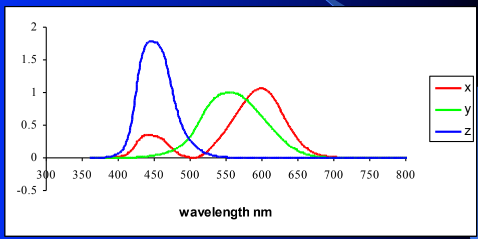
CIE-XYZ Chromaticity Space

- CIE-RGB representation not ideal
  - Colours outside 1st quadrant not achievable
  - Negative CMF function ranges
- CIE derived a different XYZ basis with better mathematical behaviour
  - $X(\lambda)$, $Y(\lambda)$, $Z(\lambda)$ basis functions (imaginary primaries)
  - X, Z have zero luminance
  - CMF for Y is spectral luminous efficiency function V (corresponds to perceived brightness)
- Known matrix A for transformation to CIE-RGB

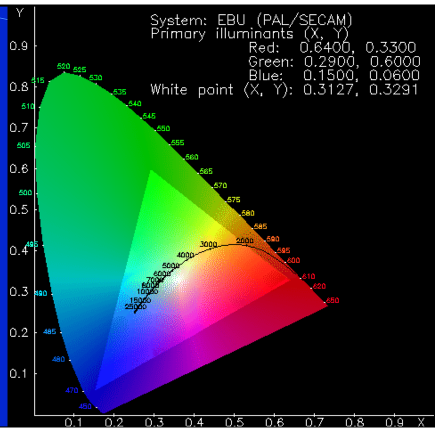
CIE-XYZ Chromaticity Space

- $C(\lambda) \approx X.\, X(\lambda) + Y.\, Y(\lambda) + Z.\, Z(\lambda)$
  - $X = \int C(\lambda) x(\lambda) d\lambda$
  - $Y = \int C(\lambda) y(\lambda) d\lambda$
  - $Z = \int C(\lambda) z(\lambda) d\lambda$
  - x,y,z are the CMFs
  - y is equivalent to V

2-deg XYZ Colour Matching Functions



CIE-XYZ Chromaticity Diagram



Converting Between XYZ and RGB

- System has primaries R(λ), G(λ), B(λ)
- How to convert between a colour expressed in RGB and vice versa?
- Derivation…R(λ), G(λ), B(λ) are physical colours and therefore can be expressed as:

$$R(\lambda) = X_R X(\lambda) + Y_R Y(\lambda) + Z_R Z(\lambda)$$
$$G(\lambda) = X_G X(\lambda) + Y_G Y(\lambda) + Z_G Z(\lambda)$$
$$B(\lambda) = X_B X(\lambda) + Y_B Y(\lambda) + Z_B Z(\lambda)$$

Converting Between XYZ & RGB

- Therefore the chromaticies are:

$$\left(\frac{X_R}{X_R+Y_R+Z_R}, \frac{Y_R}{X_R+Y_R+Z_R}\right) = (x_R, y_R)$$
$$\left(\frac{X_G}{X_G+Y_G+Z_G}, \frac{Y_G}{X_G+Y_G+Z_G}\right) = (x_G, y_G)$$
$$\left(\frac{X_B}{X_B+Y_B+Z_B}, \frac{Y_B}{X_B+Y_B+Z_B}\right) = (x_B, y_B)$$

- The RHS are usually known from manufacturer's data, but the denominators are unknown.

Converting Between RGB & XYZ

- For constants α, we can write:

$$C_R = \alpha_R(x_R, y_R, z_R)$$
$$C_G = \alpha_G(x_G, y_G, z_G)$$
$$C_B = \alpha_B(x_B, y_B, z_B)$$

- If matrix A converts from RGB to XYZ then in particular,
  - $C_R = (1,0,0)$ A
  - $C_G = (0,1,0)$ A
  - $C_B = (0,0,1)$ A

$$A = \begin{bmatrix} \alpha_R x_R & \alpha_R y_R & \alpha_R z_R \\ \alpha_G x_G & \alpha_G y_G & \alpha_G z_G \\ \alpha_B x_B & \alpha_B y_B & \alpha_B z_B \end{bmatrix}$$

Converting Between RGB & XYZ

- To determine the α, consider the white point.
- In XYZ this is (1/3,1/3,1/3) and in RGB is usually (1,1,1).
- So (1/3,1/3,1/3) = (1,1,1)A, and hence:

$$\begin{bmatrix} x_R & x_G & x_B \\ y_R & y_G & y_B \\ z_R & z_G & z_B \end{bmatrix} \begin{bmatrix} \alpha_R \\ \alpha_G \\ \alpha_B \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

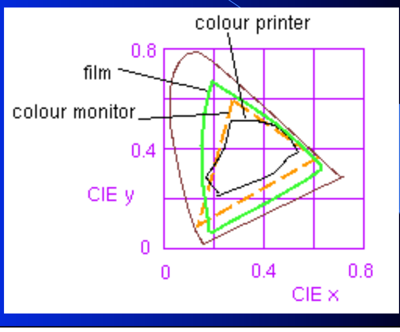
Converting Between RGB & XYZ

- Given an RGB colour on a monitor 1 that we want to reproduce on another one 2, we can use A_1 to go from RGB to XYZ on 1, and then then $(A_2)^{-1}$ to go from XYZ to monitor 2.
- Given a computed XYZ colour (e.g., in a global illumination algorithm) we can use A^{-1} to compute the intensity for a particular monitor.

Colour Gamuts and Undisplayable Colours

- Display has RGB primaries, with corresponding XYZ colours C_R, C_G, C_B
- Chromaticities c_R, c_G, c_B will form triangle on CIE-XYZ diagram
- All points in the triangle are displayable colours
  - forming the colour gamut

Some Colour Gamuts



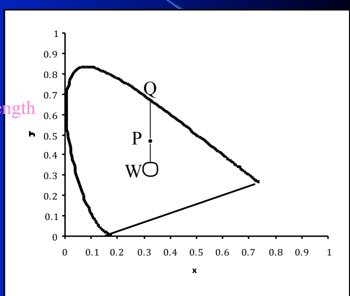
Undisplayable Colours

- Suppose XYZ colour computed, but not displayable?
- Terminology
  - Dominant wavelength
  - Saturation

XYZ with White Point

For colour at P

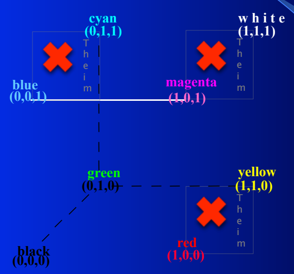
- Q dominant wavelength
- WP/WQ saturation



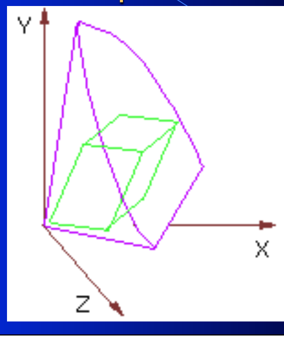
Colour might not be displayable

- Falls outside of the triangle (its chromaticity not displayable on this device)
  - Might desaturate it, move it along line QW until inside gamut (so dominant wavelength invariant)
- Colour with luminance outside of displayable range.
  - Clip vector through the origin to the RGB cube (chrominance invariant)

RGB Colour Cube



RGB Cube Mapped to XYZ Space



Summary for Rendering

- Incorrect to use RGB throughout!!!
  - Different displays will produce different results
  - RGB is not the appropriate measure of light energy (neither radiometric nor photometric).
  - But depends on application
    - Most applications of CG do not require 'correct' colours…
    - …but colours that are appropriate for the application.

For Rendering

- Algorithm should compute C(λ) for surfaces
  - means computing at a sufficient number of wavelengths to estimate C (not 'RGB').
- Transform into XYZ space
  - $X = \int C(\lambda)x(\lambda)d\lambda$
  - $Y = \int C(\lambda)y(\lambda)d\lambda$
  - $Z = \int C(\lambda)z(\lambda)d\lambda$
- Map to RGB space, with clipping and gamma correction.