Computer Animation

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Heavily based on slides by Marco Gillies

Physical simulation

Physical simulation

- Animation is about things moving
- The motion of “things” is governed by the laws of physics
- These laws of physics are very well understood and normally have computationally tractable mathematical formulae
- So to get things to move lets use physics

Physical simulation: books

- One book that is OK:
- “Physics for Game Developers” David M. Bourg – O’Reilly
- A definitive guide is:
- http://www2.cs.cmu.edu/~baraff/pbm/pbm.html

Newton’s third law

- The basis of physical simulations is Newton’s third law

$$f = ma$$

Newton’s third law

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$$f = ma$$

$$a = \frac{dv}{dt}$$

$$v = \frac{dp}{dt}$$

Newton's third law

- ! The basis of physical simulations is Newton's third law (vectors)

$$f = ma$$

$$a = \frac{dv}{dt}$$

$$v = \frac{dp}{dt}$$

Newton's third law

- ! Work out the forces acting on an object
  - –!Gravity
  - –!Impulse forces (collisions)
  - –!Forces from springs, friction, other objects
- ! Use Newton's law to work out the position of an object at each time step
- ! When we deal with forces and accelerations it is called "Dynamics"
- ! Up to now we've just used velocities and positions "Kinematics"

Integrating Newton's laws

- ! The correct formulas

$$a = \frac{f}{m}$$

$$v = \int_{t_0}^{t} a\,dt$$

$$p = \int_{t_0}^{t} v\,dt$$

Integrating Newton's laws

- ! The obvious approximation

$$a_{t'} = \frac{f_{t'}}{m}$$

$$v_{t'} = v_t + (t - t')a_{t'}$$

$$p_{t'} = p_t + (t - t')v_{t'}$$

A projectile

- ! As an example choose a ball launched from the ground under gravity
- ! Time step t=0.1
- ! Has initial conditions

$$f = [0, -9.81m, 0]$$

$$v_0 = [10, 10, 0]$$

$$p_0 = [0, 0, 0]$$

A Projectile

- ! a=-9.81

A Projectile

•! a=-9.81 v=[10,10-0.981,0]

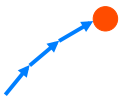
A Projectile

•! a=-9.81 v=[10,9.019,0] p=[1,0.9019, 0]

A Projectile

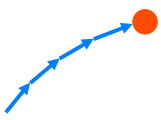
•! a=-9.81 v=[10,8.038,0] p=[2,1.7057, 0]

A Projectile



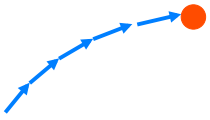
•! a=-9.81 v=[10,7.057,0] p=[3,2.4114, 0]

A Projectile



•! a=-9.81 v=[10,6.076,0] p=[4,3.019, 0]

A Projectile



•! a=-9.81 v=[10,5.095,0] p=[5,3.528, 0]

Is the method valid?

- Is this a reasonable method?

Is the method valid?

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- Taylor expansion of a function

$$y(t+\Delta t) = y(t) + \Delta t \frac{dy(t)}{dt} + \frac{(\Delta t)^2}{2!}\frac{d^2y(t)}{dt^2}\cdots$$

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- If Δt is small we can ignore higher terms

Is the method valid?

- Is this a reasonable method?
- Taylor expansion of a function

$$y(t+\Delta t) = y(t) + \Delta t \frac{dy(t)}{dt}$$

- If Δt is small we can ignore higher terms
- This is the same as our method
- Called Euler's method

Step size

- Δt is called the step size
- It is very important in this sort of calculation
- If it is too large then the solution will be inaccurate
- If it is too small then you will need a lot of processing power

Step size

- What is worse is that errors can build up over time
- So for long(ish) animations you need a very small step size
- Can get very expensive
- Can make Euler's method unusable

How can we improve it?

- Dynamically change the step size as we need it
- Use a different method

Adaptive step sizes

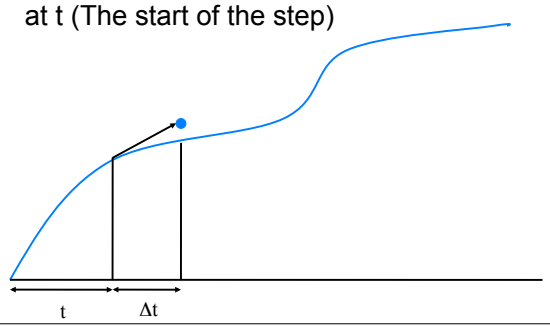
- Only use as small a step size as you need
- Every so often test the different between using step size Δt and $\Delta t /2$
- If the difference is big enough start using $\Delta t /2$
- Do the same for $\Delta t *2$

Other integration methods

- This is a vast field
- There are huge numbers of different methods you can use
- Using 2nd and higher derivatives
- Evaluating the different derivatives for different values of t
- I will describe one

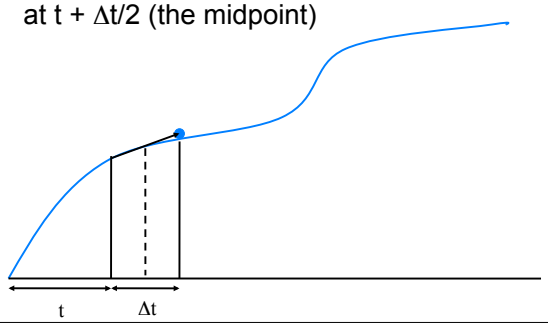
Midpoint method

- Euler's method evaluates the derivative at t (The start of the step)



Midpoint method

- Can get more accuracy by evaluating it at t + $\Delta t/2$ (the midpoint)



Midpoint method

- The formula becomes:

$$p(t+\Delta t) = p(t) + \Delta t \frac{dp}{dt}(t+\Delta t/2)$$

Midpoint method

- ! The formula becomes:

$$p(t+\Delta t) = p(t) + \Delta t \frac{dp}{dt}(t+\Delta t/2)$$

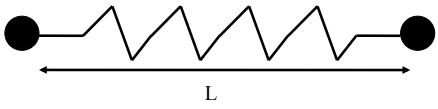
- ! This is one example from a many types of improvement
- ! Far more to go into but beyond the scope of the course

Summary

- ! A physics simulation consists of
  - –!A set of particles
  - –!A set of forces on those particles
  - –!Initial position of those particles
- ! The alogrithm:
  - –!Use the forces to update the acceleration
  - –!Integrate to update velocities and positions
- ! We need appropriate forces

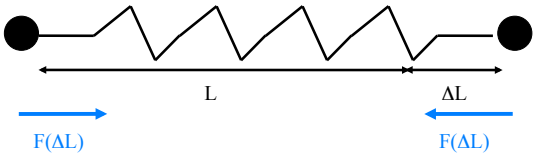
Springs

- ! Springs are a common source of forces
- ! The forces try to maintain a desired length
- ! Join two particles by a spring



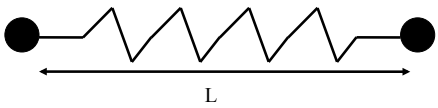
Springs

- ! Stretching the spring creates forces on both particles to restore the desired length
- ! These forces are proportional to the change in length



Springs

- ! Ditto for squashing



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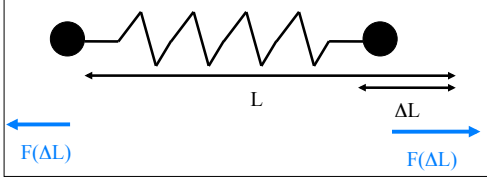


Springs

- The Formula for spring forces on the particles are given by Hooke's Law
- Have a force pointing along the length of the spring that depends on the change of length:

$$F = -k_s \Delta L$$

- Where k_s is a spring constant, which controls how springy or rigid it is

Springs

- The problem with this formula is that the spring can bounce about too much and never come to rest
- We need to add a damping force
- Damping forces reduce the motion of an object (like friction)
- They are proportional to the velocity of an object (but opposite)

Springs

- Introduce a damping force
- Proportional to the relative velocity of the two particles (and negative)

$$F = -\left[k_s \Delta L - k_d \left(v_2 - v_1\right)\right]$$

- Where k_d is a damping constant, which controls how damped it is

Using springs

- Springs are a very useful tool because they can be used to build models of interesting physical systems

Using springs

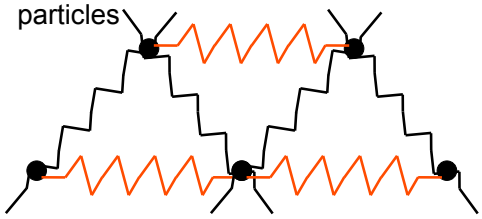
- A string of springs can simulate hair, grass etc.

Hair

- Unfortunately there is nothing to stop the hair bending
- Need to add in extra forces to prevent bending
- Many approaches but you can just add extra springs

Hair

- Putting springs between every other particle
- Counteracts bending between 3 particles



Hair

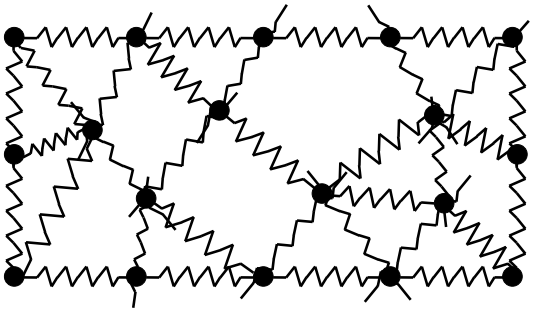
- Animating each hair would be hugely expensive
- Generally methods simulate a smallish number of strands and render each one as a clump of hair rather than an individual hair

Hair

- Many other methods
- Often use more continuous representation of hair

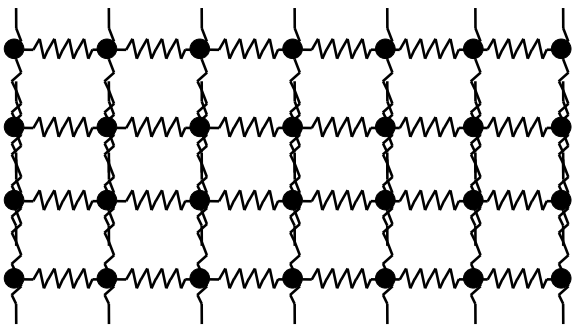
Using springs

- A lattice can simulate cloth



Using springs

- A lattice can simulate cloth

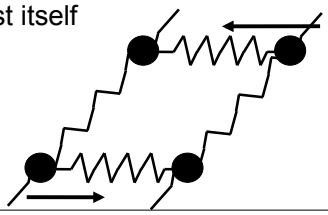


Cloth

- This simple lattice allows you to simulate the in plane stretching properties of cloth well, but there are other forces that go on in cloth
- Bending can be handled in the same way as for hair

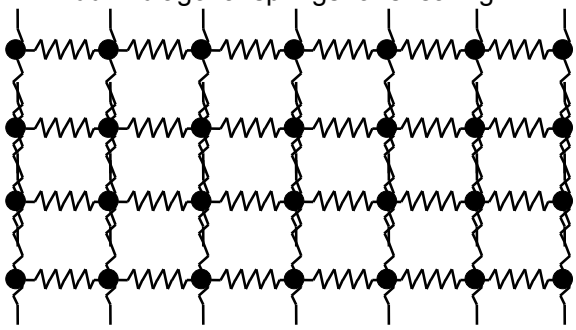
Cloth

- ! Shearing
- ! This becomes an issue going from 1D hair to 2D cloth
- ! A force that prevents the cloth pulling against itself



Cloth

- ! Add in diagonal springs for shearing



Cloth

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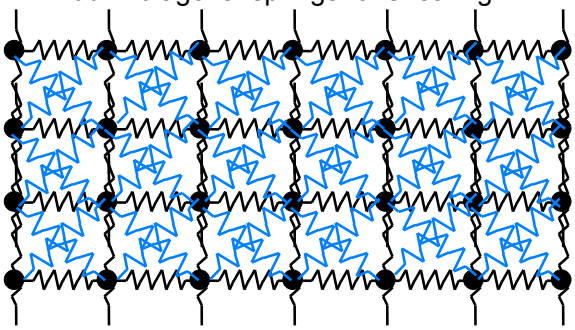
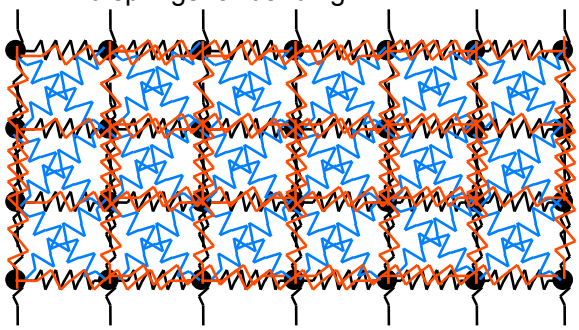


Cloth

- ! And springs for bending



Cloth

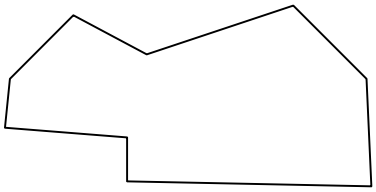
- ! Demo

Cloth

- ! Springs aren't all there is to physics, hair and clothing
- ! There are lots of other approaches, more continuous methods, more realistic force functions etc

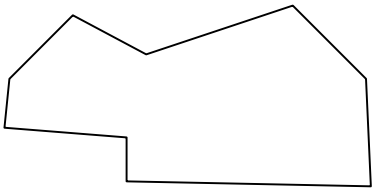
Rigid bodies

- Up to now we have dealt with forces on particles (points)
- What about more complex object?
- A rigid body:



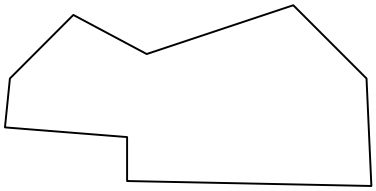
Rigid bodies

- Can move around
- But cannot deform



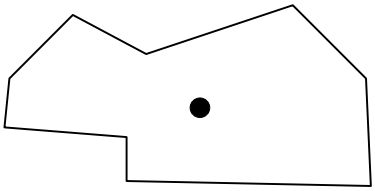
Rigid bodies

- Could make it out of springs
- But as you don't want it to deform the spring constant must be very high
- Very expensive to integrate



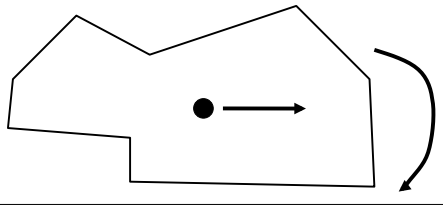
Rigid bodies

- Has a centre of mass that can be treated as a particle



Rigid bodies

- The movement of a rigid body can be decomposed into 2:
- Motion of the centre of mass
- Rotation about the centre of mass



Rigid bodies

- The motion of the centre of mass is treated the same as the particle dynamics discussed above
- The rotation adds a new set of components above the x,y,z position components
- Won't go into much detail about the maths here

Natural Phenomena

- Simulation is also very useful for modelling natural phenomena
  - Wind
  - Clouds
  - Water
  - Fire

Natural Phenomena

- All of these are physical systems and so can be simulated based on their physics
- Generally the models needed are much more complicated than the ones I've described
- However, there is one less physically based method that is often used for this sort of system

Particle Systems

- A more ad-hoc simulation method
- Good for phenomena that are not well represented by solid, rigid surfaces/ bodies
- Used for many different phenomena
  - Fire
  - Clouds
  - Explosions
  - Grass and plants

Particle Systems

- These systems can all be modelled as large sets of particles
- Not (necessarily) physically modelled in the way described above
- The structure of all particle systems are the same
- The details of particle behaviour can differ

Particle Systems – each frame

- Particles are created
- Attributes assigned to each particle
- Old particles are deleted
- Particles are moved and transformed
  - Can vary between different types of system
- Particles are rendered

Particle Creation

- Particles are randomly created
- They are all created within a certain area
- The mean number of particles created controls the size of the system
- This mean can vary over time to have the system grow or shrink

Particle Attributes

- Each new particle is randomly assigned a set of attributes
- Again mean and variance control the overall attributes of the system
- The attributes used depend on the type of system
- Examples: Velocity, acceleration, mass, colour

Particle deletion

- Particles are also deleted
- Particles have a certain lifetime, if the particle has exceeded its lifetime it is destroyed
- Also sometimes particles are deleted if they leave a certain area (visible area)

Particle motion and transformation

- This step controls what the particles actually do
- Varies depending on what you are simulating (e.g. clouds vs fire)
- Examples:
  - Particles move according to their velocity
  - Particles change colour (e.g. get brighter/ darker)

Particle rendering

- Particles can be treated as light sources
  - Don't have to worry about occulsion
- Or particles are small primitives, often partially transparent
- Can apply motion blur or other types of blurring to make individual particles less visible

Example: Smoke generation

- Particle Creation:
  - Particles are created at the source of the smoke (often a point)
  - Particles mostly go upwards with some sideways variation

Example: Smoke generation

- Particle Attributes:
  - Velocity: Upwards, slightly to the side
  - Appearance, grey and transparent

Example: Smoke generation

- Particle Deletion:
  - Delete at end of lifespan
  - Limited lifetime allows for smoke dissipation
  - Delete if they go out of the view

Example: Smoke generation

- Particle Motion and transformation:
  - Particles move according to their velocity
  - It is also useful to have a wind force
    - Accelerates the particles in the direction of the wind
  - Turbulence makes the particles more realistic
    - Random variation in wind force
  - Particles initially increase in size

Example: Smoke generation

- Particle Rendering:
  - Particles are rendered as small polygons
  - The polygons are blurred at the edges so that its less obvious that they are polygons
  - They are transparent

Example: Smoke generation