Introduction to Matlab
Course notes

Mark Herbster and Jason Kastanis

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Part I
Interface Guide

1 Overview

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2 Style of the guide

This guide has been designed to offer a short introduction to programming and the Matlab environment. The main functionality of the graphical user interface is described using example images. These images were produced on a PC running Windows and Matlab version 6.5. They might differ slightly from the version of Matlab that you are running.

Bold is used for all the icons, tools, menu items and other parts of the Matlab interface. The *italic* font is used for the introduction of basic elements of programming. Elements, such as commands, that belong in the Matlab programming language were written using the `verbatim` font.

3 A brief history of Matlab

MATrix LABoratory was originally developed by Cleve Moler in the 1970's, then chairman of the computer science department of the University of New Mexico. It was an interface for the LINPACK and EISPACK libraries, which were written in FORTRAN with the participation of Moler. Matlab was originally intended for a linear algebra course. Its aim was to simplify the use of these subroutine libraries by avoiding the complexities of FORTRAN. Matlab began gaining popularity within the applied mathematics community. The early versions were based on the command prompt and had no graphical interface. In 1983 Moler, Jack Little and Steve Bangert rewrote Matlab in C and the following year founded Mathworks to market it further. From version 6 Matlab was based on the LAPACK library which has superseded both LINPACK and EISPACK.

4 Basic elements of programming

A program is a collection of instructions for the computer to execute. It is essentially an algorithm, in that sense it has to be deterministic. Each instruction should be unambiguous. A programming language just like any language has a set of rules. These rules describe the syntax and semantics of the language. The basic elements of a programming language are described below.

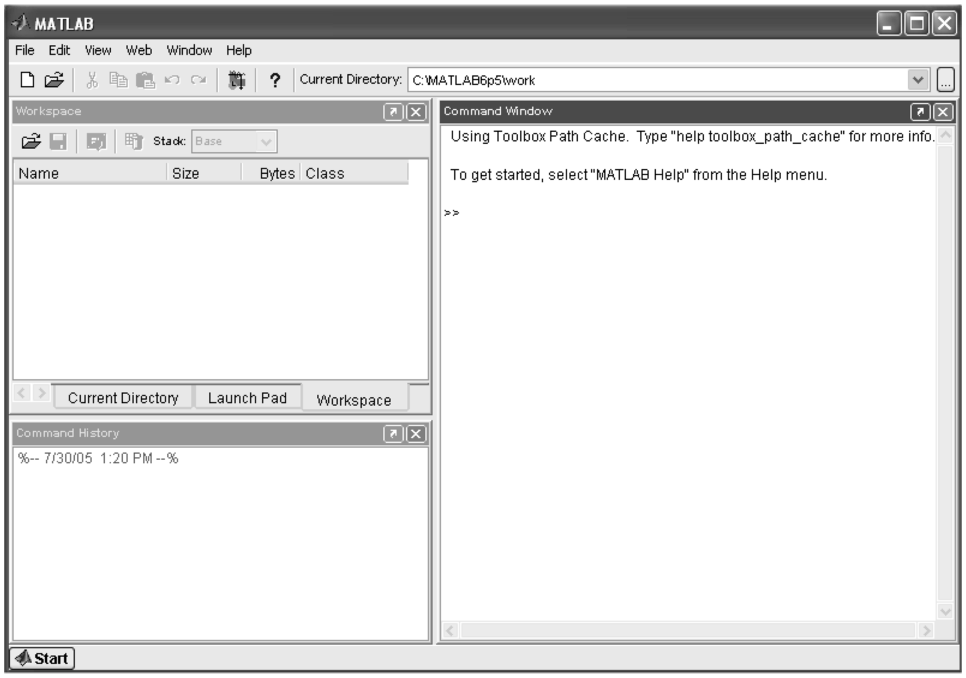
Variables are places to store values on the computer memory. The name of the variable serves as the address in the memory, where the value of this variable is held. e.g. x=5, x is the variable, it represents a particular location in the memory, where the value 5 is stored. Variables can have *types*. Types describe the kind of values a variable can accept and they are divided in primitive and composite. Primitive types are the ones provided by the programming language and they typically contain integers, floating-point numbers and characters. Composite types are made from the combination of primitive types with other primitive types or other composite types. For example an integer variable is of primitive type and it can only store integer values. In some languages the declaration of the type and dimensions of a variable is not required. Variables can be local or global, local ones are only accessible in a particular part of the program, while global ones can be accessed anywhere.

An *array* is the most basic data structure. It is a list of elements of the same type. Individual elements of an array can be accessed using a consecutive range of integers. This is referred to as the index. The index denotes the position of an element in the list. One dimensional arrays are called vectors and two dimensional are called matrices. *Assignment* is the task of storing a value in a variable. It is commonly done using the equal sign (=). Keep in mind that the equal sign in Mathematics stands for equality, in programming it stands for assignment.

Expressions compute new values from old ones. The expression 5 + 3 will calculate the sum of the values 5 and 3. In the previous expression the plus sign is an *operator*, which operates on the values 5 and 3. *Statements*, or instructions, describe what the program will do. They can contain assignments, expressions and control flow operations. *Control flow* operations are divided in two main categories, *conditionals* and *loops*. Conditionals, as the name suggests, condition the flow of the program, if a variable has a particular value (or belongs in a particular range of values), then a particular set of statements will be executed, if not another set will. Loops are used for repetition, their construction contains a rule determining how many times a set of statements will be repeated. An entire set of statements grouped together is called a *function*. A function takes variables, in this case called arguments, and can return values. Calling a function transfers the control over to the function, the group of statements which form it will be executed. When this finishes it returns (with or without a result) to the original flow of the program. Having specified these basic elements of programming, a program can be redefined as an ordered collection of statements, functions and variables.

5 Matlab main desktop

The Matlab **main desktop**, presented in fig. 1, has many areas and windows. These are discussed in the following sections.



```
Using Toolbox Path Cache.  Type "help toolbox_path_cache" for more info.

To get started, select "MATLAB Help" from the Help menu.

>>
```

```
%-- 7/30/05  1:20 PM --%
```

Figure 1: The default Matlab main desktop

5.1 Title bar

The **title bar** (fig. 2) contains the program's name and logo as well as the window control buttons. It is situated on the top of the main Matlab window.

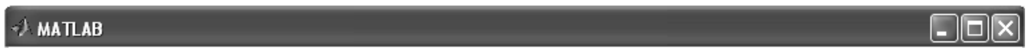


Figure 2: The title bar

5.2 Menu bar

The **menu bar** (fig. 3) is underneath the **title bar**. It has commands for **opening**, **closing files**, **preferences**, etc. Many of these commands have keyboard shortcuts, but these vary between different operating systems. The shortcut keys are commonly displayed next to the command, for example look at fig. 4

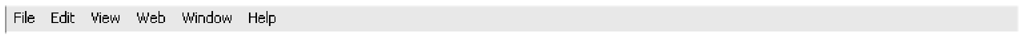


Figure 3: The menu bar

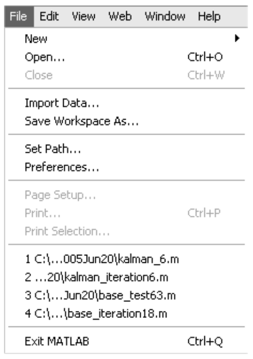


Figure 4: Example of a menu item

5.3 Desktop toolbar

The **desktop toolbar** (fig. 5) is placed underneath the **menu bar**. It contains many items found on the menus, **new file**, **open**, **copy**, **paste**, ... To find out what each icon does, leave the mouse above it for a few seconds and a small box with a tool tip will appear, e.g. fig. 6.

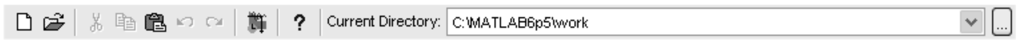


Figure 5: The desktop toolbar

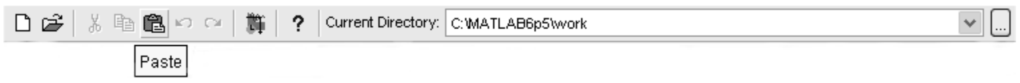
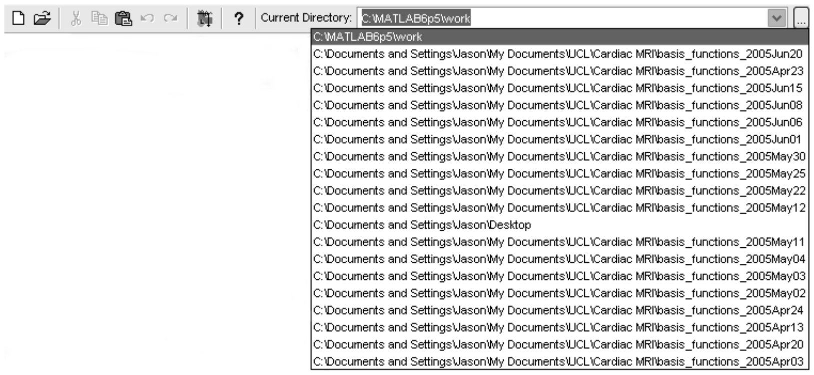


Figure 6: A tooltip

On the left side of the **desktop toolbar** there is box, which can be edited, called the **current directory**. This defines the location, where Matlab is working. It is the folder at which the user is looking at. If Matlab (or the user) can't find a particular file, it means that the file is not in the **current directory**. Commonly used or referenced directories can be setup from the menu bar using **File** → **Set Path**... . The small arrow on the right of the box will show past current directories (fig. 7). Next to that the **browse tool**, the icon with the three dots, is used for finding a folder.

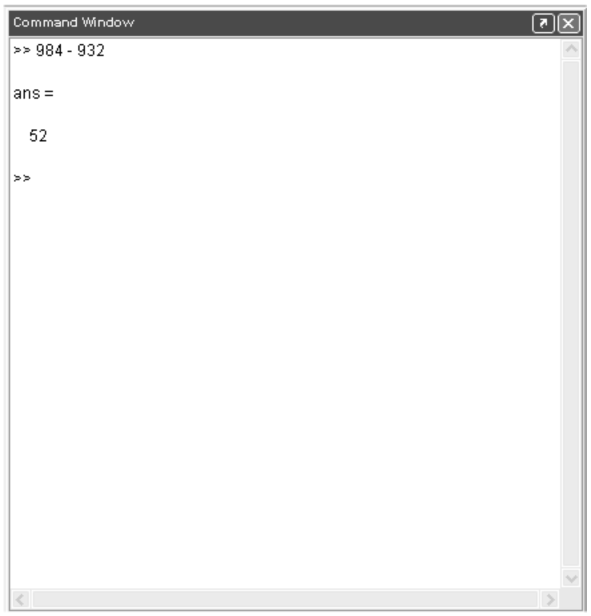
5.4 Command window

The **command window** (fig. 8) is the most important part of the Matlab **main desktop**. It is the window where input and output appears. In this window the user can enter commands and obtain results. Each new line on the **command prompt** starts with the symbol `>>`. This defines where new input can be entered. Input can be apart from Matlab commands, various DOS or Linux prompt type of commands, e.g `dir`, `ls`.



```
C:\MATLAB6p5\work
C:\MATLAB6p5\work
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Jun20
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Apr23
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Jun15
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Jun08
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Jun06
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Jun01
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005May30
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005May25
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005May22
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005May12
C:\Documents and Settings\Jason\Desktop
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005May11
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005May04
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005May03
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005May02
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Apr24
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Apr13
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Apr20
C:\Documents and Settings\Jason\My Documents\UCL\Cardiac MRI\basis_functions_2005Apr03
```

Figure 7: Past directories



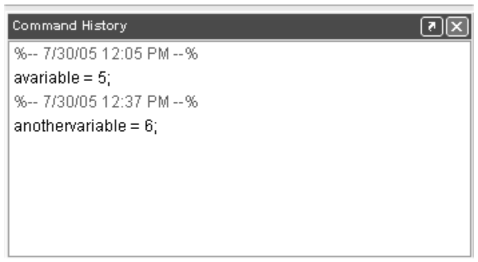
```
>> 984 - 932

ans =

  52

>>
```

Figure 8: Command window

By pressing the up arrow on the keyboard the user can scroll through all the previously entered commands. To scroll back the down arrow can be used. If a letter (or more) is typed, then the up and down arrows can be used to scroll through all the commands that have been previously typed and begin with these letters. The tab button can be used to complete the name of commands or functions. If a command does not exist or if more than one commands with the same starting letters exist, then pressing the tab will make a sound.

5.5 Command history

The **command history** window (fig. 9) contains the history of the commands entered in the **command window**. It begins on each new session with the starting date and time. Thus each session history is separated by dates. Commands from the history window can be copied and pasted, dragged and dropped.



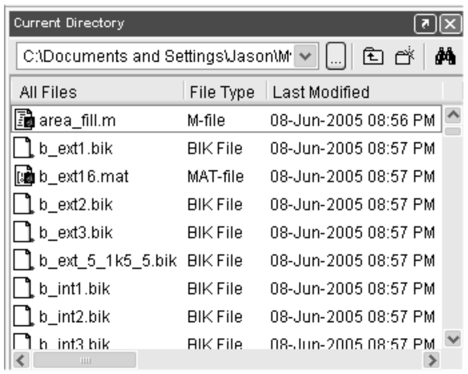
```
%-- 7/30/05 12:05 PM --%
avariable = 5;
%-- 7/30/05 12:37 PM --%
anothervariable = 6;
```

Figure 9: Command history window

5.6 Current directory (window)

The **current directory** is also visible as a window inside the Matlab **main desktop** (fig. 10). On the top of this window there is box, which contains the location of the **current directory**, same as the one in the **desktop toolbar**. File names appear on the left column, file types on the middle and last date of modification on the right column.

On the same part of the Matlab **main desktop**, the next two windows (**launch pad**, **workspace**) appear as tabs. All of these windows are separable from the **main desktop** window.



Current Directory

C:\Documents and Settings\Jason\M

All Files	File Type	Last Modified
area_fill.m	M-file	08-Jun-2005 08:56 PM
b_ext1.bik	BIK File	08-Jun-2005 08:57 PM
b_ext16.mat	MAT-file	08-Jun-2005 08:57 PM
b_ext2.bik	BIK File	08-Jun-2005 08:57 PM
b_ext3.bik	BIK File	08-Jun-2005 08:57 PM
b_ext_5_1k5_5.bik	BIK File	08-Jun-2005 08:57 PM
b_int1.bik	BIK File	08-Jun-2005 08:57 PM
b_int2.bik	BIK File	08-Jun-2005 08:57 PM
b_int3.bik	BIK File	08-Jun-2005 08:57 PM

Figure 10: Current directory window

5.7 Launch pad

The **launch pad** (fig. 11) is a way of accessing various Matlab resources, such as the **import wizard**, the **profiler**, the **Graphical User Interface (GUI) builder**, etc.

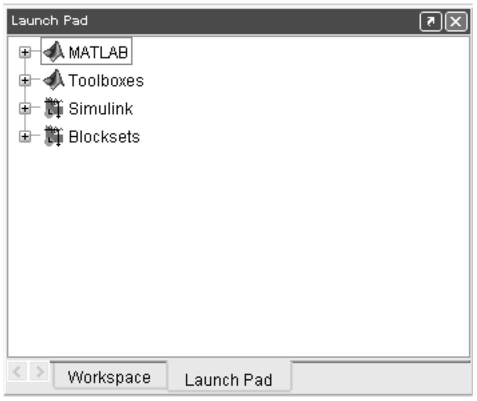


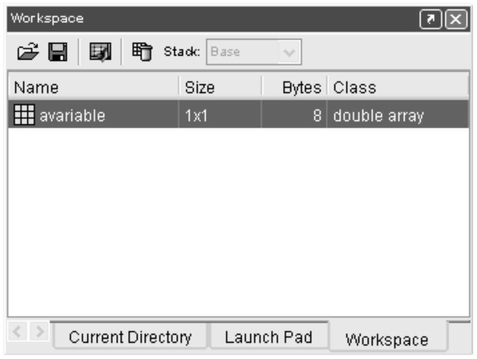
Figure 11: Launch pad

These windows appear as separate windows from the Matlab **main desktop**, some of which can be docked, that is they can be a part of the **main deskto**p, and some which cannot be docked, they are called undockable windows. The **launch pad** can also be used to launch the help and demo files of the toolboxes. As it can be seen on fig. 11 it lives on the same window as the

current directory. Those two can be switched using the tabs on the lowest part of the window, by pressing the one with the corresponding name.

5.8 Workspace

Another window that cohabits the same space as the **launch pad** and the **current directory** is the **workspace window** (fig. 12). This window displays the loaded variables of the current Matlab session, these are variables you have created and are currently loaded on the memory. It displays their name, their size, that is their dimensions, the number of bytes they take on memory and their class, that is the type of variable.



Stack: Base

Name	Size	Bytes	Class
avariable	1x1	8	double array

Figure 12: Workspace

On the top part of the **workspace window** there is toolbar with tools associated to the variables. For example the **open** icon will launch a separate window, the **array editor** (fig. 13), for viewing and editing the contents of a variable.

6 Opening and editing files

6.1 Opening files

There are a few ways the user can open a file. From the menu bar, **File** → **Open**, from the **desktop toolbar** by clicking on the **open file** icon, by typing the name of the file on the **command prompt**, selecting it, right-clicking and choosing **open** selection. Matlab can deal with a variety of file

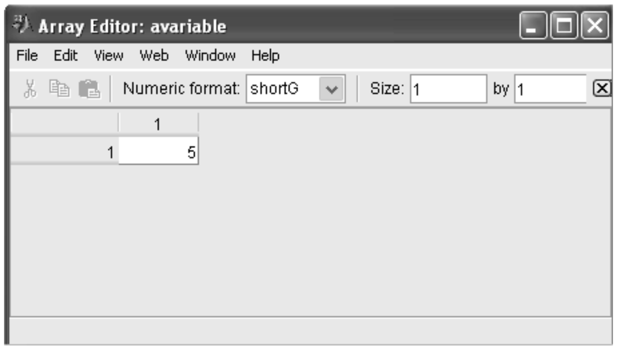


Figure 13: Array editor

formats, but only a few will be mentioned here. Workspace files store loaded variables in to a .mat file, figure files (.fig) are graphic files, and M-files contain code and finish with .m. Variables contained in a mat file will be loaded on the workspace as soon as the mat file is opened. Figure files will open on a separate window (fig. 14), this offers certain tools for editing and manipulating the figure.

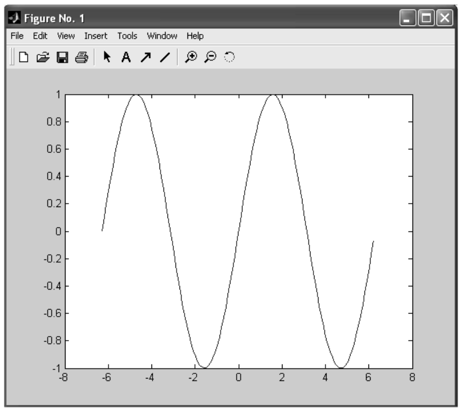
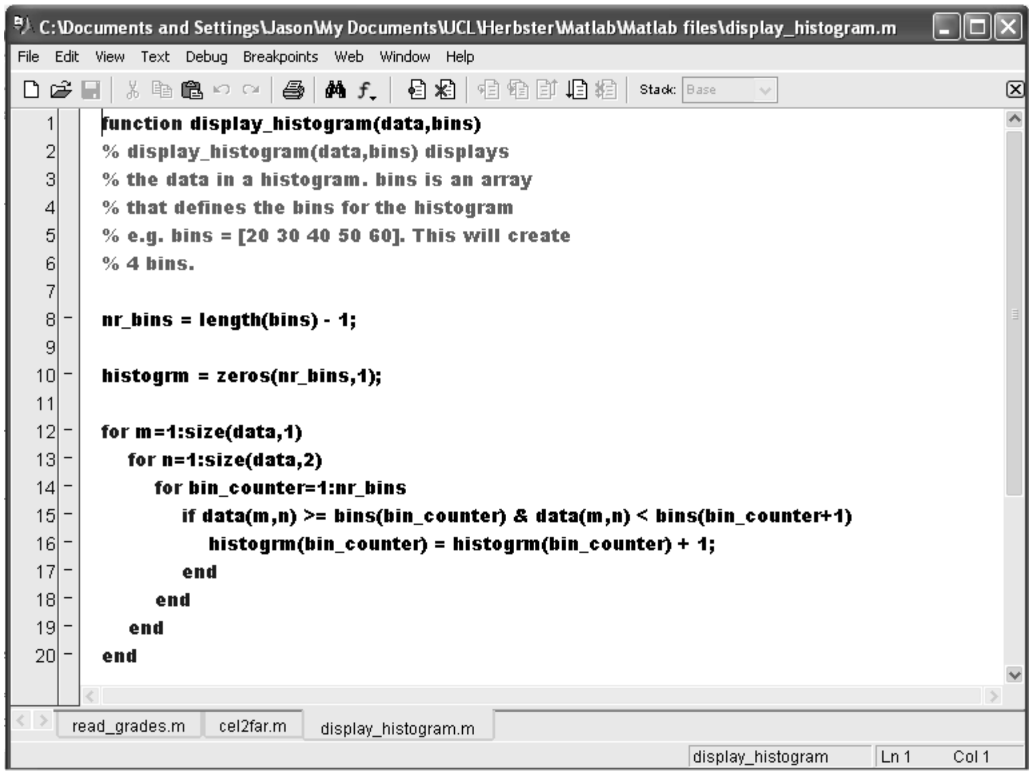


Figure 14: Figure window

6.2 Editing files

As soon as an M-file is opened the **editor window** fig. 15 will appear as a separate window.


C:\Documents and Settings\Jason\My Documents\UCL\Herbster\Matlab\Matlab files\display_histogram.m
File Edit View Text Debug Breakpoints Web Window Help
Stack: Base
read_grades.m
cel2far.m
display_histogram.m
display_histogram
Ln 1
Col 1

```
function display_histogram(data,bins)
% display_histogram(data,bins) displays
% the data in a histogram. bins is an array
% that defines the bins for the histogram
% e.g. bins = [20 30 40 50 60]. This will create
% 4 bins.

nr_bins = length(bins) - 1;

histogrm = zeros(nr_bins,1);

for m=1:size(data,1)
    for n=1:size(data,2)
        for bin_counter=1:nr_bins
            if data(m,n) >= bins(bin_counter) & data(m,n) < bins(bin_counter+1)
                histogrm(bin_counter) = histogrm(bin_counter) + 1;
            end
        end
    end
end
```

Figure 15: Editor window

This window can be docked in to the Matlab **main desktop**. Every file that is opened will appear in the same window. Each file can be in its own editor window, in most cases it is practical to keep them in one. Files are chosen by the tabs on the lower part of the **editor window** (fig. 16).

Figure 16: Editor tabs

Of course one could use any editor, but Matlab's editor offers color coding, running and debugging facilities. Similar to the **command prompt**, the user can select the name of a function file, right-click on it and open it. On the bottom right of the **editor window** (fig. 17), information about the file is displayed as well as the line and the column where the cursor is placed, this can be very useful when debugging. On the left side of the editing area the lines are numbered.

Figure 17: Information on the lower part of the editor

A toolbar (fig. 18) is displayed on the top of the **editor window**. The standard buttons, **New file**, **Open file**, **Save**, **Copy**, **Cut**, **Paste**, **Print** are placed here. Next to them the **binocular icon** represents the find tool. This is used for searching the file for a particular keyword and replacing it if required. Further to the right are the **debugging tools** for setting and clearing breakpoints. The **Run** button is also situated on the right of the debugging tools. This will execute the code contained in the active file.

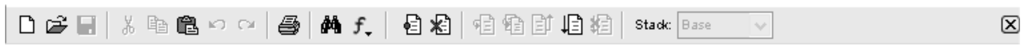


Figure 18: The editor toolbar

Files can also be executed using the **menu bar** (fig. 19) on the upper part of the **editor window** in **Debug** → **Run**. The **menu bar** contains all of the toolbar commands and many others.

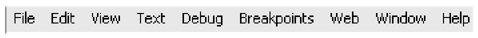


Figure 19: The editor menu bar

The name of the file is displayed on the top of the **editor window** on the title bar (fig. 20).

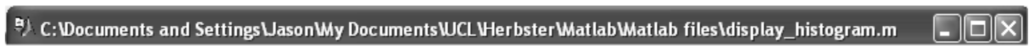

C:\Documents and Settings\Jason\My Documents\UCL\Herbster\Matlab\Matlab files\display_histogram.m

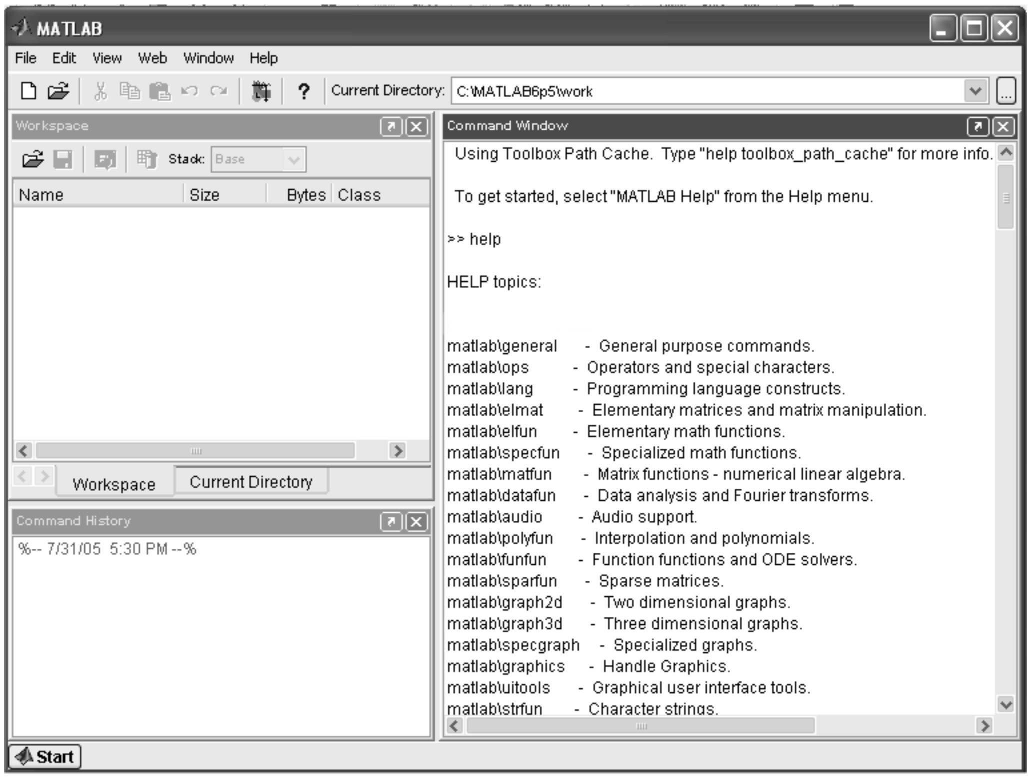
Figure 20: The editor title bar

7 Getting help

It is an essential part of programming to be able to find out information about syntax and functionality as well as to see working examples. There are three ways the user can get help in Matlab, **text based help**, **graphical help interface** and **web based help**.

7.1 Text based help

Text based help can be obtained from the **command prompt** by typing `help`. The help topics then appear as in fig. 21.



```
Using Toolbox Path Cache.  Type "help toolbox_path_cache" for more info.

To get started, select "MATLAB Help" from the Help menu.

>> help

HELP topics:


matlab\general     -  General purpose commands.
matlab\ops        -  Operators and special characters.
matlab\lang       -  Programming language constructs.
matlab\elmat      -  Elementary matrices and matrix manipulation.
matlab\elfun      -  Elementary math functions.
matlab\specfun    -  Specialized math functions.
matlab\matfun     -  Matrix functions - numerical linear algebra.
matlab\datafun    -  Data analysis and Fourier transforms.
matlab\audio      -  Audio support.
matlab\polyfun    -  Interpolation and polynomials.
matlab\funfun     -  Function functions and ODE solvers.
matlab\sparfun    -  Sparse matrices.
matlab\graph2d    -  Two dimensional graphs.
matlab\graph3d    -  Three dimensional graphs.
matlab\specgraph  -  Specialized graphs.
matlab\graphics   -  Handle Graphics.
matlab\uitools    -  Graphical user interface tools.
matlab\strfun     -  Character strings.
```

Figure 21: Text based help topics

To see the subtopics of one of the topics of fig. 21 type the name of the topic in the **command prompt**. For example:

```
>> help matlab\general
```

The command `help` can also be used to find out information about a specific function. For example:

```
>> help sin

 SIN    Sine.
    SIN(X) is the sine of the elements of X.

 Overloaded methods
    help sym/sin.m
```

In the case the function is unknown or the user wants to search for a specific keyword, the command `lookfor` can be used.

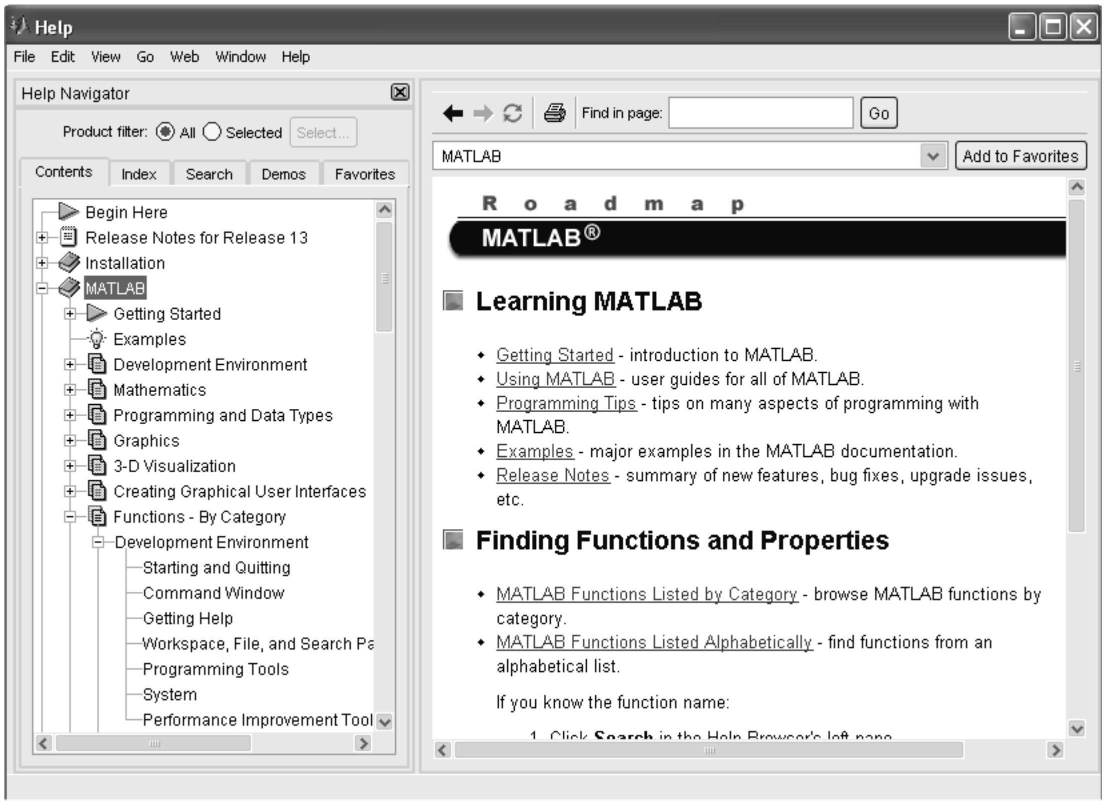
```
>> lookfor infinity
INF Infinity.
CEIL   Round towards plus infinity.
FLOOR  Round towards minus infinity.
CHOLINC  Sparse Incomplete Cholesky and Cholesky-Infinity factorizations.
ACTDEMO Demo of digital H-infinity hydraulic actuator design.
DHINF Discrete H-Infinity control synthesis (bilinear transform version).
DHINFOPT Discrete H-Infinity control synthesis via Gamma iteration.
DINTDEMO Demo of H-Infinity design of double integrator plant.
...
```

The command `lookfor` will search in all help entries. To find out the details in one of the search results `help` can be used as previously:

```
>> help INF

 INF Infinity.
    INF returns the IEEE arithmetic representation for positive
    infinity.  Infinity is also produced by operations like dividing by
    zero, eg. 1.0/0.0, or from overflow, eg. exp(1000).

    See also NaN, ISFINITE, ISINF.
```

7.2 Graphical interface help

A more extensive and user friendly option to get help is the **graphical interface help** window (fig. 22). This is a separate window and it is launched from the **menu bar**, **Help** → **Matlab help**, the **main desktop toolbar** by pressing the **question mark** or by typing `helpbrowser` in the **command prompt**.



Learning MATLAB

- Getting Started - introduction to MATLAB.
- Using MATLAB - user guides for all of MATLAB.
- Programming Tips - tips on many aspects of programming with MATLAB.
- Examples - major examples in the MATLAB documentation.
- Release Notes - summary of new features, bug fixes, upgrade issues, etc.

Finding Functions and Properties

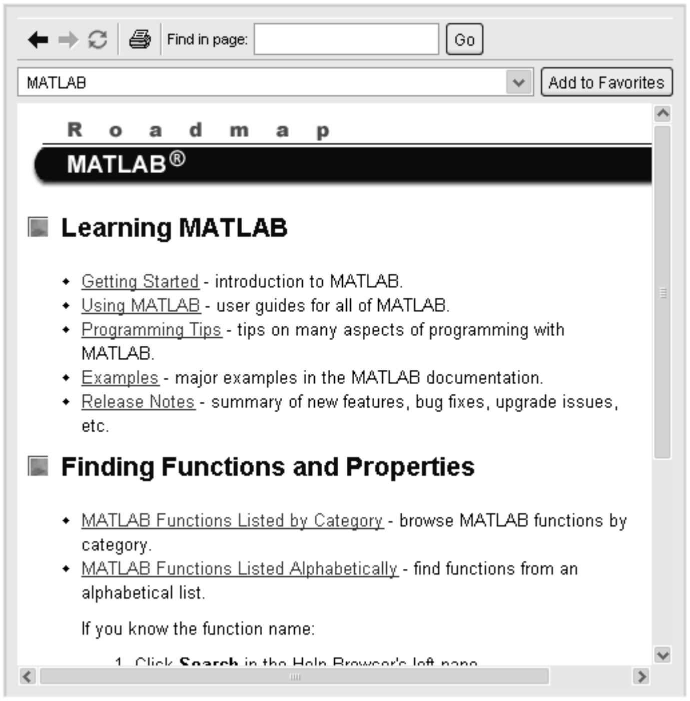
- MATLAB Functions Listed by Category - browse MATLAB functions by category.
- MATLAB Functions Listed Alphabetically - find functions from an alphabetical list.

If you know the function name:

Figure 22: Help browser

The **help browser** has a menu bar on the top and it is divided into two main areas. The area on the left is **query area** and the right one is where information appears.

On the top of the **information area** (fig. 23) there is small toolbar with **back**, **forward** and **reload** buttons, similar to a web browser, a **print** button and a **Find in page** button for searching a keyword in the current page.



R o a d m a p

MATLAB®

Learning MATLAB

- Getting Started - introduction to MATLAB.
- Using MATLAB - user guides for all of MATLAB.
- Programming Tips - tips on many aspects of programming with MATLAB.
- Examples - major examples in the MATLAB documentation.
- Release Notes - summary of new features, bug fixes, upgrade issues, etc.

Finding Functions and Properties

- MATLAB Functions Listed by Category - browse MATLAB functions by category.
- MATLAB Functions Listed Alphabetically - find functions from an alphabetical list.

If you know the function name:

1. Click **Search** in the Help Browser's left pane

Figure 23: Information area

The **query area** on the left, also called the **help navigator** (fig. 24) has five tabs, **Contents**, **Index**, **Search**, **Demos** and **Favorites**.

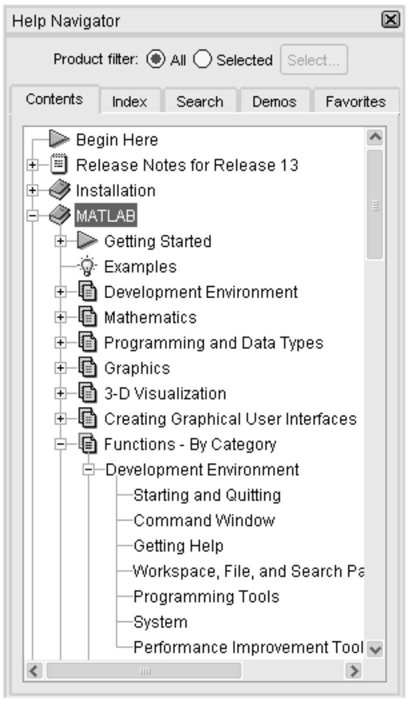
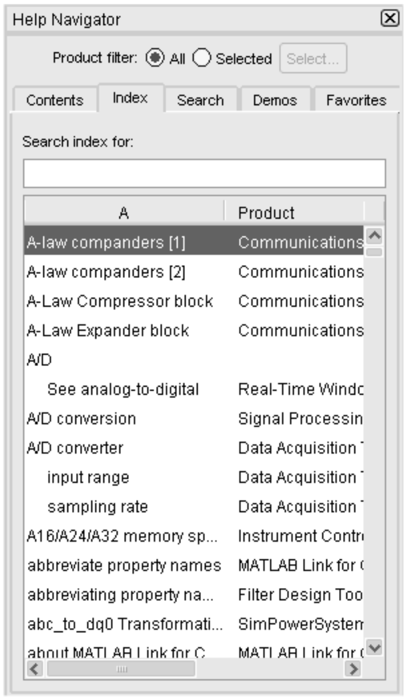


Figure 24: Help navigator

The **Contents** tab, shown in fig. 24, has a tree list of all the help information in Matlab. By clicking on a particular item, the information appears on the right area.

The next tab is the **Index** (fig. 25). This tab contains a searchable alphabetical list of all topics in Matlab.



Product filter: [✓ All] Selected

Search index for:

A	Product
A-law companders [1]	Communications
A-law companders [2]	Communications
A-Law Compressor block	Communications
A-Law Expander block	Communications
A/D	
See analog-to-digital	Real-Time Windo
A/D conversion	Signal Processin
A/D converter	Data Acquisition
input range	Data Acquisition
sampling rate	Data Acquisition
A16/A24/A32 memory sp...	Instrument Contr
abbreviate property names	MATLAB Link for
abbreviating property na...	Filter Design Too
abc_to_dq0 Transformati...	SimPowerSysten
about MATLAB Link for C	MATLAB Link for

Figure 25: Help navigator, the Index tab

The **Search** tab (fig. 26) can prove very useful. It gives the ability to search in various ways all the help information. In Matlab version 6 and 6.5 there is an option of searching for function names, unfortunately this has been removed in version 7. On the search results the left column shows the title of the topic and the right the section where this topic belongs.

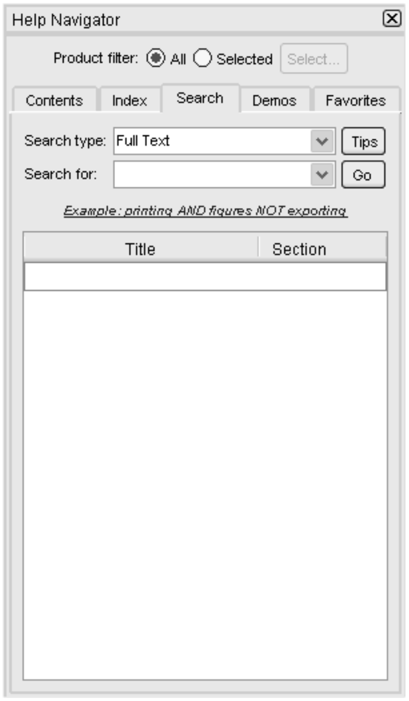


Figure 26: Help navigator, the Search tab

The **Demos** tab (fig. 27) contains all of Matlab's demo examples in a tree list.

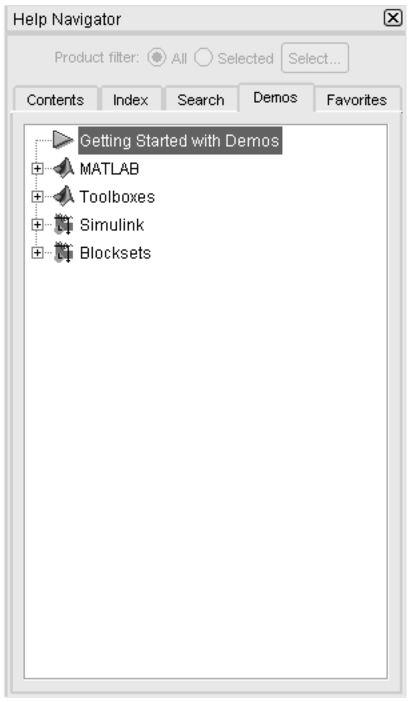


Figure 27: Help navigator, the Demos tab

The last tab on the right in the **help navigator** is the **Favorites** (fig. 28). The **Favorites** can be used to link commonly used help topics. To create a favorite click on the **Add to Favorites** button in the **information area** (fig. 23).

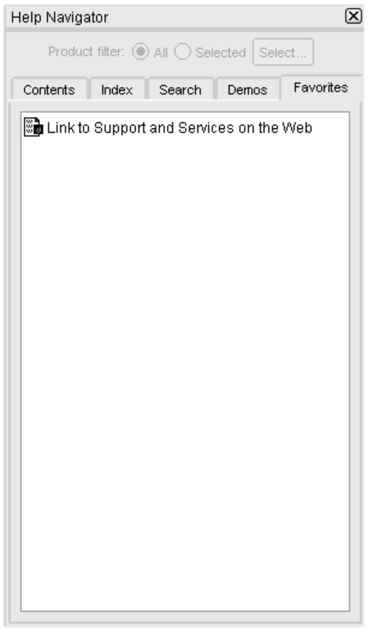

Help Navigator
Product filter: All Selected Select...
Contents
Index
Search
Demos
Favorites
Link to Support and Services on the Web

Figure 28: Help navigator, the Favorites tab

7.3 Web-based help

Help in Matlab can also be obtained from the world wide web. This appears in the **graphical help interface** by typing in the **command prompt**:

```
>> web http://www.mathworks.com
```

It can be of some interest that the **graphical help interface** can be used as web browser by replacing `http://www.mathworks.com` in the previous example. Alternatively one can launch the default system browser by typing in the command prompt:

```
>> web http://www.mathworks.com -browser
```

8 Setting the desktop layout

The setting of the **desktop layout** should be adjusted by each user according to their preferences. In the previous sections the default Matlab desktop was used as an example. There are various layouts available from the menu bar of the main desktop, in **View** → **Desktop Layout** (fig. 29).

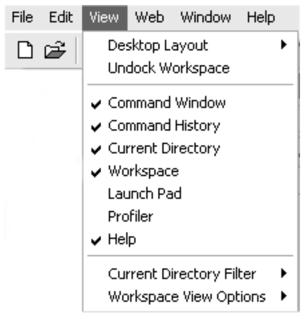


Figure 29: View menu

All these can be customized further to fit most needs. As mentioned previously most windows are dockable, this means that they can be part of the **main desktop** or they can exist as separate windows. To undock a window press the **small arrow** icon next to the **close** icon or by dragging and dropping it outside the **main desktop** area.

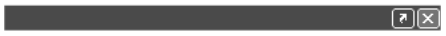


Figure 30: Undock button

To dock a window click on its menu bar on View `->` Dock ... . To make a window reappear, if it is nowhere to be seen, click on **View** → in the menu bar on the **main desktop** (fig. 29) and check if it is ticked. Keep in mind that a desktop with many smaller windows can easily become cluttered and unfriendly, as information will take longer to be found. The number and size of screens of the workstation is also an important variable on the layout of the desktop.

Part II

Lecture 1

9 Overview of Lecture 1

- Style of notes
- Recommended reading
- Introduction to Matlab
- Building matrices
- Addressing and assigning elements
- Building special matrices
- Matrix operations
- Equation solving
- User defined functions
- Plotting
- Utility commands
- Summary table of functions

10 Style of notes

These notes have been prepared to be compatible with Matlab version 6.5, for most cases this should be true for previous versions of Matlab as well as version 7. Matlab version 7 offers new features, some of those will be explained in comparison with version 6.5. It will be clearly stated, when a feature from version 7 is used.

The `verbatim` font is used for everything that is written in the Matlab programming language. For the names of variables we use small fonts for scalars, vectors and strings and capital fonts for matrices. Throughout these notes we will see examples that perform the same operation. This is intended to present some of the different ways we can work in Matlab. At the end of each set of notes we have provided a summary table of all newly introduced functions with their corresponding page numbers. Many of Matlab's functions introduced can take a different number of arguments, we have only included the basic arguments each function can take. For more details, you should look in Matlab's help.

11 Recommended reading

Matlab's help contains all the built in functions with extensive details and examples. These are well written and always worth looking at. More details and examples can be found in:

Hanselman, Duane C.: Mastering MATLAB 6 : a comprehensive tutorial and reference, Pearson Education

Kuncicky, David C.: Matlab programming, Pearson Education

Free alternatives similar to Matlab are

Scilab

http://www-rocq.inria.fr/scilab/

Rlab

http://rlab.sourceforge.net/

Octave

http://www.octave.org/

12 Introduction to Matlab

MATrix LABoratory is one of the most popular packages for scientific computing. It offers extensive libraries of numerical methods and a variety of tools for visualization. It is designed mainly for discrete computations with focus on matrices. Even though it has the capability to perform analytical tasks such as finding the derivative of a continuous function, it is not very extensive on symbolic mathematics. A more suited tool for this job is another mathematical programming package called Mathematica, developed by Stephen Wolfram.

Matlab is an interpreted programming language. The statements are translated in to machine code one by one by Matlab's interpreter. In comparison, a compiled programming language like C has a program translated as a whole in to machine code by the compiler. Interpreted languages are faster for development as they have no need for compilation and errors in the code can be found quickly. To run a piece of Matlab code on any computer it has to have Matlab installed. In the case of compiled languages there is no such requirement. Because of the interpretation Matlab tends to be slower than compiled languages.

There are 3 ways in which you can work in Matlab, from the command prompt of Matlab, writing scripts and writing functions. Matlab script and function files are called M-files. A script file takes no arguments, it is just a series of statements that will be executed sequentially. A function file can take and return arguments. To begin with, we will start with writing directly to the command prompt.

13 Building matrices

Manipulation of matrices is one of the most important tasks in Matlab.

Command	**Meaning**
[]	Matrix constructor
,	Separates matrix columns
;	Separates matrix rows
:	from-to, all

Matrices can be used to represent images, systems of linear equations and generally many types of data.

```
>> A = [2, 4; 6, 8]

A =

     2     4
     6     8

```

The commas can be replaced by spaces.

```
>> A = [2 4; 6 8]

A =

     2     4
     6     8
```

Commas and semicolons can also be used to separate statements. Commas will display the result, while the semicolon does not. It is practical to insert a semicolon at the end of each statement, when working with large matrices.

```
>> A = [2 4; 6 8], B = [1 3; 5 7]; C = [12 13; 14 15]

A =

     2     4
     6     8


C =

    12    13
    14    15
```

Matrices can be combined by using the comma or the semicolon in conjunction with the matrix constructor `[]`.

```
>> C = [A , B], D = [A ; B]
```

```
C =

     2     4     1     3
     6     8     5     7


D =

     2     4
     6     8
     1     3
     5     7
```

If the matrices we are trying to combine are not of the correct ”shapes”, i.e. the rows or columns that we are trying to combine do not match, then the following error will be produced.

```
>> C, D

C =

     2     4     1     3
     6     8     5     7


D =

     2     4
     6     8
     1     3
     5     7

>> E = [C ; D]
??? Error using ==> vertcat All rows in the bracketed expression
must have the same number of columns.
```

The problem in the construction of matrix `E` is that we are using the row separator (`;`), while the matrix `C` has 4 columns and matrix `D` has 2. The same problem would appear if we were trying to combine two matrices that do not have the same number of rows using the column separator (`,` or space).

```
>> E = [C , D]
??? Error using ==> horzcat All matrices on a row in the bracketed
expression must have the same number of rows.
```

To combine matrices with the row separator (;) the number of columns of each matrix has to be the same, that means the matrices will have to be $a \times b$ and $c \times b$. To combine matrices using the column separator (,) the number of rows of each matrix has to be the same, that is $a \times b$ and $a \times c$.

The operator `:` is used for building sequences of numbers. This is especially useful when building uniformly spaced vectors.

```
>> 1:1:5

ans =

     1     2     3     4     5
```

In this example we have created a sequence of numbers starting from 1 and ending at 5 with a step size of 1.

The generalization of the use of the colon operator (:) is $start : step : end$. The start, step and end can be any real numbers as long as: $start + (c \times step) > end, c \in \mathbb{N}$.

The colon operator (`:`) can also be used in conjunction with the matrix constructor (`[]`).

```
>> F = [1:1:5;11:2:20]

F =

     1     2     3     4     5
    11    13    15    17    19
```

14 Addressing and assigning elements

The first action we have to take in order to assign a value to a variable is to address the element of the variable we want. To address an element of a matrix the round brackets `(a,b)` are used. `a` and `b` are positive integers, i.e. $a, b \in \mathbb{N}$. The first element inside the brackets denotes the row and the second denotes the column.

```
>> F

F =

     1     2     3     4     5
    11    13    15    17    19

>> F(2,3)

ans =

    15
```

Vectors can also be used to address elements in a matrix.

```
>> F([1 ; 2] , 3)

ans =

     3
    15

>> F([1 2] , 3)

ans =

     3
    15
```

The colon : can be used to address all the elements of a row or column.

```
>> F(:,1)

ans =

     1
    11

>> F(1,:)

ans =

     1     2     3     4     5
```

Matrices can also be addressed as if they were a vector. The elements are numbered by first counting the elements of a column and then progressing to the next column (as in fig. 31).

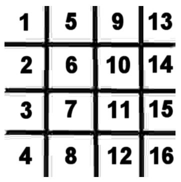


Figure 31: Numbering of a matrix as a vector

```
>> F

F =

     1     2     3     4     5
    11    13    15    17    19

>> F(3)

ans =

     2
```

```
>> F(4)

ans =

    13
```

Elements can be modified in a matrix simply by addressing the particular element and then assigning it using the equal =.

```
>> F

F =

     1     2     3     4     5
    11    13    15    17    19

>> F(1,2) = 5

F =

     1     5     3     4     5
    11    13    15    17    19

>> F([1 2] , 3) = [21 ; 22]

F =

     1     5    21     4     5
    11    13    22    17    19

>> F(:,4) = [14 ; 16]

F =

     1     5    21    14     5
    11    13    22    16    19
```

Note that if you are modifying more than one element the dimensions of what is being addressed and what is being assigned have to agree.

```
>> F([1 2] , 3) = [21 22]
???  In an assignment  A(matrix,matrix) = B, the number of rows in
B and the number of elements in the A row index matrix must be the
same.
```

In Matlab version 7 this will not return an error, as it is possible to assign without the dimensions agreeing. If only one element is being assigned and it does not belong to the matrix, then the matrix is filled with zeros and expanded in order for this element to be inside the new matrix.

```
>> F

F =

     1     5    21    14     5
    11    13    22    16    19

>> F(3,4) = 20

F =

     1     5    21    14     5
    11    13    22    16    19
     0     0     0    20     0
```

15 Building special matrices

It is convenient to be able to build special matrices without the necessity of long procedures. The following table summarizes the functions to build special matrices.

Functions	**Meaning**
`ones(a,b)`	Creates an `a` x `b` matrix with all elements equal to 1
`zeros(a,b)`	Creates an `a` x `b` matrix with all elements equal to 0
`eye(a)`	Creates an `a` x `a` identity matrix
`repmat(e,a,b)`	Replicates in `a` x `b` tiles the element `e`
`rand(a,b)`	Creates an `a` x `b` random matrix (uniform distribution in [0,1])
`randn(a,b)`	Creates an `a` x `b` random matrix (normal distribution)
`linspace(s,e,nr)`	Creates a uniformly spaced array
`logspace(s,e,nr)`	Creates a logarithmically spaced array

```
>> ones(3,5)

ans =

     1     1     1     1     1
     1     1     1     1     1
     1     1     1     1     1

>> zeros(5,3)

ans =

     0     0     0
     0     0     0
     0     0     0
     0     0     0
     0     0     0

>> rand(2,3)

ans =

    0.9501    0.6068    0.8913
    0.2311    0.4860    0.7621

>> randn(3,3)
```

```
ans =

    1.1892    0.1746   -0.5883
   -0.0376   -0.1867    2.1832
    0.3273    0.7258   -0.1364
```

The functions `ones()`, `zeros()`, `rand()`, `randn()` can be simplified for square matrices by only having one number inside the round brackets.

```
>> ones(3)

ans =

     1     1     1
     1     1     1
     1     1     1

>> zeros(3)

ans =

     0     0     0
     0     0     0
     0     0     0

>> randn(3)

ans =

   -0.0592    0.5077   -0.6436
   -1.0106    1.6924    0.3803
    0.6145    0.5913   -1.0091
```

The function `eye(a)` constructs the identity matrix, which is by definition square. The identity matrix has 1 in the elements on the diagonal and 0 everywhere else.

```
>> eye(3)
```

```
ans =

     1     0     0
     0     1     0
     0     0     1
```

`repmat(e,x,y)` can be used to replicate any element `e` (scalar, vector, or matrix) in to a matrix. The matrix returned will have dimensions equal to the first dimension of `e` times `x` by the second dimension of `e` times `y`. The element `e` will be replicated `x` times in the x axis and `y` times in the y axis.

```
>> repmat(2,3,3)

ans =

     2     2     2
     2     2     2
     2     2     2

>> A = repmat([7 8], 3, 4)

A =

     7     8     7     8     7     8     7     8
     7     8     7     8     7     8     7     8
     7     8     7     8     7     8     7     8

>> size([7 8])

ans =

     1     2

>> size(A)

ans =
     3     8

>> repmat([1 2],3,3)

ans =

     1     2     1     2     1     2
     1     2     1     2     1     2
     1     2     1     2     1     2

>> repmat([1; 2],3,3)

ans =

     1     1     1
     2     2     2
     1     1     1
     2     2     2
     1     1     1
     2     2     2

>> repmat([1 3; 2 4],3,3)

ans =

     1     3     1     3     1     3
     2     4     2     4     2     4
     1     3     1     3     1     3
     2     4     2     4     2     4
     1     3     1     3     1     3
     2     4     2     4     2     4
```

The function `size(A)` returns the number of rows and columns of `A`. To create uniformly spaced vectors, the function `linspace(s,e,nr)` can be used. The first argument (`s`) inside the round brackets is the starting point, the second (`e`) is the end point and the third one (`nr`) is the number of elements including the starting and the end point. An equivalent operation can be performed using the colon : notation, as discussed previously, with the same starting and ending points and a step size of $\frac{e-s}{nr-1}$.

```
>> linspace(1,3,5)

ans =

    1.0000    1.5000    2.0000    2.5000    3.0000
>> 1:0.5:3

ans =

    1.0000    1.5000    2.0000    2.5000    3.0000
```

In the same way we can create logarithmically spaced vectors using the function `logspace(s,e,nr)`. The difference is that the starting point `s` is defined as 10^s, the ending point as 10^e and the size of the step is $10^{\frac{e-s}{nr-1}}$

```
>> logspace(0,1,5)

ans =

    1.0000    1.7783    3.1623    5.6234   10.0000
```

16 Matrix operations

The following table has the most important matrix operations.

Matrix operations	**Meaning**
'	Transpose
+	Addition
-	Subtraction
*	Multiplication
^	Power

The transpose of a matrix A is a matrix A^T (or A'), which has the rows of A as columns and the columns as rows.

```
>> A = ones(3), B = rand(3)
```

```
A =

     1     1     1
     1     1     1
     1     1     1


B =

    0.4660    0.5252    0.8381
    0.4186    0.2026    0.0196
    0.8462    0.6721    0.6813

>> B'

ans =

    0.4660    0.4186    0.8462
    0.5252    0.2026    0.6721
    0.8381    0.0196    0.6813

>> A + B

ans =

    1.4660    1.5252    1.8381
    1.4186    1.2026    1.0196
    1.8462    1.6721    1.6813

>> A - B

ans =

    0.5340    0.4748    0.1619
    0.5814    0.7974    0.9804
    0.1538    0.3279    0.3187
```

Matrix multiplication is defined as follows: Given two matrices A and B, then their product $C = AB$ is $C_{rt} = A_{rs}B_{st}$ and an element of matrix C is defined as $c_{ij} = \sum_{k=1}^{s} a_{ik}b_{kj}$, where a_{ik}, b_{kj} are elements of A and B respectively.

```
>> A * B

ans =

    1.7309    1.3999    1.5390
    1.7309    1.3999    1.5390
    1.7309    1.3999    1.5390
```

Power of matrix is defined as follows: $\underbrace{A^k = A \times A...A}_{k\ times}$

```
>> B^2

ans =

    1.1462    0.9145    0.9719
    0.2965    0.2741    0.3682
    1.2522    1.0385    1.1866
```

Vectorized operations are also useful. These operations are performed between two corresponding elements of the two matrices. The following table contains a list of main operations.

Vectorized operations	**Meaning**
`.*`	Multiply corresponding elements
`./`	Divide corresponding elements
`.^`	Power of each element

```
>> A = [1 2; 4 6], B = [3 5; 7 9]

A =

     1     2
     4     6
```

```
B =

     3     5
     7     9

>> A .* B

ans =

     3    10
    28    54

>> A ./ B

ans =

    0.3333    0.4000
    0.5714    0.6667

>> A .^ 2

ans =

     1     4
    16    36
```

Matrix and scalar operations can be stated similarly to matrix operations. Also `+`, `-`, `/` and `*` are naturally defined between matrices and scalars.

```
>> A

A =

     1     2
     4     6

>> A + 2
```

```
ans =

     3     4
     6     8

>> A - 2

ans =

    -1     0
     2     4

>> A * 2

ans =

     2     4
     8    12

>> A / 2

ans =

    0.5000    1.0000
    2.0000    3.0000
```

17 Equation solving

Consider the following system of equations.

$$\begin{aligned} 2x_1 + x_2 - 2x_3 &= 10 \\ 6x_1 + 4x_2 + 4x_3 &= 2 \\ 10x_1 + 8x_2 + 6x_3 &= 8 \end{aligned} \tag{1}$$

This can be represented by a matrix equation $Ax = b$

```
>> A = [2 1 -2; 6 4 4; 10 8 6], b = [10 ; 2 ; 8]

A =

     2     1    -2
     6     4     4
    10     8     6



b =

    10
     2
     8
```

The solution of this equation is to invert matrix A and multiply it with b, $x = A^{-1}b$. To invert a matrix we can use `inv(A)` or simply raise the matrix to power of -1. These two are equivalent.

```
>> x = inv(A) * b

x =

    1.0000
    2.0000
   -3.0000

>> x = A^-1 * b

x =

    1.0000
    2.0000
   -3.0000
```

Another way of calculating the solution of this equation is to use the left division sign `\`. This will invert the matrix and multiply to the left. The difference of left division with the previous methods is that if the matrix is singular, then it will calculate the least squares solution.

```
>> x = A \ b

x =

    1.0000
    2.0000
   -3.0000
```

Another method of obtaining the least squares inverse also called Moore-Penrose pseudoinverse is to use `pinv(A)`.

```
>> x = pinv(A) * b

x =

    1.0000
    2.0000
   -3.0000
```

The pseudoinverse should be used when we know that the matrix does not have a proper inverse.

If the matrix is square and non-singular then the function **pinv(A)** is a computationally expensive way of calculating the inverse. In this case we should use the proper inverse (**inv** or raise to the power of -1).

18 User defined functions

To write our own function we will have to open a new M-file. This can be done in Matlab's editor or in any external editor, as discussed previously. An M-file can contain a script or a function. A script is a series of statements that will be executed sequentially and has the same effect as typing them in the command prompt. The first line of a function should be in the following form:

```
function [val1, val2, ...] = filename(arg1,arg2,...)
```

Note that the name of the function has to match the name of the file. The first commented lines after the first line, which defines the function, are displayed when requesting Matlab's help. Create a function called `my_function` with the following code.

```
function [rtn] = my_function(a,b)
% rtn = my_function(a,b) takes two variables and adds them

rtn = a + b;
```

```
>> my_function(2,4)

ans =

     6
>> help my_function

  rtn = my_function(a,b) takes two variables and adds them
```

Using the first lines for information on the function, explaining what the function does, what arguments it takes, what variables it returns, version number, etc. can prove to be very useful in large projects.

Remember that all variables inside a function are local and cannot be accessed from the command prompt or any other functions. All variables inside a script file can be accessed from the command prompt and other scripts.

Next is an example of a function `myint` that integrates a given function `f` on an closed interval `[a,b]` according to accuracy `n`. The higher `n` is the more samples `myint` will take in the interval `[a,b]` for the function `f`.

```
function s = myint(f,a,b,n)
% function s = myint('f',a,b,n)
%
%  This function numerically integrates f from a to b
%  by summing n approximating rectangles

invs = linspace(a,b,n+1) ;
fx = feval(f,invs(1:n)) ;
s =((b-a)/n)*sum(fx) ;
```

On the command prompt we can type:

```
>> myint('sin',0,pi,100)

ans =

    1.9998
```

The function `feval(f,values)` takes a string `f` defining the function and an array `values` with the values of where the function `f` should be evaluated. `feval` can take any predefined, both user created and built in Matlab, function as the `f` argument.

19 Plotting

The main function for simple plots is `plot(x,y)`, it takes two arguments, the x values and the y values.

```
>> x_values = linspace(0,4*pi,100);y_values = sin(x_values);
>> plot(x_values,y_values);
```

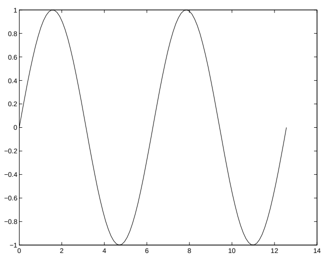


Figure 32: Sin plot

3D plots can be performed using the `plot3(X,Y,Z)` function. This works similar to `plot(x,y)` with the addition of the `Z` coordinates. 3D plots can also be displayed using the `mesh(X,Y,Z)` function. `X`, `Y` and `Z` are the x, y and z coordinate matrices. Define the following function:

```
function rtn = funky(x,y)
% rtn = funky(x,y)

rtn = sin(sqrt(x.^2 + y.^2))./(x.^2 + y.^2 + 0.0001);
```

Note that we have used vectorized power in the function definition. If we pass arrays to this functions the elements will be multiplied individually instead of performing a matrix multiplication. On the command prompt we write the following statements:

```
>> [X Y] = meshgrid(linspace(-8,8,30),linspace(-8,8,30));
>> Z = funky(X, Y);
>> mesh(X,Y,Z);
```

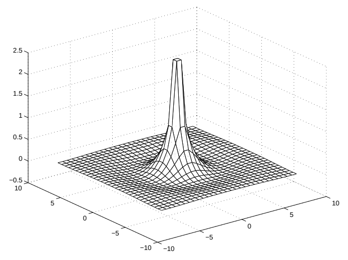


Figure 33: Mesh plot of the funky function

The function `[X Y] = meshgrid(A,B)` will create the grids `X` and `Y` for a 3D plot according to the vectors `A` and `B`.

20 Utility commands

Command	**Meaning**
whos	Gives the sizes and types of all loaded variables
save `'my_work'`	Save the current workspace on to `'my_work'`
load `'my_work'`	Load `'my_work'` on to the current workspace
diary on	Starts diary
diary off	Stops diary
close all	Closes all figure windows
clear all	Clears from memory all loaded variables
clc	Clear command window
Control + c	Stops execution of a program
Control + i	Smart indent selected text
Control + r	Comment selected text
Control + t	Uncomment selected text
quit	Exit Matlab

21 Summary table of functions

Function	p.	Meaning
`[]`	27	Matrix constructor
`,`	27	Separates matrix columns
`;`	27	Separates matrix rows
`:`	27	from-to, all
`()`	31	Addressing elements in matrix
`ones(a,b)`	35	Creates an `a` x `b` matrix with all elements equal to 1
`zeros(a,b)`	35	Creates an `a` x `b` matrix with all elements equal to 0
`eye(a)`	35	Creates an `a` x `b` identity matrix
`repmat(A,a,b)`	35	Replicates in `a` x `b` tiles the element `A`
`rand(a,b)`	35	Creates an `a` x `b` random matrix (uniform distribution in [0,1])
`randn(a,b)`	35	Creates an `a` x `b` random matrix (normal distribution)
`linspace(s,e,nr)`	35	Creates a uniformly spaced array
`logspace(s,e,nr)`	35	Creates a logarithmically spaced array
`size(A)`	38	Returns the number of rows and columns of `A`
`'`	39	Transpose
`+`	39	Addition
`-`	39	Subtraction
`*`	39	Multiplication
`^`	39	Power
`.*`	41	Multiply corresponding elements
`./`	41	Divide corresponding elements
`.^`	41	Power of each element
`inv(A)`	44	Invert matrix `A`
`A \ B`	44	Invert the matrix `A` and multiply it with `B`
`pinv(A)`	45	Pseudoinverse of matrix `a`
`feval(f,values)`	47	Evaluate the string function `f` at `values`
`plot(x,y)`	47	Plot `x` and `y` values
`plot3(X,Y,Z)`	47	3D plot `X`, `Y` and `Z` values
`mesh(X,Y,Z)`	47	3D mesh plot of `X`,`Y` and `Z` values
`meshgrid(A,B)`	48	Create `X` and `Y` grid matrices for a 3D plot

22 Lab exercises 1

Programming exercises 1

A. Create the following matrices in Matlab:

```
A =

     5     3     1     0
     2     4     7     2
     6     4     3     1

B =

     1     7
     3     4
     2     3

C =

     6     6     0     5
     9     2     1     8
```

1. Combine matrices `A`, `B` and `C` in all possible ways (horizontally and vertically) using the constructor `[]` and the operators `;` and `,` .
2. Using the `:` create the following arrays:

```
a =

     1     2     3     4     5     6

b =
     2     2.5   3     3.5   4     4.5   5

c =
    3       2.75   2.5    2.25   2     1.75   1.5    1.25    1
```

3. Write code that will add the 1st, 3rd and 6th element of the arrays `a`, `b` and `c`. This sum should should be placed as an extra element at the end of each array.

4. For matrix

   ```
   D =

        1     5     2     9     6
        2     4     3     2     7
   ```

   write code that will add the elements of the 1st row to elements of the 2nd row and place them on a 3rd row in matrix `D`.

5. Using the **`rand`** function add a maximum 10% random error on each element of the matrices `A`, `B` and `C`. 10 % random error means that you should add to each element a number that is at most 10 % of the element's value.

B. Create the matrix `E` as follows:

```
E =

     4     6     0
     5     1     3
```

1. Find at least 2 ways of adding the value `1` to each element of matrix `E`.

2. Using the **`linspace`** function create the following array:

   ```
   d =
        1      1.2     1.4     1.6     1.8      2
   ```

3. Using the **`repmat`** function create a 3x6 matrix with the array `d` as each row.

4. Using the `repmat` function create a 6x4 matrix with the array `d` as each column. Note you will have to transpose the array `d`.

5. Using the `meshgrid` function create two matrices `X` and `Y` with the array `d`. Recreate those two matrices using the `repmat` function.

6. Type the function `funky` from the notes 1 on p.48 and save it. Construct an array `e` starting at -5 and ending at 5. The step size is your own choice. Using the array `e` and the `repmat` or the `meshgrid` function create the `X` and `Y` coordinate matrices for the calculation of the `funky` function. Evaluate the function from -5 to 5 and plot it using `plot3` and `mesh`. Note how the plot changes if you change the step size in the array `e`.

C. Using the `:` operator, element addressing and the matrix constructor `[]` create the following matrix. Note you might need to create an intermediate array holding all of the values of the matrix.

```
F =

     1     2     3     4
     5     6     7     8
     9    10    11    12
```

1. Using the `reshape` function and an intermediate array, that holds all the values, create the matrix `F` with the same values as previously. Use Matlab's help to find out the syntax and functionality of `reshape`.

2. Using matrix multiplication multiply the 1st column of `F` with 3, the 2nd with 2, the 3rd with 5 and the 4th with 7.

3. Using matrix multiplication multiply the 1st row of `F` with 3, the 2nd with 2 and the 3rd with 5.

4. Using vectorized multiplication perform the same operations to matrix `F` as the previous two exercises.

5. Using matrix multiplication find the sum of each row and column of the matrix `F`.

D. Given the following linear systems:

$$\begin{aligned} 2x_1 + 3x_2 - x_3 + 4x_4 &= 23 \\ 1x_1 + 1x_2 - 3x_3 + 5x_4 &= 11 \\ 7x_1 + x_2 + 3x_3 + 4x_4 &= 12 \\ 5x_1 + 4x_2 + 3x_3 - 11x_4 &= 14 \end{aligned} \tag{2}$$

$$\begin{aligned} 2x_1 + 3x_2 - x_3 + 4x_4 + 3x_5 &= 23 \\ 1x_1 + 1x_2 - 3x_3 + 5x_4 + 5x_5 &= 11 \\ 7x_1 + x_2 + 3x_3 + 4x_4 + 7x_5 &= 12 \\ 5x_1 + 4x_2 + 3x_3 - 11x_4 + 11x_5 &= 14 \end{aligned} \tag{3}$$

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 23 \\ 1x_1 + 1x_2 - 3x_3 &= 11 \\ 7x_1 + x_2 + 3x_3 &= 12 \\ 5x_1 + 4x_2 + 3x_3 &= 14 \end{aligned} \tag{4}$$

1. Solve all of the above systems.
2. Test how good your solutions are by replacing the x variables into the equations.

E.

1. Write a function `my_calculations` that takes two variables, adds, subtracts, multiplies and divides them and returns all these results.
2. Write a function `my_cos_sin` that calculates the following expression:

$$y = 2\cos(x) + 3\sin(2x)$$

given `x` and returns `y`. Keep in mind that the functions `sin` and `cos` take radians as arguments.

3. Calculate the values of `y` for $x \in [0, 2\pi)$.
4. Use the `feval` function to calculate `y` for $x \in [0, 2\pi)$.
5. Plot the `sin()`, `cos()` and `my_cos_sin()` functions on the correct x-axes.

Programming exercises 2

Files for this exercise are available from
`http://www.cs.ucl.ac.uk/staff/M.Herbster/GI03/`

1. In this first exercise we will produce a simple visualization of a gradient descent algorithm. Consider the function,

$$f(x, y) = (x - 2)^2 + 2(y - 3)^2$$

```
>> [X,Y] = meshgrid(linspace(0,5,15),linspace(0,5,15)) ;
>> mesh(X,Y,fcarg(X,Y)) ;
```

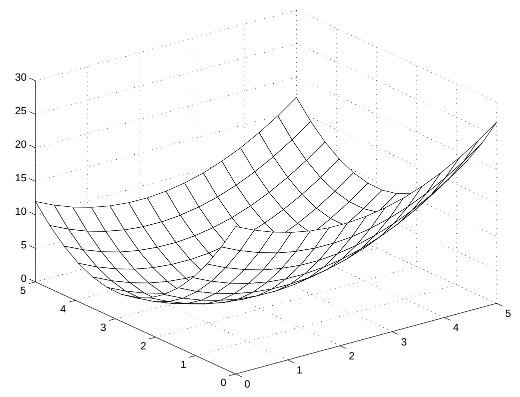


Figure 1

Algebraically, we see that $(2, 3)$ is the minima of this function. Numerically we may use the matlab function *graddesc.m* to calculate the minima.

```
>> graddesc('fc','dfc',[0,0],0.1,0.1)
ans =
    1.9550    2.9995
```

Suppose we start a gradient descent algorithm at (0,0) on the way to the minima we traverse a series of points,

$$\{(0, 0, f(0, 0)), (x_2, y_2, f(x_2, y_2)), (x_3, y_3, f(x_3, y_3)), \ldots, \sim (2.0, 3.0, 0.0))\} \tag{5}$$

Visualizing this path in three dimensions we have,

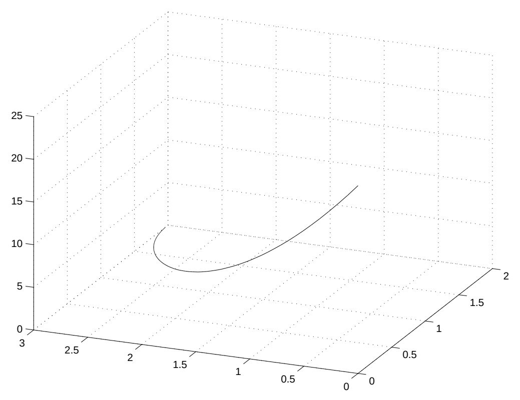


Figure 2

Projecting down to the xy plane we have,

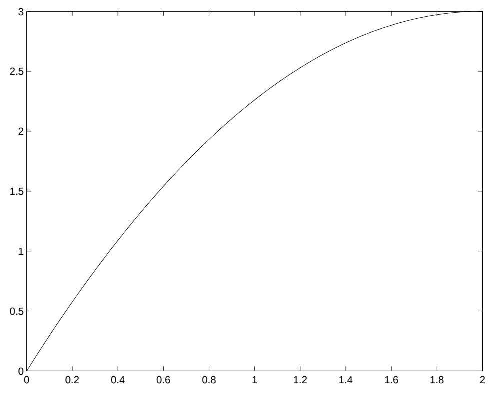


Figure 3

(a) Produce a plot similar to Figure 1.

(b) Modify the function *graddesc.m* to produce a sequence of points as in Equation 1.

  i. Modify the code.

  ii. Produce a plot similar to Figure 2. (Hint: to do this it will be necessary to massage the sequence of points into a form usable by `plot3()`, then to produce the grid use the command `grid on`.

  iii. Produce a plot similar to Figure 3.

2. In this exercise we will use *gradient descent* to perform linear regression (linear least squares). Consider the matrix equation,

$$Ax = b$$

if there is no exact solution then for any potential solution x we may define a column vector of error terms

$$e = Ax - b.$$

The sum of the errors squared is then

$$e^T e$$

in matlab this is just $e' * e$. Thus the least squares solution is the x that minimizes

$$(Ax - b)^T (Ax - b).$$

In Matlab

```
>> A\b
```

computes the least square solution directly.

   (a) Give a matlab function to compute the least squares solution by gradient descent. The arguments should include the matrix A, the column vector b, an initial guess, a step size, and a tolerance (i.e., a convergence criteria). For example: `mydescent(A,b,guess,step,tol)`. *Note: that standard gradient is a very inefficient method to compute the least squares solution to a set of equations.* Also observe in order to code `mydescent.m` you will need to determine the symbolic solution of $\nabla_x[(Ax - b)^T (Ax - b)]$ where ∇_x is the gradient with respect to x.

   (b) Use the above function to give a least squares solution to the equations

$$\begin{aligned} x_1 - x_2 &= 1 \\ x_1 + x_2 &= 1 \\ x_1 + 2x_2 &= 3 \end{aligned}$$

(c) Visualize the above solution with a plot as in Figure 3.

3. This exercise explores the *convergence* of gradient descent in a single variable. Gradient descent can be defined as follows. Given $f(x)$, let f' denote the first derivative, let λ denote the step size and let x_0 denote the initial point, hence gradient descent can be defined as a sequence of iterates of $G_{f,\lambda}(x) = x - \lambda f'(x)$ which we will denote as follows

$$G^{(0)} = x_0,\ G^{(1)} = x_0 - \lambda f'(x_0),\ G^{(2)} = x_0 - \lambda f'(x_0) - \lambda f'(x_0 - \lambda f'(x_0)),\ \ldots \tag{6}$$

hence gradient descent on f with step size λ and starting point x_0 converges to x^* if $G^{(n)} \to x^*$ as $n \to \infty$.

For the following exercises experimental arguments will receive a some credit however the best responses will give/include mathematical arguments. Notation: let $|x|$ denote the absolute value of x.

(a) Does there exist a nontrivial starting point x_0 and step size λ such that gradient descent on $f(x) = |x-1|^3$ converges to 1. What is your evidence?

(b) Does there exist a nontrivial starting point x_0 and step size λ such that gradient descent on $f(x) = \sqrt{|x-1|}$ converges to 1. What is your evidence?

(c) For what values of $\lambda > 0$ does there exist an $x_0 \neq 0$ such that gradient descent on $f(x) = x^4 + 5x^2$ converges to 0. Why?

(d) Formulate a set of sufficient conditions for gradient descent to converge in one variable.

Part III
Lecture 2

23 Overview of Lecture 2

- Relational operators
- Logical operators
- Control flow
  - `for` loops
  - `while` loops
  - `if-else-end`
  - `switch-case-otherwise-end`
- Precision issues
- Additional data types
  - Strings
  - Cell arrays
  - Structures
- Input/Output (I/O)
- Formatted Input/Output
- Summary table of functions

24 Relational operators

The following table contains a list of the relational operators in Matlab.

Operator	**Meaning**
`<`	Less than
`<=`	Less than or equal to
`>`	Greater than
`>=`	Greater than or equal to
`==`	Equal to
`~=`	Not equal to

$\boxed{\text{Relational operators return a Boolean value, that is 1 if true and 0 if false.}}$

```
>> 4 > 5

ans =

     0

>> 4 < 5

ans =

     1
```

Relational operators can be applied to any size vectors. Next is an example of how to compare two vectors of equal size.

```
>> a = [1:5], b = 5 - a

a =

     1     2     3     4     5


b =

     4     3     2     1     0
```

```
>> a > b

ans =

     0     0     1     1     1
```

The answer is an array of the original size with each position representing the result of the comparison between corresponding elements of the arrays. Relational operators can also be used to compare arrays with scalars.

```
>> a > 4

ans =

     0     0     0     0     1
```

The function `find(A)` finds indices and values of nonzero elements of an array `A`.

```
>> b

b =

     4     3     2     1     0

>> locs = find(b.^2 > 5)

locs =

     1     2

>> b(locs)

ans =

     4     3
```

25 Logical operators

The following table lists the logical operators in Matlab. Note that `A` and `B` are not considered purely as matrices but as logical expressions.

Operator	**Function Form**	**Meaning**
`&`	`and(A,B)`	and
`|`	`or(A,B)`	inclusive or
	`xor(A,B)`	exclusive or
`~`	`not(A)`	not

The operator `and` (`&`) requires both expressions `A`, `B` to be true in order to return true, all other combinations return false. The inclusive or (`|`) returns true if one or both of the expressions are true, while the exclusive or (`xor(A,B)`) returns true only if one expression is true and returns false if both expressions are true or false. The operator not (`~`) returns true if the expression is false and false if the expression is true. This can be summarized in a truth table.

`A`	`B`	`A&B`	`A|B`	`xor(A,B)`	`~A`
0	0	0	0	0	1
1	0	0	1	1	0
0	1	0	1	1	1
1	1	1	1	0	0

These operators are used in the following way. Consider `A` and `B` to be two logical expressions with a Boolean value, that is 1 for true and 0 for false.

```
>> A = 0; B = 1;
>> A & B

ans =

     0

>> A | B

ans =

     1
```

```
>> xor(A , B)

ans =

     1

>> ~A

ans =

     1
```

Next are some examples on the use of these operators in combination with the relational operators.

```
>> a = [1:8]

a =

     1     2     3     4     5     6     7     8

>> a > 4 & a <= 6

ans =

     0     0     0     0     1     1     0     0

>> a > 4 | mod(a,2) == 0

ans =

     0     1     0     1     1     1     1     1

>> xor(a > 4, mod(a,2))

ans =
     1     0     1     0     0     1     0     1

>> a > 4 & (~mod(a,2) == 0 )

ans =

     0     0     0     0     1     0     1     0
```

The `mod(a,b)` function calculates the modulus after the division `a/b`. The not operator can usually be replaced by directly negating the expression. The last expression on the previous example can be rewritten as:

```
>> a > 4 & (mod(a,2) ~= 0 )

ans =

     0     0     0     0     1     0     1     0
```

It is also possible to build expressions combining multiple logical operators.

```
>> (a > 4 & a <= 6) | (a == 1)

ans =

     1     0     0     0     1     1     0     0
```

26 Control flow

In the following sections the control flow of loops (`for`, `while - end`) and conditional statements (`if - else`, `switch - case`) will be introduced.

26.1 for loops

The most common way of repeating a sequence of statements for a specific number of times is to use a `for` loop. Consider the following example:

```
>> for x=1:2
disp('This statement is repeated')
end
```

The statement between `for` and `end` will be repeated two times. The output on the screen will be:

```
This statement is repeated
This statement is repeated
```

On the first line of the loop, starting with `for`, we define how many times the statements will be repeated. If we simply change the `2` from the previous example to `3`, the statements will be repeated three times.

```
>> for x=1:3
disp('This statement is repeated')
end
```

The function `disp(a)` displays `a` on to the screen. The output on the screen will be:

```
This statement is repeated
This statement is repeated
This statement is repeated
```

The first line of the loop contains an assignment of an array. The number of columns of this array defines how many times the statements inside the loop will be executed. On every repetition `x` will be equal to a column of the array it was assigned to. The variable `x` will take consecutive values, the columns, of the assigned array. Consider the following example:

```
>> for x = [1 3 5]
disp('x equals'),disp(x)
end
```

The output on the screen will be:

```
x equals
     1

x equals
     3

x equals
     5
```

The statement `disp('x equals'),disp(x)` was repeated three times, that is the number of columns of the array `[1 3 5]` used in the assignment, each time taking the value of the column of the array `[1 3 5]`. A matrix can also be used instead of vector. In this case `x` will be a column vector instead of a single number.

```
>> for x = [1 2 3; 4 5 6]
disp('x equals'),disp(x)
end
```

The output on the screen will be:

```
x equals
     1
     4

x equals
     2
     5

x equals
     3
     6
```

The general syntax of the `for` loop is as follows:

```
for x=array

    statements % executed once for each column in array
               % x is the columns consecutively
end
```

The first line defines how many times `statements` will be executed. If `array` is a $m \times n$ matrix then `statements` will be executed n times. Each time `x` will be equal to the corresponding column.

The following example calculates and displays the mean and the sum of each column of a matrix.

```
>> A=[1:12]; B = reshape(A,4,3)

B =

     1     5     9
     2     6    10
     3     7    11
     4     8    12

>> for x=B
disp('Mean:'); disp(mean(x)); disp('Sum:'); disp(sum(x));
end
Mean:
    2.5000

Sum:
    10

Mean:
    6.5000

Sum:
    26

Mean:
   10.5000
```

```
Sum:
    42
```

The function `reshape(A,r,c)` reshapes `A` in to `r` rows and `c` columns. The functions `mean(a)` and `sum(a)` calculated the mean and sum of `a`. If `a` is a matrix then they calculate the mean or sum on each column of `a`.

In the case where `x` takes consecutive integer values it can be used as an index in array.

```
>> a = [1,2,3];
>> for x=a
b(x) = x^2;
end
>> b

b =

     1     4     9

>> x_values = [0:0.1:2*pi - 0.1];
>> for index=1:length(x_values)
y_values(index) = sin(x_values(index));
end
>> figure;plot(x_values,y_values);
```

Question: How can we simplify the above example and avoid using a `for` loop?

The function `length(a)` returns the size of the largest dimension of `a`. The last statement will produce the plot in fig. 34.

26.2 while loops

Often we wish to repeat a sequence of statements while a given condition remains true. The general syntax for a while loop is the following.

```
while expression

    statements  % are executed while expression is true
```

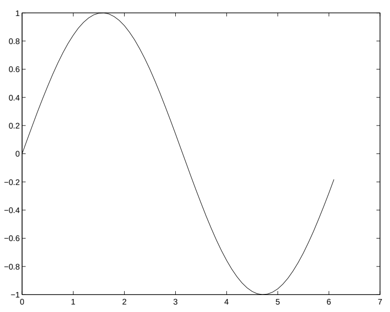


Figure 34: Sin plot

```
                % if the expression returns an array all
                % must be true
end
```

The first line defines when the repetition is going to stop. `statements` will be executed repeatedly, while `expression` remains true, that is while `expression` returns the value 1.

The `while` loop can be used exactly as a `for` loop, if we define a variable to count the number of repetitions. This variable is commonly referred to as the counter.

```
>> a_counter = 0;
>> while a_counter < 3
a_counter = a_counter + 1;
disp('The counter is equal to: '),disp(a_counter);
end
```

The output on the screen will be:

```
The counter is equal to:
     1

The counter is equal to:
     2

The counter is equal to:
     3
```

Next is an example that finds the first non zero entry of an array or a matrix and stores its index.

```
>> A = zeros(10,1);
>> A(5) = 1;
>> m = 1;
>> while A(m) == 0
m = m + 1;
end
>> m

m =

     5
```

Question: How can the above example be accomplished using the function `find()`?

The difference between the two types of loops is that **for** loops can only repeat statements based on the number of columns of the assigned array. **while** loops can repeat statements based on any expression. Thus **while** loops are more general than **for** loops. The most common use of **for** loops is for counting type of repetitions, their advantage there is that they have the counter build in to the syntax.

26.3 if-else-end

To execute different sequences of statements according to some conditions we can use `if-else-end`. The general syntax is as follows:

```
if expression1
    statements1 % executed if expression1 is true
elseif expression2
    statements2 % executed if expression1 is false and expression2 is true
elseif expression3
    statements3 % executed if all previous expressions are false
....            % and expression3 is true
else
    statements4 % executed if no expression is true
end
```

Note that `expression2` will only be checked if `expression1` is false.

The next expression will only be checked if the previous one is false. That means that if two or more expressions are true, only the first one is going to be executed.

To demonstrate this a simple example of `if-else` follows next.

```
>>if 1<2
disp('First condition');
elseif 1<3
disp('Second condition');
end

First if-else
```

The following function uses `if-else` to separate an array of numbers into numbers divisible by 2, divisible by 3 and not divisible by 2 or 3.

```
function [div_2, div_3, not_div] = myseparate(A)
% rtn = mysort(A) takes a scalar vector as input
% and sorts it out to numbers that are divisible by 2 (div_2),
% numbers that are divisible by 3 (div_3) and
% other numbers that are not divisible by 2 or 3.
% If a number is divisible by 2 and 3 then it will go to both lists.

count_div_2 = 0; count_div_3 = 0; count_not_div = 0;

for m=1:length(A)
    if mod(A(m),2) == 0 & mod(A(m),3) == 0
        count_div_2 = count_div_2 + 1;
        div_2(count_div_2) = A(m);
        count_div_3 = count_div_3 + 1;
        div_3(count_div_3) = A(m);
    elseif mod(A(m),2) == 0
        count_div_2 = count_div_2 + 1;
        div_2(count_div_2) = A(m);
    elseif mod(A(m),3) == 0
        count_div_3 = count_div_3 + 1;
        div_3(count_div_3) = A(m);
    else
        count_not_div = count_not_div + 1;
        not_div(count_not_div) = A(m);
    end
end
```

Question: How can we accomplish the above without using `if-else` and `for` loop in three statements?

On the command prompt we will type:

```
>> A = [2 3 4 45 53 42 1 3 9]

A =

     2     3     4    45    53    42     1     3     9

>> [div2 div3 notdiv] = myseparate(A)

div2 =

     2     4    42


div3 =

     3    45    42     3     9


notdiv =

    53     1
```

We can also have `if-else` statements inside an `if-else` sequence. The previous function can be rewritten as:

```
function [div_2, div_3, not_div] = myseparate(A)
% rtn = mysort(A) takes a scalar vector as input
% and sorts it out to numbers that are divisible by 2 (div_2),
% numbers that are divisible by 3 (div_3) and
% other numbers that are not divisible by 2 or 3.
% If a number is divisible by 2 and 3 then it will go to both lists.

count_div_2 = 0; count_div_3 = 0; count_not_div = 0;

for m=1:length(A)
    if mod(A(m),2) == 0
        count_div_2 = count_div_2 + 1;
        div_2(count_div_2) = A(m);
        if mod(A(m),3) == 0
            count_div_3 = count_div_3 + 1;
            div_3(count_div_3) = A(m);
        end
    elseif mod(A(m),3) == 0
        count_div_3 = count_div_3 + 1;
        div_3(count_div_3) = A(m);
    else
        count_not_div = count_not_div + 1;
        not_div(count_not_div) = A(m);
    end
end
```

26.4 switch-case-otherwise-end

Another method of executing statements according to some conditions is to use `switch-case-otherwise-end`.

```
switch expression
    case result1
        statements1 % executed if the result from expression
                    % is equal to result1
    case result2
        statements2 % executed if the result from expression is not equal
                    % to result1 and it is equal to result2
    case result3
```

```
        statements3 % executed if the result from expression is not equal to
                    % any of the previous results and is equal to result3
    ...
    otherwise
        statements4 % executed if the result from expression is
                    % not equal to any of the previous results
end
```

Similarly to `if-else`, further `statements` will only be executed if the `results` from the previous are not equal to the `result` of `expression`.

The following function converts the month number in to the name of the month.

```
function [rtn] = disp_month (num_month)
% [rtn] = disp_month (num_month)
% takes a number as an argument (num_month), displays
% the equivalent month with each name and
% and returns 1 if the month exists and 0 otherwise

rtn = 1;
switch num_month
    case 1
        disp('Month entered is January');
    case 2
        disp('Month entered is February');
    case 3
        disp('Month entered is March');
    case 4
        disp('Month entered is April');
    case 5
        disp('Month entered is May');
    case 6
        disp('Month entered is June');
    case 7
        disp('Month entered is July');
    case 8
        disp('Month entered is August');
    case 9
```

```
        disp('Month entered is September');
    case 10
        disp('Month entered is October');
    case 11
        disp('Month entered is November');
    case 12
        disp('Month entered is December');
    otherwise
        disp('Month entered does not exist');
        rtn = 0;
end
```

On the command prompt we can type:

```
>> true_false = disp_month(8);
Month entered is August
>> true_false

true_false =

     1
```

The main difference between `if-else` and `switch-case` is that the latter can only decide on one expression, namely the one after `switch`.

27 Precision issues

By default Matlab declares variables as double precision and up to version 6.5 it can only perform double precision arithmetic. The computing precision is not the displaying precision. Matlab will display results using short precision, by default, which uses only 5 digits. This can be changed using `format(type)` function.

```
>> 3/7

ans =

    0.4286
```

```
>> format('long')
>> 3/7

ans =

   0.42857142857143

>> format('long', 'e')
>> 3/7

ans =

    4.285714285714286e-001
```

Variables can also be defined to be of a specific type (`single(A)`, `double(A)`, `int8`, `int16(A)`, `int32(A)`, `int64(A)`, `uint8(A)`, `uint16(A)`, ...). These are all primitive types. `int8` uses 8 bits and it can take any of the 2^8 possible values of signed integers. That means that it can take values from -128(-2^7) to 127($2^7 - 1$). `uint8` also uses 8 bit, but it can only take unsigned values from 0 to 255($2^8 - 1$). `int16` uses 16 bits for signed integers from -2^{15} to $2^{15} - 1$. `uint16` also uses 16 bits for unsigned integers and takes values from 0 to $2^{16} - 1$. The rest of the integer data types are defined in the same way.

double is the only one, which can be used for mathematical operations up to Matlab version 6.5.

In Matlab version 7 mathematical operations can be performed between variables that belong to the same type, or between variables that belong to `double` and variables that belong to one other type (that can be any type as long as everything that is not `double` belongs to one type). Mathematical operations between different types, with the exception of `double` mentioned above, can not be performed.

The next example is in Matlab version 7:

```
>> a = int16(1); b = int16(2);
>> a_b = a + b

a_b =
      3

>> c = double(3);
>> b_c = b + c

b_c =

      5

>> d = int8(4);
>> a_d = a + d
??? Error using ==> plus Integers can only be combined with
integers of the same class, or scalar doubles.

>> whos
  Name      Size                    Bytes  Class

  a         1x1                         2  int16 array
  a_b       1x1                         2  int16 array
  b         1x1                         2  int16 array
  b_c       1x1                         2  int16 array
  c         1x1                         8  double array
  d         1x1                         1  int8 array

Grand total is 6 elements using 17 bytes
```

In Matlab version 6.5 we would get:

```
>> a = int16(1); b = int16(2);
>> a_b = a + b
??? Error using ==> + Function '+' is not defined for values of
class 'int16'.

>> c = double(3);
>> b_c = b + c
??? Error using ==> + Function '+' is not defined for values of
class 'int16'.
```

This error can appear when reading in data from a file, like an image and then trying to perform a mathematical operation. The solution is simply to convert them to `double`.

```
>> a = double(a), b = double(b)

a =

     2


b =

     3

>> a + b

ans =

     5
```

Matlab can be inexact sometimes. Consider the following example:

```
>> (-0.08 + 0.5 - 0.42) == (0.5 - 0.42 - 0.08)

ans =

     0

>> (-0.08 + 0.5 - 0.42) ~= (0.5 - 0.42 - 0.08)

ans =

     1

>> (-0.08 + 0.5 - 0.42) - (0.5 - 0.42 - 0.08)

ans =
   -1.387778780781446e-017
```

According to Matlab these two operations are not equal. We can define a function

```
function rtn = toleq(A, B, e)
% Determines equality according to some precision
rtn = abs(A - B) < e;
```

This function determines whether two arrays (`A`, `B`) of equal size have equal values according to some precision `e`.

```
>> toleq((-0.08 + 0.5 - 0.42), (0.5 - 0.42 - 0.08), eps)

ans =

     1
```

This returns the correct result. `eps` is the smallest number in Matlab that when added to 1 produces a larger number. The following is an example of the properties of `eps`.

```
>> [eps eps 1 1 10 10 0.1 0.1] <
[eps 2*eps 1+eps 1+eps/2 10+4*eps 10+5*eps 0.1+eps/31 0.1+eps/32]

ans =

     0     1     1     0     0     1     1     0
```

The first four results follow directly from the definition of `eps`. It is interesting to see that when adding `eps` to numbers like 10 or 0.1 it does not behave as it does with 1. In the case of 10 we need to add more than `eps` (`4*eps`) to get a larger number. In the case of 0.1 we need to add less (`eps/31`) to get a larger number.

28 Additional data types

So far we have used mainly arrays that hold scalar values. In Matlab we can also define strings (arrays of characters), cell arrays and structures. Cell arrays and structures are composite types.

28.1 Strings

A string is defined with the use of the `''` operator. The general syntax is:

```
the_string = 'somecharacters'
```

An example of this is:

```
>> t = 'a string'

t =

a string
```

In a string the space is also considered to be a character.

Various functions that apply to scalar arrays also apply to strings.

```
>> length(t)

ans =

     8

>> t == 'a string'

ans =

     1     1     1     1     1     1     1     1

>> inv_t = t(length(t):-1:1)
```

```
inv_t =

gnirts a
```

A simpler way to invert a list of elements is to use `end`.

```
>> inv_t = t(end:-1:1)

inv_t =

gnirts a
```

A string can hold numbers, but they are considered to be characters.

```
>> t = '1 2 3'

t =

1 2 3

>> eval(['sum([' t '])'])

ans =

     6
```

The function `eval(a)` executes a Matlab expression held in the string `a`. To convert strings to numbers we can use `str2num(a)`.

```
>> t_scal = str2num(t)

t_scal =

     1     2     3

>> sum(t_scal)
```

```
ans =

     6
```

It is also possible to convert numbers to strings using `num2str(a)`.

```
>> t_scal = [1 2 3]

t_scal =

     1     2     3

>> t = num2str(t_scal)

t =

1  2  3
```

28.2 Cell arrays

Cell arrays can hold multiple data types. It is possible for example to have a string, various types of matrices as well as a cell array inside a cell array. Cell arrays are constructed with the use of the curly brackets `{}`, this is analogous to the square bracket `[]` constructor of arrays.

```
>> A = {[1 2 3 ; 4 5 6] 'hello world' ; 4 {'red' [10 11 12]}}

A =

    [2x3 double]    'hello world'
    [         4]       {1x2 cell}

>> celldisp(A)

A{1,1} =

     1     2     3
     4     5     6



A{2,1} =

     4



A{1,2} =

hello world


A{2,2}{1} =

red


A{2,2}{2} =

    10    11    12
```

Space or comma defines the next column and semicolon defines the next row. This works exactly the same way as in normal arrays. To display cell array `A` we have to use the `celldisp(A)` function. To access a specific cell of a cell array the curly brackets are used as in the next example.

```
>> A{1,1}

ans =

     7     8     9
    10    11    12

>> A{1,2}
```

```
ans =

hello world

>> A{2,2}

ans =

    'red'    [1x3 double]
```

To access a specific element of any data type inside a cell, we will use the equivalent operator immediately after the curly brackets. If it is an array or a string we will use round brackets `()`, if it is another cell we will use curly brackets `{}`.

```
>> A{1,1}

ans =

     1     2     3
     4     5     6

>> A{1,1}(2,2)

ans =

     5

>> A{2,2}

ans =

    'red'    [1x3 double]

>> A{2,2}{2}([1 3])

ans =
    10    12
```

It is also possible to construct arrays of cell arrays and cell arrays of cell arrays.

```
>> {A ; A}

ans =

    {2x2 cell}
    {2x2 cell}

>> [A ; A]

ans =

    [2x3 double]    'hello world'
    [         4]       {1x2 cell}
    [2x3 double]    'hello world'
    [         4]       {1x2 cell}
```

28.3 Structures

Another method of having multiple data types within a single variable is to use structures. Structures have a field and each field can hold any data type. A field is defined after the full stop(.) .

```
>> data.the_values = [1 2 3; 4 5 6; 7 8 9];
>> data.the_max = max(max(data.the_values));
>> data.the_min = min(min(data.the_values));
>> data.the_mean = mean(mean(data.the_values));
>> data

data =

    the_values: [3x3 double]
```

```
       the_max: 9
       the_min: 1
      the_mean: 5
```

It is also possible to build arrays of structures

```
>> data(2).the_values = randn(2,5);
>> data(2).the_max = max(max(data(2).the_values));
>> data(2).the_min = min(min(data(2).the_values));
>> data(2).the_mean = mean(mean(data(2).the_values));
>> data

data =

1x2 struct array with fields:
    the_values
    the_max
    the_min
    the_mean
```

To access a particular element of a field of the structure, we use the standard notation as follows:

```
>> data(1).the_values

ans =

     1     2     3
     4     5     6
     7     8     9

>> data(1).the_values(3,2)

ans =

     8
```

We can construct arrays and cell arrays of all corresponding elements of a field using the square brackets `[]` or the curly brackets `{}`, respectfully.

```
>> [data.the_max], [data.the_mean]

ans =

    9.0000    2.1832


ans =

    5.0000    0.2310

>> {data.the_max}, {data.the_mean}

ans =

    [9]    [2.1832]


ans =

    [5]    [0.2310]
```

The difference between cell arrays and structures is mainly a conceptual one. Structures are more organized because of the field names. Cell arrays can be very useful because all the different cells can be accessed by number, which makes them easy to use within a loop.

29 Input/Output (I/O)

We have already been introduced to the `disp(a)` function. `disp(a)` takes one argument, which can be an array of numbers or a string.

```
>> disp('a string')
a string
>> disp([1 2 3])
     1     2     3
>> a_str = 'a string';
>> disp(a_str)
a string
>> a = [1 2 3];
>> disp(a)
     1     2     3
>> disp([a_str, 'and ' , num2str(a)])
a stringand 1  2  3
```

In the last example we have combined two strings and a numerical variable, which was converted into a string using `num2str`. To obtain an input from the user, we can use the `input(a)`, where `a` is string. The user can input any kind of data.

```
>> A = [1 2 3];
>> B = input('Enter a matrix: ');
Enter a matrix: [100:100:300]
>> A + B

ans =

   101   202   303

>> B = input('Enter a matrix: ');
Enter a matrix: A
>> A + B

ans =

     2     4     6
```

30 Formatted Input/Output

For formatted input/output and general file handling we have `fopen`, `fclose`, `fread`, `fwrite`, `fprintf`, `fscanf`, `sprintf`, `sscanf`, `ferror`, `feof` and `fseak`. All these functions are equivalent to the programming language ANSI C. These functions will prove very useful when reading data from a file or writing data to a file.

Function	**Meaning**
`fopen('filename','flag')`	Open a file
`fclose(fid)`	Close a file
`fread(fid)`	Read binary data from a file
`fwrite(fid,A,'precision')`	Write binary data `A` to a file
`fprintf(fid,format,A)`	Write formatted data to a file
`fscanf(fid,format,A)`	Read formatted data from a file
`sprintf(format,A)`	Write formatted data `A` to a string
`sscanf(s,format,A)`	Read string `s` under format control
`ferror(fid)`	Query about file I/O errors
`feof(fid)`	Test for end of file
`fseek(fid,offset,origin)`	Set the file position indicator

The functions `fopen('filename','flag')` and `fclose(fid)` open and close a file. To be able to read data from a file, a file identifier (fid) is required. fid is returned from `fid=fopen('filename')`. Note that `'filename'` is a string. The second argument of `fopen('filename','flag')` is a flag determining how we are going to process the file we have opened (read only, write,...). A file should be closed as soon as we are done reading or manipulating data from this file. To close the file we will use `fclose(fid)`.

The function `fwrite(fid,A,'precision')` can be used to write data in a file. The `fid` is obtained from the `fopen` function, `A` is the data to be written and `'precision'` is a string defining the number of bits to be written. `'precision'` defines the type of data. In the following example we will use `'char'` to write characters on to a file. For more alternatives for the `'precision'` string look in Matlab's help.

```
>> a = [1 2 3 8 4 5];
>> fid = fopen('some_data_1.txt','w');
>> fwrite(fid,num2str(a),'char');
>> fclose(fid);
```

In the call of the `fopen` function the flag `'w'` was used. This defines that the file will be opened, or created if it does not exist, for reading and writing discarding any data stored in that file. We have converted the numerical variable `a` to a string using `num2str`, in order to be compatible with the `'char'` type. The previous example created a file with the following contents:

```
1 2 3 8 4 5
```

To read in data from a file, we can use `fread(fid)`. Note that the flag `'w'` in the `fopen` function call should not be used for reading files as it will delete all contents of the file. If the second argument of `fopen` is omitted, a file will be opened for reading only. This is exactly the same to the `'r'` flag. Next is a simple example that reads in numbers from the file `some_data_1.txt` created in the previous example.

```
1 2 3 8 4 5
```

```
>> fid = fopen('some_data_1.txt');
>> A = fread(fid);
>> fclose(fid);
>> b = char(A');
>> b

b =

1 2 3 8 4 5

>> b_num = str2num(b);
>> b_num

b_num =

     1     2     3     8     4     5

>> b_num + 3

ans =

     4     5     6    11     7     8
```

It is important to close the file, using the `fclose(fid)` function, as soon as we are done reading the data from the file to avoid any unwanted changes on it. Note that `A` (returned from `fread(fid)`) has to be converted from binary using `char` and then from characters to numbers using `str2num(a)` in order to process it. The command `fread` is capable of reading data files with more than one row. The commands are exactly the same as before and the data file named `some_data_2.txt` has the following contents:

```
1 2 33 8 4 5
9 12 23 72 51 98
23 3 1 5 1 17

>> fid = fopen('some_data_2.txt');
>> A = fread(fid);
>> fclose(fid);
>> B = char(A');
>> B = str2num(B);
>> B

B =

     1     2    33     8     4     5
     9    12    23    72    51    98
    23     3     1     5     1    17
```

The function `fscanf(fid,format,A)` can be used instead of `fread` to read data from a file. It offers a lot more control in what is being read. `fid` is the file identifier, `format` is a string defining the format of the data. We will only consider the main entries to `format`, width and precision field, and conversion character. Width and precision field define the minimum number of digits to be read left and right from the decimal point respectively. A typical string for `format` is `'%6.3f'`, the percentage sign denotes the start of the conversion specification, 6 is the minimum number of digits to be read, 3 is the number of digits to be read on the right of the decimal point and `f` implies fixed-point notation. We can also use `i` for signed integers and `s` for characters. The final argument `A` defines the size of what we are reading. In the next example we will read the file `some_data_1.txt` with the same contents as before.

```
1 2 3 8 4 5

>> fid = fopen('some_data_1.txt');
>> a_1 = fscanf(fid,'%i',1)

a_1 =
     1

>> a_2 = fscanf(fid,'%i',1)

a_2 =

     2

>> a_3 = fscanf(fid,'%i',1)

a_3 =

     3

>> a_4 = fscanf(fid,'%i',1)

a_4 =

     8

>> a_5 = fscanf(fid,'%i',1)

a_5 =

     4

>> a_6 = fscanf(fid,'%i',1)

a_6 =

     5

>> fclose(fid);
```

Every time the `fscanf` function is called the file position indicator moves one position, thus the next number on the file is read. The `%i` conversion character indicates that we are reading an integer. To simplify the code we can put this in a loop and read the data file in to an array.

```
>> fid = fopen('some_data_1.txt');
>> for m=1:6
a(m) = fscanf(fid,'%i',1);
end
>> fclose(fid);
>> a

a =

     1     2     3     8     4     5
```

This approach has the disadvantage that we have to know in advance how many numbers we are reading from the data file. We can use the function `feof(fid)` and test when the file position indicator has reached the end of the file. This function returns 1 if the file position indicator is at the end of the file and 0 otherwise.

```
>> the_counter = 0;
>> fid = fopen('some_data_1.txt');
>> while ~feof(fid)
the_counter = the_counter+1;
a(the_counter) = fscanf(fid,'%i',1);
end
>> fclose(fid);
>> a

a =

     1     2     3     8     4     5
```

In the same way we can read the file `some_data_2.txt`, which contains more than one row.

```
>> the_counter = 0;
>> fid = fopen('some_data_2.txt');
>> while ~feof(fid)
the_counter = the_counter+1;
```

```
a(the_counter) = fscanf(fid,'%i',1);
end
>> fclose(fid);
>> a

a =

  Columns 1 through 10

     1     2    33     8     4     5     9    12    23    72

  Columns 11 through 18

    51    98    23     3     1     5     1    17
```

If we wish to read the data from the file in to a matrix, we will need to know the number of rows in the data file.

```
>> fid = fopen('some_data_2.txt');
>> a = fscanf(fid,'%i',[3 inf]);
>> fclose(fid);
>> a

a =

     1     8     9    72    23     5
     2     4    12    51     3     1
    33     5    23    98     1    17
```

The `inf` in the `fscanf` function call stands for infinite, and it is used in order to read as many columns as there are in file.

The next example is a function that reads data from a file containing the names of the columns and rows and the data and loads them in to three arrays. The function is defined as follows:

```
function [column_name, row_name, data] = read_data(filename)
% [column_name row_name data] = read_data(filename)
% takes a string (filename) for the name
% and reads in the data in to three arrays. column_name is a
% string containing the names of the columns, row_name is a
% string containing the names of the rows and data is
% an array containing the data

fid = fopen(filename);

column_names_total =  fscanf(fid, '%s',1);

start_pos = 0; end_pos = 0; count = 0;
for m=1:size(column_names_total,2)
    if column_names_total(m) == ','
        count = count + 1;
        start_pos = end_pos + 1;
        end_pos = m;
        column_name(count,1:end_pos-start_pos)
        = column_names_total(start_pos:end_pos-1);
    end
end
nr_col = count;

count = 0;
while ~ feof(fid)
    count = count + 1;
    temp = fscanf(fid, '%s', 1);
    row_name(count,1:length(temp)) = temp;
    data(count,:) = fscanf(fid,'%g',nr_col)';
end

fclose(fid);
```

The `%s` and `%g` conversion characters are used for characters and floating-point numbers respectively. The latter is capable of reading both integer and floating-point numbers.

Given the following data file called `grades.txt`:

```
3C71,4C65,3C24,4C54,2C44,4D98,3C70,3C65,
John 58 48 74 58 76 43 65 68
```

```
Bob 72 63 59 51 60 54 62 52
Jane 58 74 84 81 76 59 76 63
Mark 45 51 73 75 74 61 52 59
Lisa 47 42 51 53 48 57 64 55
Dan 55 57 75 61 68 49 51 72
Phil 59 61 63 65 72 74 73 87
Daisy 77 81 79 75 71 70 76 84
Mary 56 61 55 68 64 78 72 71
Peter 54 68 81 47 49 51 58 59
Anna 81 73 69 63 73 71 62 55
Jason 43 50 52 41 38 33 48 37
```

We can type on the command prompt, which will display the following:

```
>> [column_name, row_name, data] = read_data('grades.txt');
>> whos
  Name               Size                   Bytes  Class

  column_name        8x4                       64  char array
  data              12x8                      768  double array
  row_name          12x5                      120  char array

Grand total is 188 elements using 952 bytes
```

The function `fprintf(fid,format,A)` can be used for writing data to a file or the screen. This function can take 3 arguments in the following syntax `fprintf(fid,format,A)`. `fid` and `format` have been introduced previously. The final argument `A` are the variables to be printed. In the case of reading in data (`fscanf(fid,format,A)`, `sscanf(str,format,A)`) `fid` and `format` are used in the same way. The only difference in the arguments between `fprintf` and `fscanf` is the third argument `A`. In the case of `fscanf` `A` defines the shape of the data we are reading in, while in `fprintf` `A` is the variable to be printed.

Continuing from the previous example of reading data in from a file, we can output maxima, minima and means of the data to the screen or a file. Next is the `print_data_screen` function:

```
function print_data_screen(column_name, row_name, data_row)
```

```
% function print_data_screen(column_name, row_name, data_row)
% prints data on the screen

fid = 1;
disp('------------------------------------------------------------');
for m=1:size(data_row,2)
    the_mean = mean(data_row(:,m));
    the_max = max(data_row(:,m));
    the_min = min(data_row(:,m));
    fprintf(fid,'Subject : %6s   ',column_name(m,:) );
    fprintf(fid,'Average:  %.2f  Best: %g  Worst:
                 %g \n', the_mean, the_max, the_min);
end

disp('------------------------------------------------------------');

for m=1:size(data_row,1)
    the_mean = mean(data_row(m,:));
    the_max = max(data_row(m,:));
    the_min = min(data_row(m,:));
    fprintf(fid,'Student : %6s   ',row_name(m,:) );
    fprintf(fid,'Average:  %.2f  Best: %g  Worst:
                 %g \n', the_mean, the_max, the_min);
end

disp('------------------------------------------------------------');
```

Note that the `\n` in the `fprintf` function is the symbol for new line. On the command prompt we will type:

```
>> print_data_screen(column_name, row_name, data);
-------------------------------------------------------
Subject :   3C71   Average:  58.75  Best: 81  Worst: 43
Subject :   4C65   Average:  60.75  Best: 81  Worst: 42
Subject :   3C24   Average:  67.92  Best: 84  Worst: 51
Subject :   4C54   Average:  61.50  Best: 81  Worst: 41
Subject :   2C44   Average:  64.08  Best: 76  Worst: 38
Subject :   4D98   Average:  58.33  Best: 78  Worst: 33
Subject :   3C70   Average:  63.25  Best: 76  Worst: 48
Subject :   3C65   Average:  63.50  Best: 87  Worst: 37
-------------------------------------------------------
Student :   John   Average:  61.25  Best: 76  Worst: 43
Student :    Bob   Average:  59.13  Best: 72  Worst: 51
Student :   Jane   Average:  71.38  Best: 84  Worst: 58
Student :   Mark   Average:  61.25  Best: 75  Worst: 45
Student :   Lisa   Average:  52.13  Best: 64  Worst: 42
Student :    Dan   Average:  61.00  Best: 75  Worst: 49
Student :   Phil   Average:  69.25  Best: 87  Worst: 59
Student :  Daisy   Average:  76.63  Best: 84  Worst: 70
Student :   Mary   Average:  65.63  Best: 78  Worst: 55
Student :  Peter   Average:  58.38  Best: 81  Worst: 47
Student :   Anna   Average:  68.38  Best: 81  Worst: 55
Student :  Jason   Average:  42.75  Best: 52  Worst: 33
-------------------------------------------------------
```

To output these tables on to a file we can modify the `print_data_screen` function to accept a filename as an argument. This is defined as `print_data_file`.

```
function print_data_file(column_name, row_name, data_row,filename)
% function print_data_file(column_name, row_name, data_row) prints
% data on a file

fid = fopen(filename,'w'); % 'w' used for writing in to a file

for m=1:size(data_row,2)
    the_mean = mean(data_row(:,m));
    the_max = max(data_row(:,m));
    the_min = min(data_row(:,m));
    fprintf(fid,'Subject : %6s   ',column_name(m,:) );
    fprintf(fid,'Average:  %.2f  Best: %g  Worst:
                 %g \n', the_mean, the_max, the_min);
end



for m=1:size(data_row,1)
```

```
    the_mean = mean(data_row(m,:));
    the_max = max(data_row(m,:));
    the_min = min(data_row(m,:));
    fprintf(fid,'Student : %6s   ',row_name(m,:) );
    fprintf(fid,'Average:  %.2f  Best: %g  Worst:
                 %g \n', the_mean, the_max, the_min);
end
fclose(fid);
```

We will type on the command prompt:

```
>> print_data_file(column_name, row_name, data,'grade_results.txt');
```

This will generate a file called `grade_results.txt` in the current directory. Another example of output to the screen is the following script called `cel2far`, which converts temperatures from Celsius to Fahreneit and uses `fprintf(fid,format,A)` to print numbers to the screen.

```
bounds = input('Enter start temp, end temp, increment [s e i] ');

celsius = [bounds(1):bounds(3):bounds(2)];

fahr = (celsius .* 1.8) + 32;

temptable = [celsius ; fahr];

disp('Temperature chart');

disp('-------------');

for pair = temptable

    fprintf(1,'%5.1fC %5.1fF\n', pair(1), pair(2));

end

disp('-------------');
```

Note that `fid` is equal to 1 as in `print_data_screen` example, when the `fprintf(fid,format,A)` is used. 1 is reserved for printing to the screen. This function can easily be converted to write in a file by using `fopen(filename)`, obtaining a fid and then using that in `fprintf(fid,format,A)`. On the command prompt we will type `cel2far` to start the script.

```
>> cel2far
Enter start temp, end temp, increment [s e i] [0 50 10]
Temperature chart
-------------
  0.0C  32.0F
 10.0C  50.0F
 20.0C  68.0F
 30.0C  86.0F
 40.0C 104.0F
 50.0C 122.0F
-------------
```

Next is a script file with a modified version of `cel2far` to output the chart on to a file.

```
bounds = input('Enter start temp, end temp, increment [s e i] ');

celsius = [bounds(1):bounds(3):bounds(2)];

fahr = (celsius .* 1.8) + 32;

temptable = [celsius ; fahr];

fid = fopen('C:\cel2far_data.txt', 'w');

for pair = temptable

    fprintf(fid,'%5.1f %5.1f\n', pair(1), pair(2));

end

fclose(fid);
```

From the command prompt we can read the data in the following way.

```
>> fid = fopen('cel2far_data.txt');
>> A = fscanf(fid, '%g %g', [2 inf]);
>> A'

ans =

     0    32
    10    50
    20    68
    30    86
    40   104
    50   122

>> fclose(fid);
```

Similarly to a previous example we have used `[2 inf]` as the third argument of `fscanf(fid,format,A)` for reading in the data. This will read 2 rows and infinite number of columns. The function `string = sprintf(format, A)` works exactly the same way as `fprintf` with the exception that it writes to a string and therefore does not require a fid. `sscanf` also works very similarly to `fscanf`.

```
>> str  = sprintf('%5.0f ',[10:17])

str =

   10     11     12     13     14     15     16     17

>> A = sscanf(str, '%f');
>> A'

ans =

    10    11    12    13    14    15    16    17

>> A = sscanf(str, '%f',[2 4]);
>> A

A =

    10    12    14    16
    11    13    15    17
```

In this example we have read data is in 2 rows and 4 columns. If we want to save variables that are already on the workspace we can use the function `save('filename','var1','var2',...)`. Remember that `'filename'` and the variables are strings. If the variables `var1,var2, ...` are omitted, then all variables on the workspace will be saved.

```
>> A = [1 2 3]; s = 'hello'; B = [1 2 ; 3 4];
>> save('alles.mat');
>> save('some.mat', 's', 'B');
>> save('B.mat', 'B');
>> whos -file alles.mat
  Name      Size                   Bytes  Class

  A         1x3                       24  double array
  B         2x2                       32  double array
  s         1x5                       10  char array

Grand total is 12 elements using 66 bytes

>> whos -file some.mat
  Name      Size                   Bytes  Class

  B         2x2                       32  double array
  s         1x5                       10  char array

Grand total is 9 elements using 42 bytes

>> whos -file B.mat
  Name      Size                   Bytes  Class
  B         2x2                      32  double array

Grand total is 4 elements using 32 bytes
```

To load variables that have been saved using `save` we can use `load`. The syntax of `load('filename','var1','var2',...)` is very similar to `save`.

```
>> load('B.mat');
>> B

B =

     1     2
     3     4

>> clear all;
>> load('alles.mat');
>> A, s, B

A =

     1     2     3


s =

hello


B =

     1     2
     3     4

>> clear all;
>> load('alles.mat','B', 's');
>> s, B
```

```
s =

hello


B =

     1     2
     3     4
```

We can use `load` to read all the variables from a file into a structure.

```
>> c = load('alles.mat')

c =

    B: [2x2 double]
    s: 'hello'
    A: [1 2 3]

>> c.B, c.s

ans =

     1     2
     3     4


ans =

hello
```

`load` can also be used to read data file. Consider a file called `cel2far_data2.txt`, which has the following contents.

```
0.0  32.0
10.0  50.0
20.0  68.0
```

```
30.0  86.0
40.0 104.0
50.0 122.0
```

Then we can load it on to a variable `d` by typing the following on the command prompt.

```
>> d = load('cel2far_data2.txt')

d =

     0    32
    10    50
    20    68
    30    86
    40   104
    50   122
```

There is a number of available functions to read specialized formats, e.g. images.

```
>> copen = imread('copenhagen.jpg');
>> copen_inv = uint8(256 - double(copen));
>> imwrite(copen_inv, 'copenhagen_inv.tif', 'TIFF');
```

Images are loaded and saved as `uint8`. To be able to manipulate the image we need to convert it to `double`. To write an image in to a file we need to convert it back to `uint8`.

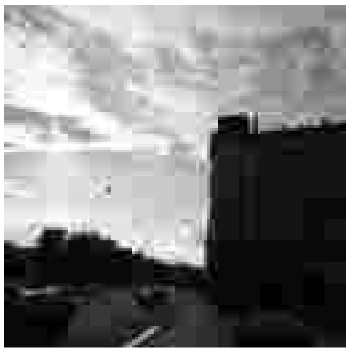


Figure 35: Copenhagen image (copenhagen.jpg)

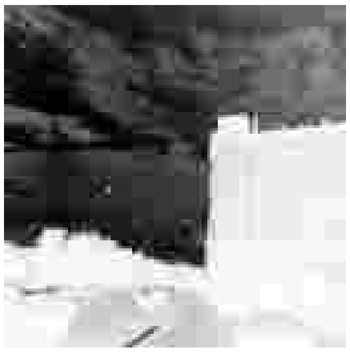


Figure 36: Inverted Copenhagen image (copenhagen_inv.tiff)

31 Summary table of functions

Function	p.	Meaning
`<`	61	Less than
`<=`	61	Less than or equal to
`>`	61	Greater than
`>=`	61	Greater than or equal to
`==`	61	Equal to
`~=`	61	Not equal to
`find(A)`	62	Find indices of `A` that are nonzero
`&`, `and(A,B)`	63	AND
`|`, `or(A,B)`	63	inclusive OR
`xor(A,B)`	63	exclusive OR
`~`, `not(A)`	63	NOT
`mod(a,b)`	65	Modulus division of `a` and `b`
`disp(a)`	66	Display `a` to the screen
`for - end`	68	Execute statements a specified number of times
`reshape(A,r,c)`	69	Reshapes `A` in to `r` rows and `c` columns
`mean(a)`	69	Calculate the mean of the array `a`
`sum(a)`	69	Sum the elements of the array `a`
`length(a)`	69	Calculate the size of the largest dimension of `a`
`while - end`	70	Execute statements while some condition is true
`if - else - end`	72	Conditionally execute statements
`switch - case - end`	75	Conditionally execute statements
`format(type)`	76	Change the display format of numbers to `type`
`double(A),uint8(A),...`	77	Change the type of `A`
`eval(a)`	82	Execute a Matlab expression held in `a`
`str2num(a)`	82	Convert string `a` to a number
`num2str(a)`	83	Convert a number `a` to a string
`{}`	83	Cell array constructor
`celldisp(A)`	84	Display cell array
`.`	86	Defines the field in a structure
`input(a)`	89	User input
`fopen('filename','flag')`	90	Open a file
`fclose(fid)`	90	Close a file
`fread(fid)`	90	Read binary data from a file
`fwrite(fid,A,'precision')`	90	Write binary data `A` to a file
`fprintf(fid,format,A)`	90	Write formatted data to a file
`fscanf(fid,format)`	90	Read formatted data from a file
`sprintf(format,A)`	90	Write formatted data `A` to a string
`sscanf(s,format)`	90	Read string `s` under format control
`ferror(fid)`	90	Query about file I/O errors
`feof(fid)`	90	Test for end of file
`fseek(fid,offset,origin)`	90	Set the file position indicator
`save('filename', 'var1',...)`	103	Saves `var1,...` in to a file
`load('filename','var1',...)`	104	Load `var1` from a file
`imread('filename')`	106	Read an image from a file
`imwrite(A,'filename',format)`	106	Write an image `A` to a file

32 Lab exercises 2

Programming exercises 1

A. Given an array `r`:

```
>> r

r =

     9     1     4     6     8
```

1. Write a `for` loop that will display 5 times the sentence `'This sentence is being repeated'`. Use the function `disp` to display the sentence. The output should look like this:

```
This sentence is being repeated
This sentence is being repeated
This sentence is being repeated
This sentence is being repeated
This sentence is being repeated
```

2. Write a `for` loop that will display 5 times the sentence `'The number of repetitions of this sentence is'` followed by how many times it has been repeated. Use the `disp` and the `num2str` functions. The output should look like this:

```
The number of repetitions of this sentence is 1
The number of repetitions of this sentence is 2
The number of repetitions of this sentence is 3
The number of repetitions of this sentence is 4
The number of repetitions of this sentence is 5
```

3. Write a `for` loop that will display 5 times the sentence `'The number of repetitions of this sentence left is'` followed by how many times it should be repeated more. Use the `disp` and the `num2str` functions. The output should look like this:

```
The number of repetitions of this sentence left is 4
The number of repetitions of this sentence left is 3
The number of repetitions of this sentence left is 2
The number of repetitions of this sentence left is 1
The number of repetitions of this sentence left is 0
```

4. Write a `for` loop that will display all the elements of the array `r` one at a time (starting with the first element (`9`) and finishing with the last element (`8`)).

5. Write a `for` loop that will display all the elements of the array `r` one at a time in reverse order (starting with the last element (`8`) and finishing with the first one (`9`)).

6. Write a `for` loop that will display elements of `r` that are larger than 5. Use `if-else` to check if an element of `r` is larger than 5.

7. Repeat the previous using a `while` loop instead of a `for` loop.

B. Given vector `a` and matrix `A` as follows:

```
>> a

a =

     4     1     3     2     1     0     7

>> A

A =

     3     4     5     2     3     6
    12    95    23     0    29    39
    57    64    72    41     8    91
    47    28    31    82    84    37
    28    40    39    45    64    69
```

1. Use a `for` loop to check if any element of vector `a` is larger than 3.
2. Calculate the mean of vector `a`.
3. Use a `for` loop to check if any element of vector `a` differs more than 2 from the mean.
4. Use two `for` loops to check if any element of matrix `A` is larger than 5.
5. Calculate the mean of matrix `A`.
6. Use two `for` loops to check if any element of matrix `A` differs more than 50 from the mean.

C. Given the same matrix `A` as before.

1. Write code that will find all numbers in `A` larger than 5 and smaller than 90. This has to be done in one statement. Hint: use the logical operators to combine statements and the `find` function.
2. Write code that will find all numbers in `A` that are larger than 5 and smaller than 90 **or** they differ less than 50 from the mean.

D. Given matrix `A` as before

1. Use `for` loops to display all the elements of `A` using the `disp()` command. Elements should be displayed with their corresponding locations on the matrix. This should be in the following format:

```
Row :1 Column :1 Value :3
Row :1 Column :2 Value :4
Row :1 Column :3 Value :5
Row :1 Column :4 Value :2
Row :1 Column :5 Value :3
Row :1 Column :6 Value :6
Row :2 Column :1 Value :12
Row :2 Column :2 Value :95
Row :2 Column :3 Value :23
Row :2 Column :4 Value :0
Row :2 Column :5 Value :29
Row :2 Column :6 Value :39
...
```

2. Use `for` loops and `if-else` to find all elements that are larger than 5 and smaller than 90 **or** they differ less than 50 from the mean. Note that you should not use the logical operators in this exercise. Can you do this using `switch-case` instead of `if-else`?

3. Same as before but use `while` loops instead of `for` loops.

E. Given `A` as before and a function defined in a string as follows $y = x_1^2 + 4x_2 + \frac{x_3}{2} + x_4 x_5 + x_6$. The index denotes the columns on the matrix, i.e. x_1 takes values from the first column of `A`, x_2 takes values from the second column of A, etc.

1. Define function y in a string. Using the `eval(a)` write a loop that will calculate this function on each row of matrix `A`.

2. Given functions $y_1 = x_1^2 + 4x_2 + \frac{x_3}{2} + x_4 x_5 + x_6$, $y_2 = x_1 x_2 + x_3 x_4 + \frac{x_5}{3} + x_6^2$ and $y_3 = x_1 x_2 x_3 + 5x_4 + \frac{x_5 x_6}{2}$ save them as strings in a structure named `functs`. It should have three fields `y_1`, `y_2` and `y_3`, one for each function. Write a loop that will calculate all functions on each row of `A`.

3. Given functions y_1, y_2 and y_3 as before, write a loop that saves results in to three fields of `functs` structure as follows:

   ```
   low       if result >=10 and result < 1000
   mid       if result >=1000 and result < 10000
   high      if result >=10000 and result < 1000000
   ```

   Use `if-else` for this exercise.

4. Same as before, but use `switch-case` instead of `if-else`. Hint you might need to use the commands `floor(a)` and `log10(a)`.

F. Create a file named functsdata.txt containing the definitions of the functions (y_1, y_2 and y_3) in the first row separated by commas without spaces and underneath the elements of matrix `A`. This file should look like this:

```
x(1)^2+4*x(2)+x(3)/2+x(4)*x(5)+x(6),
                            x(1)*x(2)+x(3)*x(4)+x(5)/3+x(6)^2,
                                   x(1)*x(2)*x(3)+5*x(4)+(x(5)*x(6))/2,
3     4     5     2     3     6
12    95    23     0    29    39
57    64    72    41     8    91
47    28    31    82    84    37
28    40    39    45    64    69
```

1. Read in the data from the file using `fscanf`. Functions and the data matrix should be read in a cell array.

2. Using the code you written for the previous exercise evaluate all the functions for each row of the data matrix and separate the results in to three types as in the previous exercise.

3. Print the results using `fprintf` on to the screen. You should print out the definition of the function, the row number and the type of the result. The type of the result should be in words (low,mid,high).

   ```
   Function : x(1)^2 + 4 * x(2) + x(3)/2 + x(4)*x(5) + x(6)
    Row :  1 Type :  low
   Function : x(1)*x(2)+x(3)*x(4)+x(5)/3+x(6)^2
   ```

```
 Row :  1 Type :  low
Function : x(1)*x(2)*x(3)+5*x(4)+(x(5)*x(6))/2
 Row :  1 Type :  low
Function : x(1)^2 + 4 * x(2) + x(3)/2 + x(4)*x(5) + x(6)
 Row :  2 Type :   low
Function : x(1)*x(2)+x(3)*x(4)+x(5)/3+x(6)^2
 Row :  2 Type :   mid
 ...
```

4. Print the results in a file named results.txt.
5. Print the results in to three files named results1.txt, results2.txt and results3.txt. Each file should contain results of one type (low,mid,high). You might need to use the `'a'` flag in the command `fopen(filename,'a')`. This flag will open (or create if it does not exist) a file, keep the contents and append data to the end of the file.
6. Repeat the previous exercise without using any conditional operators (`if-else`,`switch-case`).

G. Given any gray scale JPEG image.

1. Read it in using `imread`
2. Rotate the image 90 degrees clockwise.
3. Write the rotated image in to a JPEG file using `imwrite`.

Questions from the lecture

1. How can we simplify the next example and avoid using a `for` loop?

```
>> x_values = [0:0.1:2*pi - 0.1];
>> for index=1:length(x_values)
y_values(index) = sin(x_values(index));
end
```

2. How can the next example be accomplished using the command `find()`?

```
>> A = zeros(10,1);
>> A(5) = 1;
>> m = 1;
>> while A(m) == 0
m = m + 1;
end
>> m

m =

     5
```

3. How can we accomplish the next function without using `if-else` and `for` loop in three statements?

```
function [div_2, div_3, not_div] = myseparate(A)
% rtn = mysort(A) takes a scalar vector as input
% and sorts it out to numbers that are divisible by 2 (div_2),
% numbers that are divisible by 3 (div_3) and
% other numbers that are not divisible by 2 or 3.
% If a number is divisible by 2 and 3 then it will go to both lists.

count_div_2 = 0; count_div_3 = 0; count_not_div = 0;

for m=1:length(A)
    if mod(A(m),2) == 0 & mod(A(m),3) == 0
```

```
            count_div_2 = count_div_2 + 1;
            div_2(count_div_2) = A(m);
            count_div_3 = count_div_3 + 1;
            div_3(count_div_3) = A(m);
        elseif mod(A(m),2) == 0
            count_div_2 = count_div_2 + 1;
            div_2(count_div_2) = A(m);
        elseif mod(A(m),3) == 0
            count_div_3 = count_div_3 + 1;
            div_3(count_div_3) = A(m);
        else
            count_not_div = count_not_div + 1;
            not_div(count_not_div) = A(m);
        end
    end
```

Programming exercises 2

Aim: The aim of this exercise is to implement linear regression with basis functions and to visualize the phenomena of overfitting.

Linear regression overview: Given a set of data:

$$\{(\boldsymbol{x}_1, y_1), (\boldsymbol{x}_2, y_2), \ldots, (\boldsymbol{x}_\ell, y_\ell)\} \tag{7}$$

where $\boldsymbol{x} = (x_1, \ldots, x_n)$ is a vector in $\Re^n$ and y is a real number. Linear regression finds a vector $\boldsymbol{u} \in \Re^n$ such that the sum of squared errors

$$\text{SSE} = \sum_{t=1}^{\ell} (y_t - \boldsymbol{u} \cdot \boldsymbol{x}_t)^2 \tag{8}$$

is minimized. This is expressible in matrix form by defining X to be the $\ell \times n$ matrix

$$X = \begin{pmatrix} x_{1,1} & x_{1,2} & \ldots & x_{1,n} \\ x_{2,1} & x_{2,2} & \ldots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{\ell,1} & x_{\ell,2} & \ldots & x_{\ell,n} \end{pmatrix}, \tag{9}$$

and defining $\boldsymbol{y}$ to be the column vector $\boldsymbol{y} = (y_1, \ldots, y_\ell)$. The vector $\boldsymbol{u}$ then minimizes

$$(X\boldsymbol{u} - \boldsymbol{y})^T (X\boldsymbol{u} - \boldsymbol{y}).$$

In linear regression with basis functions we fit the data sequence with a linear combination of basis functions $(f_1, f_2, \ldots, f_k)$. This is done by transforming the data as follows

$$\{((f_1(\boldsymbol{x}_1), \ldots, f_k(\boldsymbol{x}_1)), y_1), \ldots, ((f_1(\boldsymbol{x}_\ell), \ldots, f_k(\boldsymbol{x}_\ell)), y_\ell)\}, \tag{10}$$

and then applying linear regression above to this transformed data set. Linear regression on the transformed dataset thus finds a k-dimensional vector $\boldsymbol{u} = (u_1, \ldots, u_k)$ such that

$$\sum_{t=1}^{\ell} (y_t - \sum_{i=1}^{k} u_i f_i(\boldsymbol{x}_t))^2 \tag{11}$$

is minimized.

A common basis used in practice is a polynomial basis $\{1, x, x^2, x^3, \ldots, x^{k-1}\}$ of dimension k (order $k-1$) in the figure below we give a simple fit of four points produced by a linear ($k = 2$) and cubic ($k = 4$) polynomial.

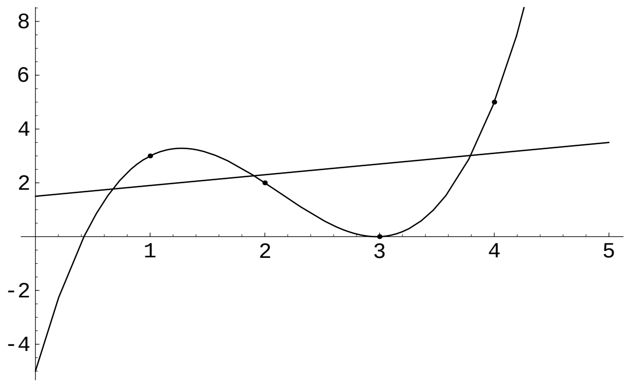


Figure 1: Data set $\{(1,3),(2,2),(3,0),(4,5)\}$ **fitted with basis** $\{1,x\}$ **and basis** $\{1,x,x^2,x^3\}$

1. For each of the polynomial bases of dimension $k = 1,2,3,4$ fit the data set of Figure 1 $\{(1,3),(2,2),(3,0),(4,5)\}$.

   (a) Produce a plot similar to Figure 1, superimposing the four different curves corresponding to each fit over the four data points. (Use the matlab commands `hold on` and `hold off` to superimpose plots)

   (b) Give the equations corresponding to the curves fitted for $k = 1,2,3$. The equation corresponding to $k = 4$ is $-5 + 15.17x - 8.5x^2 + 1.33x^3$.

   (c) For each fitted curve $k = 1,2,3,4$ give the mean square error where $\text{MSE} = \frac{\text{SSE}}{\ell}$.

2. In this part we will illustrate the phenomena of *overfitting*. First we need to define the normal distribution with mean μ and variance σ^2.

$$N(\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{12}$$

   (a) The matlab function `randn` generates random numbers with the distribution $N(0,1)$. Write a function that takes μ and σ as parameters and generates random numbers with a distribution $N(\mu,\sigma)$.

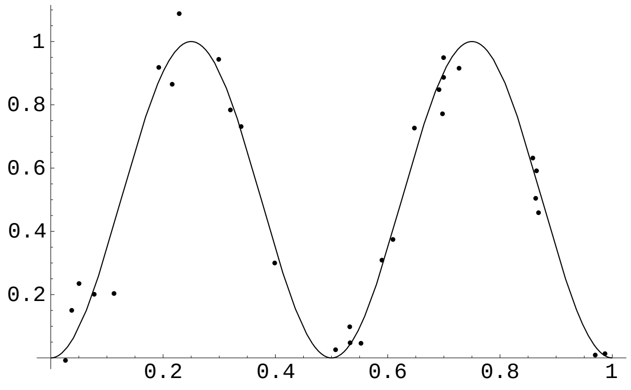
   (b) Define

$$g_\sigma(x) = \sin^2(2\pi x) + N(0,\sigma). \tag{13}$$

sample uniformly at random from the interval $[0, 1]$ 30 times creating $(x_1, \ldots, x_{30})$ and apply $g_{0.07}$ to each x creating the data set

$$S_{0.07,30} = \{(x_1, g_{0.07}(x_1)), \ldots, (x_{30}, g_{0.07}(x_{30})\}. \tag{14}$$

i. Plot the function $\sin^2(2\pi x)$ from $x = 0..1$ with the points of the above data set superimposed. The plot should resemble



ii. Fit the data set with a polynomial bases of dimension $k = 2, 5, 10, 15, 20$ plot each of these 5 curves superimposed over a plot of data points.

(c) Let the training error $\mathrm{te}_k(S)$ denote the MSE of the fitting of the data set S with polynomial basis of dimension k. Plot the log of the training error versus the polynomial dimension $k = 1, \ldots, 25$ (this should be a decreasing function).

(d) Generate a test set T of a thousand points,

$$T_{0.07,1000} = \{(x_1, g_{0.07}(x_1)), \ldots, (x_{1000}, g_{0.07}(x_{1000})\}. \tag{15}$$

Define the test error $\mathrm{tse}_k(S, T)$ to be the MSE of the test set T on the polynomial of dimension k fitted from training set S. Plot the log of the test error versus the polynomial dimension $k = 1, \ldots, 20$. Unlike the training error this is not a decreasing function. This is the phenomena of *overfitting*. Although the training error decreases with growing k the test error eventually increases since rather than fitting the function, in a loose sense, we begin to fit to the noise.

(e) For any given set of random numbers we will get slightly different training curves and test curves. It is instructive to see these curves smoothed out. For this part repeat items (*c*) and (*d*) but instead of plotting the results of a single "run" plot the average results of a 100 runs (note: plot the log(avg) rather than the avg(log)).

3. In this part we will use as a basis

$$\sin(1\pi x), \sin(2\pi x), \sin(3\pi x), \ldots, \sin(k\pi x).$$

Repeat the experiments in 2 (c-e) with the above basis.

Part IV
Lecture 3

33 Overview of Lecture 3

- Matlab performance tuning
- Set functions
- User defined functions 2
- Plotting 2
- Summary table of functions

34 Matlab performance tuning

In order to assess the performance of our code we can use Matlab's profiler. The profiler gives details in html format of how much time was spent on each function and how many times a functions was called. To enable the profiler we must type `profile on` at the beginning of our code. To produce the html report we will need to type `profile report` at the end of our code. To switch off the profiler we will type `profile off`. Next is an example that uses a user defined function `graddesc` for gradient descent. This function uses another function called `dfb`.

```
>> profile on
>> sol = graddesc('fb', 'dfb', [0.0 0.0], 0.001, 0.001);
>> profile report
>> profile off
>> profile resume % restarts profiler
>> profile clear  % clears profiler
```

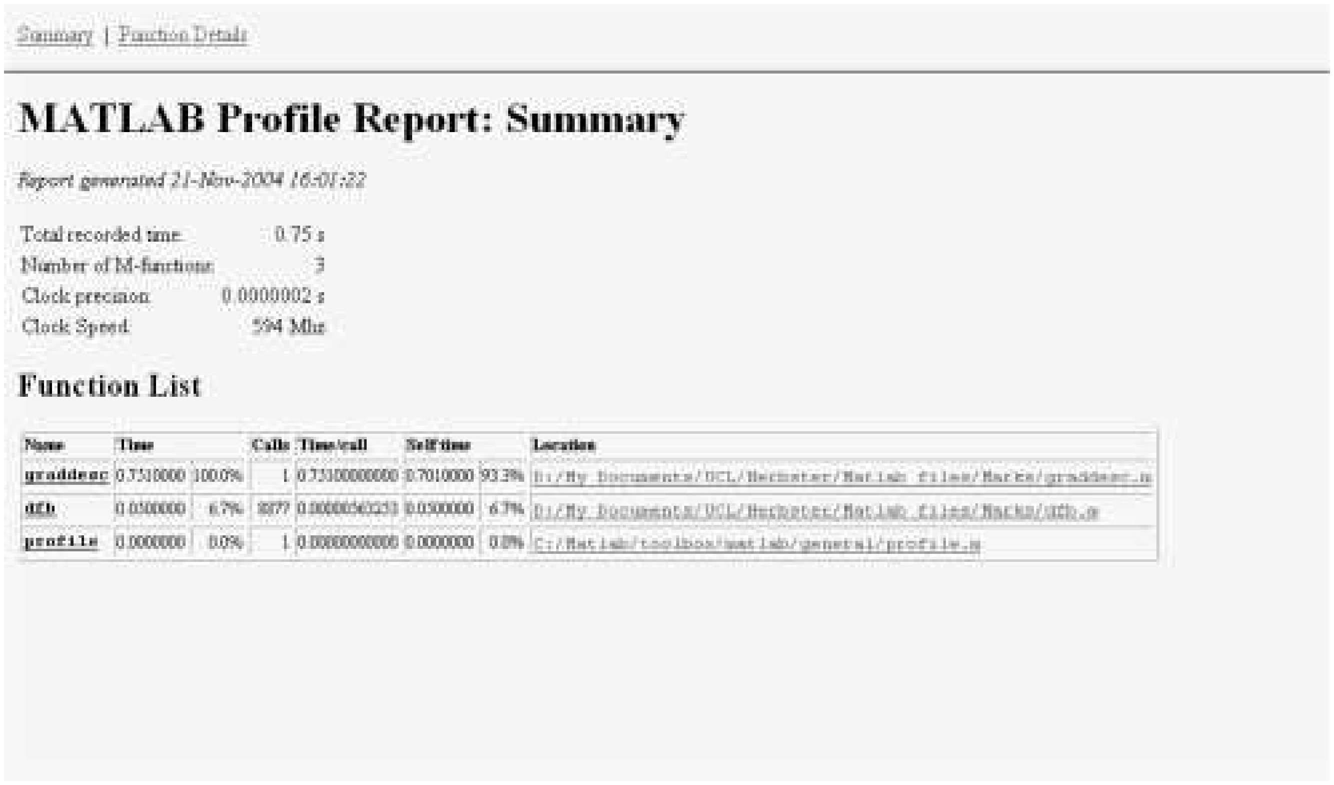
The command `profile resume` will resume the profiler and `profile clear` will clear all the data that the profiler saved. The resulting report from the profiler is shown in fig. 37. We can use this information to see how much time is spent on each function and determine inefficiencies in our code. Note that on the top of the summary report, the precision of the clock is displayed. Any time lower than that will be displayed as 0.

A simpler way to time our functions is to use a pair of the commands `tic` and `toc`.

Command	**Meaning**
`tic`	Starts timer
`toc`	Returns time

We will type `tic` immediately before our function and then type `toc` immediately after the function.

```
>> tic; a = [1:1000000]; toc

elapsed_time =

    0.1210
```


Summary | Function Details

MATLAB Profile Report: Summary

Report generated 21-Nov-2004 16:01:22

Total recorded time:	0.75 s
Number of M-functions:	3
Clock precision:	0.0000002 s
Clock Speed:	594 Mhz

Function List

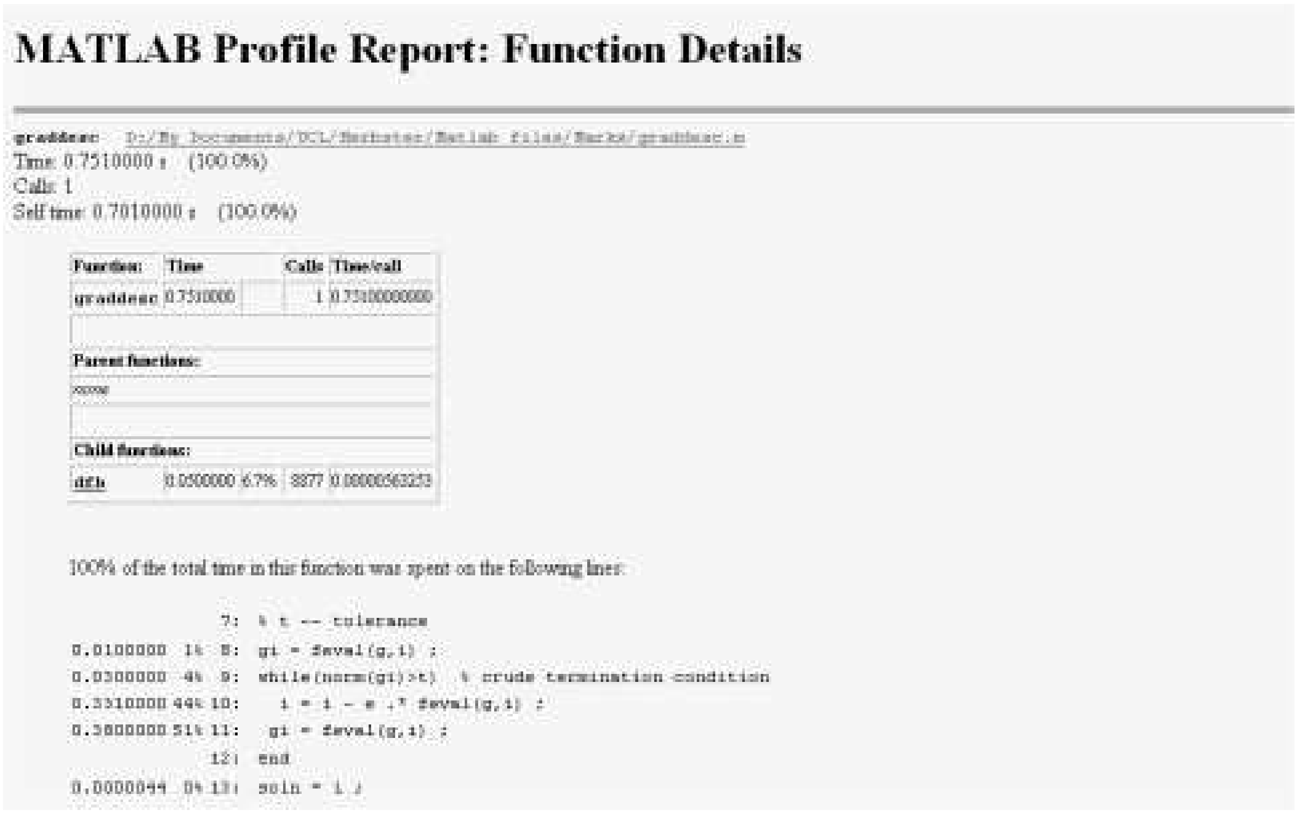
Name	Time		Calls	Time/call	Self time		Location
graddesc	0.7510000	100.0%	1	0.75100000000	0.7010000	93.3%	D:/My Documents/UCL/Herbster/Matlab files/Marks/graddesc.m
dfb	0.0500000	6.7%	8877	0.00000563233	0.0500000	6.7%	D:/My Documents/UCL/Herbster/Matlab files/Marks/dfb.m
profile	0.0000000	0.0%	1	0.00000000000	0.0000000	0.0%	C:/Matlab/toolbox/matlab/general/profile.m

Figure 37: Profile Report: Summary

Consider a simple example that calculates the sin function for 10000 points.

```
>> m = []; y_vals = []; % clears the variables
>> tic; for m=1:10000
y_vals(m) = sin(m);
end;toc

elapsed_time =

    0.8010
```

The first thing we can do to speed up our code is to preallocate arrays.



MATLAB Profile Report: Function Details

graddesc D:/My Documents/UCL/[illegible]/Matlab files/[illegible]/graddesc.m
Time: 0.7510000 s (100.0%)
Calls: 1
Self time: 0.7010000 s (100.0%)

Function:	Time		Calls	Time/call
graddesc	0.7510000		1	0.7510000000
Parent functions:				
none				
Child functions:				
dfh	0.0500000	6.7%	3877	0.0000056323

100% of the total time in this function was spent on the following lines:

```
                 7:  % t -- tolerance
0.0100000  1%  8:  gi = feval(g,i) ;
0.0300000  4%  9:  while(norm(gi)>t)  % crude termination condition
0.3310000 44% 10:    i = i - e .* feval(g,i) ;
0.3800000 51% 11:   gi = feval(g,i) ;
                12:  end
0.0000044  0% 13:  soln = i ;
```

Figure 38: Profile Report: Function details

```
>> m = []; y_vals = [];
>> tic; y_vals = zeros(1,10000); for m=1:10000
y_vals(m) = sin(m);
end;toc

elapsed_time =

    0.0700
```

The second thing and most important is to vectorize the loop.

```
>> m = []; y_vals = [];
>> tic; m = 1:10000 ; y_vals(m) = sin(m);toc
```

```
elapsed_time =

    0.0100
```

Loops in Matlab are very time consuming and should be vectorized when possible. Using array preallocation and vectorization we achieved 80 times speedup.

Method	**Time**
Simple	0.8010
Preallocation	0.0700
Preallocation and vectorization	0.0100

To further improve the performance of our code, we can use sparse matrices. A sparse matrix stores only the non-zero elements. As well as reducing memory requirements, there are special algorithms for sparse matrices which run faster than the equivalent full matrix. Sparse matrices make a noticeable difference if we use large matrices with many elements equal to zero. To initialize a sparse matrix we will use the function `sparse(a,b)`, which creates an `a` x `b` sparse matrix.

```
>> A = sparse(10,10);
>> A

A =

   All zero sparse: 10-by-10
>> B = eye(10);
>> B_spar = sparse(B);
>> whos B
  Name      Size                    Bytes  Class

  B        10x10                      800  double array

Grand total is 100 elements using 800 bytes

>> whos B_spar
```

```
  Name          Size                    Bytes  Class

  B_spar       10x10                      164  double array (sparse)

Grand total is 10 elements using 164 bytes

>> B_spar = speye(10); % A sparse identity matrix
>> C = sparse(1:10,1:10,ones(1,10))

C =

   (1,1)        1
   (2,2)        1
   (3,3)        1
   (4,4)        1
   (5,5)        1
   (6,6)        1
   (7,7)        1
   (8,8)        1
   (9,9)        1
  (10,10)       1
```

Elements of a sparse matrix can be accessed in the same way as a full matrix. If we perform an operation between a sparse matrix and a full matrix, the result will be a full matrix.

```
>> B_spar(1,1)

ans =

     1

>> B_spar(1,3)

ans =

     0
```

```
>> C = B_spar + zeros(10,10);
>> whos B_spar
  Name         Size                      Bytes  Class

  B_spar      10x10                        164  double array (sparse)

Grand total is 10 elements using 164 bytes

>> whos C
  Name      Size                      Bytes  Class

  C        10x10                        800  double array

Grand total is 100 elements using 800 bytes
```

A few useful functions for sparse matrices are listed in the following table.

Function	**Meaning**
`sparse(a,b)`	Create an `a` x `b` sparse matrix
`speye(a)`	Create an `a` x `a` sparse identity matrix
`spfun(function,A)`	Apply function to sparse matrix
`nnz(A)`	Number of non-zero elements of `A`
`full(A)`	Convert sparse matrix to full

The function `spfun(function,A)` applies a function to all the elements of a sparse matrix. `nnz(A)` counts all the non-zero elements and `full(A)` converts a sparse matrix in to a full one.

```
>> A = speye(10);
>> A_sin = spfun('sin', A);
>> nnz(A)

ans =

    10

>> A_ful = full(A);
```

The function `find(A)` is also very useful with sparse matrices as it returns all the non-zero elements.

By default all numeric operands are double precision floats. In Matlab version 7 we can use integer type (`int8`, `int16`, `int32`, `uint8`, `uint16` and `uint32`) variables. These will reduce both memory and computational requirements. If the data we are using cannot be represented with integers, we can also use single precision (by declaring variables with `single`) instead of using the default `double`. Single precision uses 32 bits instead of 64 bits of double precision. Arithmetic in Matlab 7 is only well-defined in the following two cases,

1. all operands are of the same type or
2. the operands are double precision and one other type

35 Set functions

Next is a table of useful set functions in Matlab.

Function	**Meaning**
`unique(a)`	Find unique elements of a vector `a`
`union(a,b)`	Set union of two vectors
`intersect(a,b)`	Set intersection of two vectors
`ismember(a,b)`	Detects members of a set
`setdiff(a,b)`	Set difference of two vectors

The function `unique(a)` returns all the elements that belong to `a` without repeating any of them. The function `union(a,b)` returns a vector of the union of `a` and `b`. The function `intersect(a,b)` returns all the common elements of `a` and `b`. The function `ismember(a,b)` returns a Boolean vector that is the size of `a` with 1 if the corresponding element of `a` is in `b`. The function `setdiff(a,b)` returns the elements of `a` that are not in `b`.

```
>> unique([4 2 2 4 7])

ans =

     2     4     7

>> a = [1:5], b = [7:-1:4]
```

```
a =

     1     2     3     4     5


b =

     7     6     5     4

>> union(a,b)

ans =

     1     2     3     4     5     6     7

>> intersect(a,b)

ans =

     4     5

>> ismember(a,b), a(ismember(a,b))

ans =

     0     0     0     1     1


ans =

     4     5

>> ismember(b,a), b(ismember(b,a))

ans =

     0     0     1     1
```

```
ans =

     5     4

>> setdiff(a,b), setdiff(b,a)

ans =

     1     2     3


ans =

     6     7
```

36 User defined functions 2

Variables inside functions are by default local, that means only the function, where the variables were declared, can access them and as soon as the function terminates they are deleted from memory. We can also define variables inside a function to be **global** or **persistent**. **persistent** variables can only be accessed from the function they are declared in, but they persist in memory between function calls. **global** variables are both **persistent** and they can be accessed by other functions which declare them **global**.

As an example we have created two functions `mytic` and `mytoc`:

```
function mytic

global my_time

my_time = cputime;

function rtn = mytoc

global my_time
```

```
rtn = cputime - my_time;
```

Then we can type on the command prompt:

```
>> m = [];
>> mytic;m=1:100000;sin(m);mytoc

ans =

    0.0700
```

Next is an example of a persistent variable. The next function is called `counter`

```
function rtn = counter

persistent count

if isempty(count)
    count = 1;
else
    count = count + 1;
end

rtn = count;
```

The function `isempty(A)` tests whether the variable held in `A` has been created or not. On the command prompt we will type:

```
>> counter

ans =

     1

>> counter

ans =
     2
```

We can also create functions from strings via `inline(expr_string,arg1,arg2,...)`, where `expr_string` is a string which defines the function and `arg1`, `arg2`, . . . are strings naming the arguments of the function.

```
>> h = inline('A+2*B+4*C','A','B','C')

h =

     Inline function:
     h(A,B,C) = A+2*B+4*C

>> h([1 0 ; 0 1],[1 0 ; 0 1],[0 1 ; 1 0])

ans =

     3     4
     4     3

>> g = inline('(x(1)-5).^2 + 3*(x(2)-2).^2','x')

g =

     Inline function:
     g(x) = (x(1)-5).^2 + 3*(x(2)-2).^2

>> g([1 , 2]), g([5 , 2])

ans =

    16


ans =

     0
```

The number of arguments in a function does not need to be predetermined. We can use `varargin` as an argument that means we can have a variable number of arguments of any type. To determine how many arguments the user passed to the function we will use `nargin`. Note that `nargin` counts the total number of input arguments, not just the ones corresponding to `varargin`. The following example is a function that prints any number of arguments on to the screen.

```
function print_various_arguments(head, varargin)

for m=1:nargin-1
    fprintf(1,head,m);
    disp(varargin{m});
end
```

Then on the command prompt we type:

```
>> print_various_arguments('arg %d is ',7,[1 2 3],'a string')
arg 1 is      7

arg 2 is      1     2     3

arg 3 is a string
```

The examples `print_data_screen` and `print_data_file` in the previous set of notes can be combined using `varargin`. The new function can take 3 or 4 arguments which determines whether it prints to the screen or to a file.

```
function print_data(column_name, row_name, data_row,varargin)
% function print_data(column_name, row_name, data_row,varargin)

if nargin == 3
    fid = 1;
elseif nargin == 4
    filename = varargin{1};
    fid = fopen(filename,'w');
end
```

```
if nargin == 3
    disp('-----------------------------------------------------');
end
for m=1:size(data_row,2)
    the_mean = mean(data_row(:,m));
    the_max = max(data_row(:,m));
    the_min = min(data_row(:,m));
    fprintf(fid,'Subject : %6s   ',column_name(m,:) );
    fprintf(fid,'Average:  %.2f  Best: %g  Worst:
                 %g \n', the_mean, the_max, the_min);
end
if nargin == 3
    disp('-----------------------------------------------------');
end
for m=1:size(data_row,1)
    the_mean = mean(data_row(m,:));
    the_max = max(data_row(m,:));
    the_min = min(data_row(m,:));
    fprintf(fid,'Student : %6s   ',row_name(m,:) );
    fprintf(fid,'Average:  %.2f  Best: %g  Worst:
                 %g \n', the_mean, the_max, the_min);
end
if nargin == 3
    disp('-----------------------------------------------------');
end

if nargin == 4;
    fclose(fid);
end
```

On the command prompt we can type the following to print to the screen:

```
>> print_data(column_name, row_name, data);
-------------------------------------------------------
Subject :   3C71   Average:  58.75  Best: 81  Worst: 43
Subject :   4C65   Average:  60.75  Best: 81  Worst: 42
Subject :   3C24   Average:  67.92  Best: 84  Worst: 51
Subject :   4C54   Average:  61.50  Best: 81  Worst: 41
Subject :   2C44   Average:  64.08  Best: 76  Worst: 38
Subject :   4D98   Average:  58.33  Best: 78  Worst: 33
Subject :   3C70   Average:  63.25  Best: 76  Worst: 48
Subject :   3C65   Average:  63.50  Best: 87  Worst: 37
-------------------------------------------------------
Student :   John   Average:  61.25  Best: 76  Worst: 43
Student :    Bob   Average:  59.13  Best: 72  Worst: 51
Student :   Jane   Average:  71.38  Best: 84  Worst: 58
Student :   Mark   Average:  61.25  Best: 75  Worst: 45
Student :   Lisa   Average:  52.13  Best: 64  Worst: 42
Student :    Dan   Average:  61.00  Best: 75  Worst: 49
Student :   Phil   Average:  69.25  Best: 87  Worst: 59
Student :  Daisy   Average:  76.63  Best: 84  Worst: 70
Student :   Mary   Average:  65.63  Best: 78  Worst: 55
Student :  Peter   Average:  58.38  Best: 81  Worst: 47
Student :   Anna   Average:  68.38  Best: 81  Worst: 55
Student :  Jason   Average:  42.75  Best: 52  Worst: 33
-------------------------------------------------------
```

In order to produce a file with these results we would simply type:

```
>> print_data(column_name, row_name, data,'grade_results.txt');
```

Similarly to `varargin` we can use `varargout` to return any number of variables. To count the number of variables that the user expects to be returned, that is the number of variables in square brackets on the left side of the equal sign in an assignment, we use `nargout`. Note that `nargout` counts the total number of output arguments, not just the arguments corresponding to `varargout`. Consider the following function, which can output any number of variables.

```
function [varargout] = anynumber_varout
% [varargout] = anynumber_varout

for m=1:nargout
    varargout{m} = m;
end
```

Note that the function takes no arguments as input. Its functionality is to set any number of output variables equal to their location in the square brackets. We can type on the command prompt:

```
>> [a] = anynumber_varout

a =

     1

>> [a b] = anynumber_varout

a =

     1


b =

     2

>> [a b c] = anynumber_varout

a =

     1


b =

     2


c =

     3
```

37 Plotting 2

Next is a table with some useful plotting functions.

Function	**Meaning**
`figure`	New figure window
`figure(n)`	Select figure window n
`title(string)`	Give a title to the figure window
`gtext(string)`	Add text with the mouse
`legend(string)`	Add a legend
`grid on/off`	Turn grid on/off
`subplot(m,n,p)`	Create multiple plots
`hold on/off`	Hold the current graph in the figure

`figure` opens up a new window for plotting and `figure(n)` will select the nth figure window if it exists or open a new one otherwise. `title` will define the title on the top of the figure window. The function `gtext(string)` will add text with mouse interaction. `legend(string)` will add legends. `grid on/off` turns on and off a rectangular grid on the figure window. The function `subplot(m,n,p)` will plot in a figure divided in to m by n plots in the pth position. Using the `hold` command we can overlay plots in the current figure window. When `hold on` is entered each successive `plot` function overlays the plots within the figure. When `hold off` is entered each successive `plot` function replaces the previous plot.

```
>> x = 0:0.1:10;
>> f1 = sin(x); f2 = cos(x); f3 = exp(-x); f4 = 0.028 * x.*x;
>> figure;plot(x,f1,x,f2)
```

This code will generate fig. 39.
Fig. 39 can also be generated in the following way:

```
>> figure;plot(x,f1);
>> hold on;
>> plot(x,f2);
>> hold off;
```

If we type the following set of statements, fig. 39 will be changed to fig. 40.

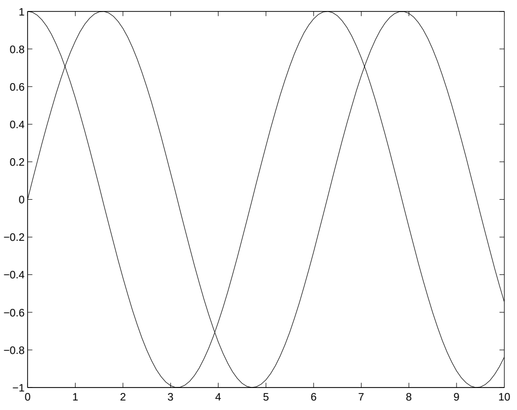


Figure 39: Sin and cos plot

```
>> title('sin(x) and cos(x)')
>> grid on
>> gtext('sin(x)')
>> legend('sin(x)', 'cos(x)')
```

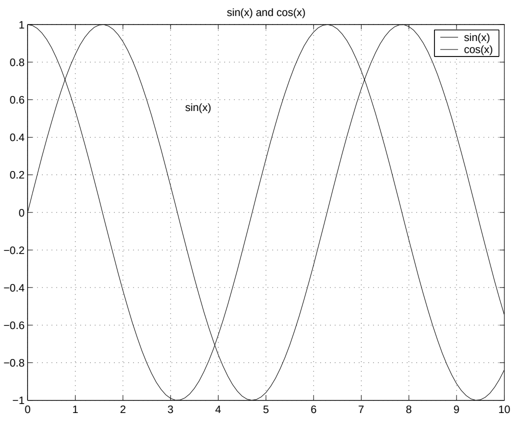


Figure 40: Sin and cos plot with text

Next is an example on the use of `subplot` .

```
>> figure
>> subplot(2,2,1);plot(x,f1);title('sin(x)')
>> subplot(2,2,2);plot(x,f2);title('cos(x)')
>> subplot(2,2,3);plot(x,f3);title('exp(-x)')
>> subplot(2,2,4);plot(x,f4);title('scaled quadratic')
```

These statements will generate the fig. 41.

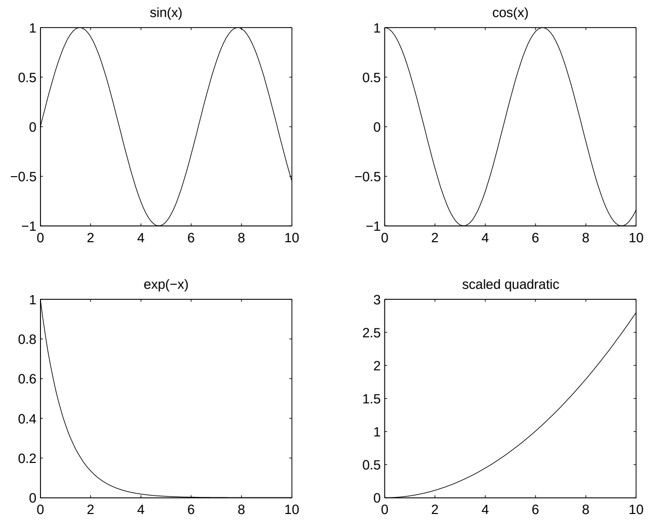
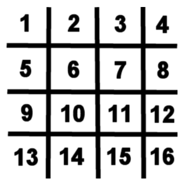


Figure 41: 4 plots in the same figure using subplot

Note that the `subplot` function counts the plots by first counting across a row and then moving to the next one (see fig. 42).



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Figure 42: Numbering of the subplots

We can also define the style of the lines in a plot by placing `linestyle` as a third argument of `plot(x,y,linestyle)`. The following table contains different line styles.

`.`	point	`-`	solid
`o`	circle	`:`	dotted
`x`	cross	`-.`	dash-dot
`+`	plus	`--`	dash-dash

The next example illustrates a number of `linestyles` (see fig. 43).

```
>> figure; plot(x,f1,'o')
>> figure; plot(x,f1,'-',x,f2,':');legend('sin(x)','cos(x)')
```

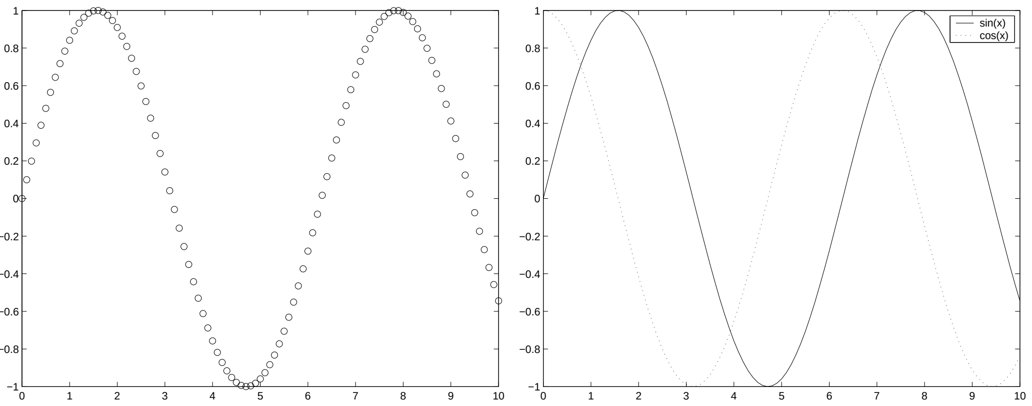


Figure 43: Plot using different line styles

A histogram plot presents a view of data by a partition into discrete ”bins”. The number of items in each bin is then plotted. For example, recall the student grades data set, we will use a histogram to see the number of students who achieved a certain grade class, in this case between 40 and 50, 50 and 60, etc. This function creates a histogram given the data and a set of bins.

```
function display_histogram(data,bins)
% display_histogram(data,bins) displays
% the data in a histogram. bins is an array
% that defines the bins for the histogram
% e.g. bins = [20 30 40 50 60]. This will create
% 4 bins.

nr_bins = length(bins) - 1;

histogrm = zeros(nr_bins,1);

for m=1:size(data,1)
    for n=1:size(data,2)
        for bin_counter=1:nr_bins
            if data(m,n)>=bins(bin_counter) & data(m,n) < bins(bin_counter+1)
                histogrm(bin_counter) = histogrm(bin_counter) + 1;
            end
        end
    end
end


x_axis = bins(1:end-1);
figure;
hold on;
for m=1:nr_bins
    line([x_axis(m); x_axis(m)], [0 ; histogrm(m)], 'LineWidth',7);
end
hold off;
```

On the command prompt we type:

```
>> display_histogram(data,[40:10:100])
```

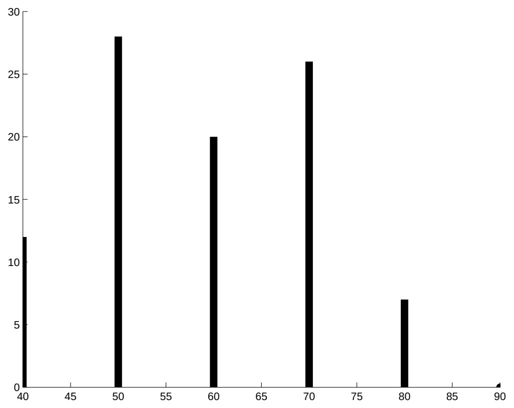


Figure 44: Histogram plot

This will create fig. 44. To make the plot clearer we have drawn horizontal lines using `line(X,Y)`, where `X` and `Y` are vectors holding the starting and ending point in x and y. After `X,Y` we can define any of the style properties of this line. In this particular example we have used `'LineWidth'`, which can also be used in the `plot` function. `'LineWidth'` is followed by a scalar defining the width of the line.

To create 3D plots we can use `meshgrid`, which generates X and Y matrices for the coordinates.

```
>> dualgauss
  = inline('exp(-(x.^2+y.^2))+0.66 .*exp(-3*((x-3).^2+(y+2.5).^2))','x','y')

dualgauss =

     Inline function:
     dualgauss(x,y) = exp(-(x.^2+y.^2))+0.66 .*exp(-3*((x-3).^2+(y+2.5).^2))

>> [X Y] = meshgrid(linspace(-4,4,30),linspace(-4,4,30));
>> Z = dualgauss(X,Y);
>> figure;meshc(X,Y,Z);
>> figure;surfc(X,Y,Z);
>> figure;contour(X,Y,Z,15);
```

The functions `meshc(X,Y,Z)`, `surfc(X,Y,Z)` and `contour(X,Y,Z)` will create a combined mesh and contour plot, a combined surface and contour plot and a contour plot respectively, shown in figs. 45, 46 and 47.

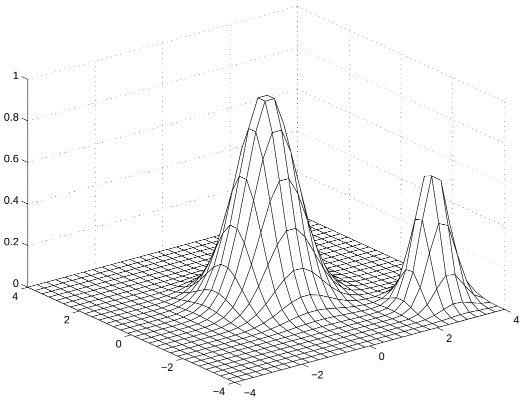


Figure 45: Colored mesh plot

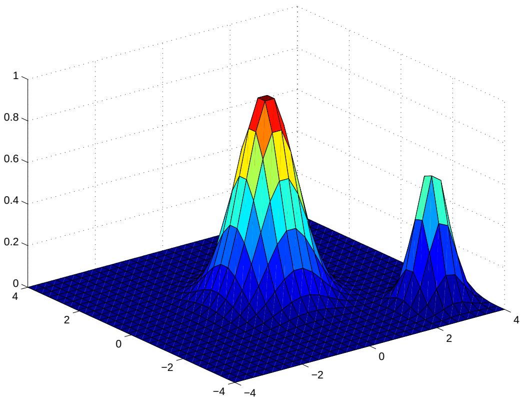


Figure 46: Surface plot

We can also create movie files from figures. Next is an example, which takes two gray scale images of the same size and blends one in to the other by linear interpolation.

```
blair = double(imread('blair.bmp'));
```

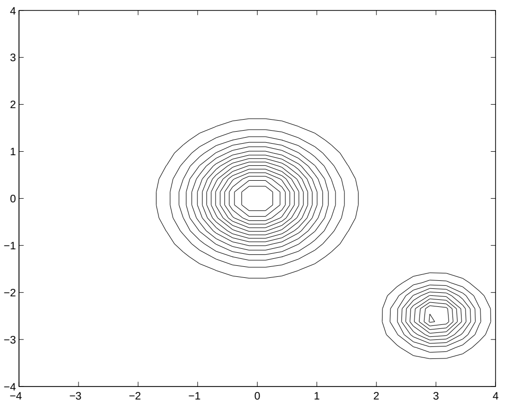


Figure 47: Contour plot

```
bush = double(imread('bush.bmp'));
tic;
for frames=1:100
    for m=1:size(blair,1)
        for n=1:size(blair,2)
            frame_array(m,n)=(bush(m,n) - blair(m,n))*(frames/100)+blair(m,n);
        end
    end
    figure;imagesc(frame_array);colormap(gray);mov_frames(frames)=getframe;
end
figure;movie(mov_frames,1,25);
movie2avi(mov_frames,'blair2bush.avi');
toc;
```

This takes approximately 45 sec on a regular PC. The problem is that we have to display all the figures using `imagesc(A)` and then use `getframe` to grab each frame from the current figure. `colormap(map)` specifies the color map to be used by `imagesc(A)`. `movie(A,n,fps)` displays the movie `A` `n` times with `fps` frames per second. `movie2avi(mov_frames,filename)` creates from the frames (`mov_frames`) a movie file named `filename`. Note that `filename` is a string.

We can avoid using this method by creating a structure, which is compatible with the `movie` function and filling it in with the appropriate data. We can also vectorize the loops to improve performance.

```
tic;
frame_array = zeros(100,size(blair,1),size(blair,2));
for frames=1:100
    frame_array(frames,:,:)=(bush(:,:) - blair(:,:)).*(frames/100)+blair(:,:);
end

for m=1:100
    mov_frames(m).cdata(:,:,1) = uint8(frame_array(m,:,:));
    mov_frames(m).cdata(:,:,2) = uint8(frame_array(m,:,:));
    mov_frames(m).cdata(:,:,3) = uint8(frame_array(m,:,:));
end
mov_frames(1).colormap = [];
figure;movie(mov_frames,1,25);
movie2avi(mov_frames,'blair2bush.avi');
toc;
```

This reduces the time to approximately 7 seconds. The initial images are presented in fig. 48. Some intermediate steps of the blending are shown in fig. 49.

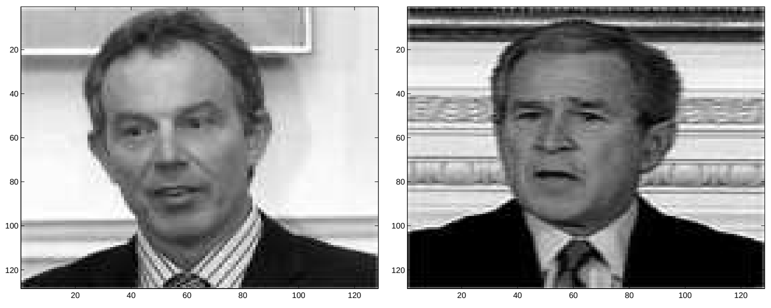


Figure 48: (a) Initial image. (b) Final image

Smoothing an image or any data set can easily be performed in Matlab using the built in convolution function. In the case of a 2D data set like an image we can use `conv2(A,B)`, which computes the 2D convolution between `A` and `B`. The following script performs Gauss smoothing, using a Gauss mask on the image.

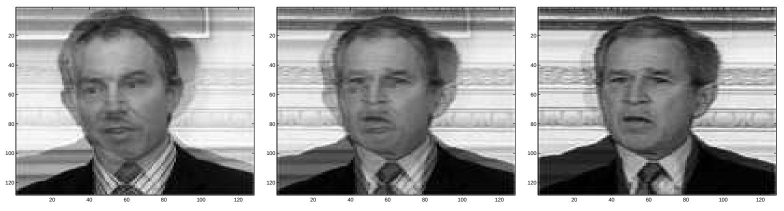


Figure 49: Intermediate steps of the linear interpolation movie example

```
data = double(imread('lena.bmp'));
x = [-5.5:0.5:5.5];
y = [-5.5:0.5:5.5];

for m=1:length(x)
    for n=1:length(y)
        gauss_filter(m,n) = gauss2d(x(m),y(n),1);
    end
end
blur_data = conv2(data,gauss_filter,'same');
figure;imagesc(blur_data);colormap(gray)
```

The third argument (`'same'`) in the `conv2` function retains the size of the convolved matrix to that of the original. The previous script will produce the right image in fig. 50.

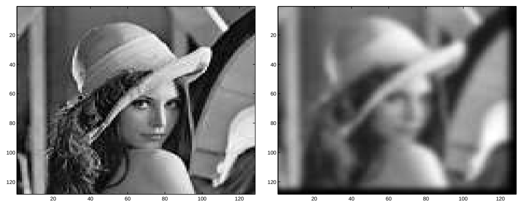


Figure 50: (a) Original Lena image. (b) Gauss smoothed Lena image

With 2D convolution we can even calculate an edge map of an image. This is done by approximating the gradient operator with a discrete convolution mask such as the Sobel operator.

```
data = imread('lena.bmp');
```

```
sobelx = [-1 0 1; -2 0 2; -1 0 1];
sobely = [1 2 1; 0 0 0; -1 -2 -1];

x_conv = conv2(data,sobelx);
y_conv = conv2(data,sobely);
edge_map = sqrt(x_conv.^2 + y_conv.^2);
figure;imagesc(data);colormap(gray);
figure;imagesc(x_conv);colormap(gray);
figure;imagesc(y_conv);colormap(gray);
figure;imagesc(edge_map);colormap(gray);
```

This script will produce the images in fig. 51.

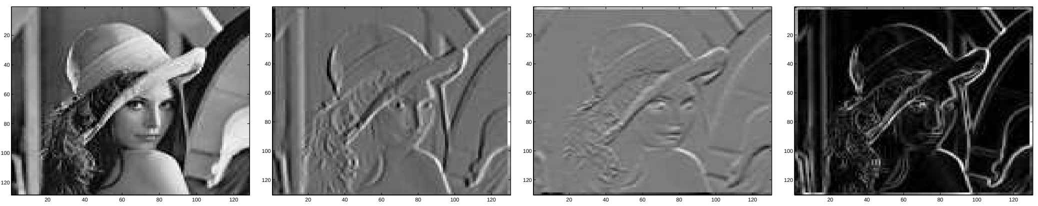


Figure 51: (a) Original Lena image. (b) Convolved with the x direction filter. (c) Convolved with the y direction filter. (d) Edge map

38 Summary table of functions

Function	p.	Meaning
`profile on/off`	122	Turn profiler on/off
`profile report`	122	Generate profiler report
`profile resume`	122	Resume profiler
`tic - toc`	122	Timer
`sparse(a,b)`	125	Create an `a` x `b` sparse matrix
`speye(a)`	127	Create an `a` x `a` sparse identity matrix
`spfun(function,A)`	127	Apply function to sparse matrix
`nnz(A)`	127	Number of non-zero elements of `A`
`full(A)`	127	Convert sparse matrix to full
`unique(A)`	128	Find unique elements of a vector `A`
`union(A,B)`	128	Set union of two vectors
`intersect(A,B)`	128	Set intersection of two vectors
`ismember(A,B)`	128	Detects members of a set
`setdiff(A,B)`	128	Set difference of two vectors
`global`	130	Create global variable
`persistent`	130	Create persistent variable
`inline(expr_string,arg1,...)`	132	Create inline function
`figure`	137	New figure window
`figure(n)`	137	Select figure window n
`title(string)`	137	Give a title to the figure window
`gtext(string)`	137	Add text with the mouse
`legend(string)`	137	Add a legend
`grid on/off`	137	Turn grid on/off
`subplot(m,n,p)`	137	Create multiple plots
`hold on/off`	137	Hold the current graph in the figure
`line(X,Y)`	142	Draw a line in a figure
`meshc(X,Y,Z)`	143	Create a colored mesh plot
`surfc(X,Y,Z)`	143	Create a colored surface plot
`contour(X,Y,Z)`	143	Create a contour plot
`imagesc(A)`	144	Scale and display an image
`colormap(map)`	144	Specifies the color map to used by `imagesc(A)`
`getframe`	144	Grab a frame from the current figure
`movie(A,n,fps)`	144	Play movie `A` `n` times with `fps` frames per second
`movie2avi(A,filename)`	144	Create a movie file
`conv2(A,B)`	145	2D convolution between `A` and `B`

39 Lab exercises 3

Programming exercises 1

A. Given the 2D Gauss function $z = e^{-\frac{x^2+y^2}{2}\sigma^2}$.

1. Write a function that accepts three arguments, an array holding x values, an array holding y values and sigma, calculates the 2D Gauss function given above and returns the z values.
2. Write a script which uses `for` loops to calculate the 2D Gauss function for an array of x and y values.
3. Calculate the function for `x = -2.5:0.01:2.5`, `y = -2.5:0.01:2.5` and `sigma = 1`. Use the `profiler` to find out how long it takes to calculate this.
4. Use the `tic-toc` commands to time your function.
5. Use array preallocation and vectorize the loop. You might find the `meshgrid` command useful. Check how long it takes to calculate this function and compare it with the previous non-optimized version.
6. Extend this function using `varargin` to accept two or three arguments. If only two arguments are entered then `sigma` will be equal to 1.
7. Write a function that makes use of `persistent` to count how many z values are above 0.5. You might find useful the `counter` example from the lecture notes.

B. Given the x,y values from the previous exercise.

1. Calculate the 2D Gauss function for sigma equals to 0.5, 1, 1.5, 2, 3, 4.
2. Using the `subplot` and the `meshc` command plot in one figure all six graphs.

C. Two sets are equal if all of their elements correspond regardless of the order. Keep in mind that multiplicity of elements in a set is ignored.

1. Write a function that determines whether two sets are equal using the set functions.

2. Using `varargin` extend this function for any number of sets.

D. Given the `display_histogram` function in the lecture notes.

1. Modify the function to display the histogram horizontally as in fig. 52.

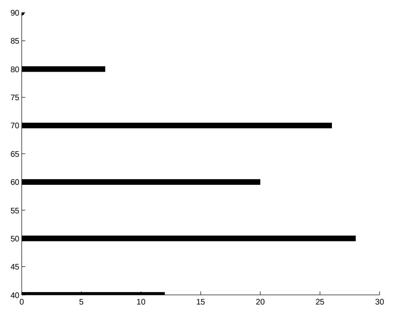


Figure 52: Horizontal histogram

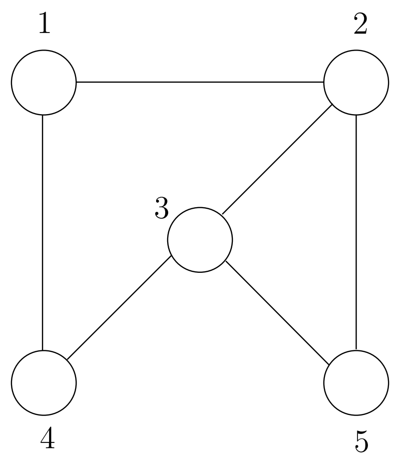
2. Extend the function using `varargin` to be able to accept an additional argument `LineWidth` to define a new width for the `line` command.

D. Given two gray scale images of the same sizes.

1. Create a movie that starts from one image and blends it to the other image by linear interpolation.
2. Create a movie that starts from one image and moves it to the right, while at the same time inserting the right part of the other image on the empty space on the left. The effect should a scrolling marquee between two images.

Programming exercises 2

1. It is common to represent the adjacency structure of graph



in an adjacency matrix A as follows,

$$A = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 & 1 \\ 4 & 1 & 0 & 1 & 0 & 0 \\ 5 & 0 & 1 & 1 & 0 & 0 \end{array}.$$

Give a Matlab function to compute the number of paths from a specified vertex i to each other vertex in the graph of length n (a path may visit the same vertex twice, e.g., a length three path from vertex 1 to 2 is 1-2-1-2). For example,
>> **allpaths(A,5,3) ;**

```
The number of paths from vertex 5 to vertex 1 of length 3 is 2
The number of paths from vertex 5 to vertex 2 of length 3 is 4
The number of paths from vertex 5 to vertex 3 of length 3 is 4
The number of paths from vertex 5 to vertex 4 of length 3 is 2
The number of paths from vertex 5 to vertex 5 of length 3 is 2
```

Hint: *the algorithm to compute all paths of length n between all vertices may be expressed in under 10 characters.*

2. In many machine learning algorithms it is important to generate a random permutation of a dataset. Matlab provides a function *randperm(n)* that returns a random permutation of the numbers $1 \ldots n$ in an array such that each of the $n!$ permutations have *equal probability* (with the usual proviso of pseudo-randomness of the base random number generator).

   Write a function *myrandperm(n)* with functionally similar behavior also assuring that each of the $n!$ possible permutations have equal probability of being generated. The best marks will be given for a function whose running time is linear in n.

3. In this problem you will implement some simple drawing functions. The top-level function developed will be able to plot a figure consisting of lines, triangles, rectangles, and circles. The function should take one argument, the figure filename. In the file each line contains a single character denoting the element to be drawn and the data needed to specify the element, i.e.,

```
L x1 y1 x2 y2
T x1 y1 x2 y2 x3 y3
R x1 y1 x2 y2    % these are the opposite corners of the rectangle
C x1 y1 radius
```

   Demonstrate your function with the following file *afig.txt* which may also be found from the class web page.

```
C 5 17  0.5
R 6 0   8 1
L 1 9   1 7
C 4 15  4
T 3.5 15  4.5 15  4 16
L 4 4   4 11
L 4 4   6 1
C 3 17  0.5
R 0 0   2 1
```

```
L 1 2   4 4
L 1 19  7 19
T 3 19  4 22  5 19
R 3 12.5 5 13.5
```