Fracturing artefacts into 3D printable puzzles to enhance audience engagement with heritage collections

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(a) 3D scanned urn model (b) Generated puzzle pieces and core (c) Fabricated puzzle in museum exhibition

Fig. 1. Workflow for the development of physical puzzles of pottery archaeological artefacts.

Three-dimensional (3D) puzzles of heritage artefacts are typically used to engage audiences in the interpretation of archaeological objects in a museum gallery. The reason for this is that a puzzle can be seen as an enjoyable educational activity in the form of a game but also as a complex activity that archaeologists undertake when re-assembling fragments, for instance of broken pottery. Until now the creation of this type of experiences is mostly a manual process and the artefacts used rarely reflect those in the collection due to the complex nature of the process. The contribution of this paper is a novel digital worfklow for the design and fabrication of 3D puzzles which overcomes these limitations. The input to the workflow is an authentic artefact from a heritage collection, which is then digitised using technologies such as 3D scanning and 3D modelling. Thereafter, a puzzle generator system produces the puzzle pieces using a cell fracture algorithm and generates a set of puzzle pieces (female) and a single core piece (male) for fabrication. Finally, the pieces are fabricated using 3D printing technology and post-processed to facilitate the puzzle assembly. To demonstrate the feasibility of the proposed novel workflow, we deployed it to create a puzzle activity of the Saltdean urn, which is exhibited at the Archaeology Gallery of the Brighton Museum and Art Gallery. The workflow is also used with further artefacts in order to demonstrate its applicability to other shapes. The significance of this research is that it eases the task of creating puzzle-like activities and maintaining them in the long term within a busy public space such as a museum gallery.

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CCS Concepts: • **Computing methodologies → Shape modeling**; Mesh geometry models; • **Applied computing →** *Computer-aided design*; *Fine arts*.

Additional Key Words and Phrases: cultural heritage, 3D printing, gallery design, hands-on activities, educational puzzles

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1 INTRODUCTION

The technological developments over the last years in 3D printing along with the attention that its applications have attracted from various communities, have resulted in making digital fabrication a popular topic of research, practice and discussion. Even though there is still a need to deal with several related obstacles, such as design knowledge, cost and available materials, before the widespread adoption of digital fabrication in people's everyday lives, the Cultural Heritage (CH) domain has proved to be a valuable field to try digital fabrication technologies. These technologies have already been implemented in a variety of processes in the CH sector from conservation and exhibition planning to packaging and creative or educational activities [21, 23, 31, 32].

This paper is concerned with the development of an application of digital fabrication which aims to contribute to the educational and communicational aspect of the CH experience. In particular, it examines how digital 3D models of artefacts can be re-purposed in creative ways in order to expand the benefits of the digitisation process. As such, the paper proposes the playful use of a 3D puzzle to enable users to experience the physical pieces or shards of a pot in a similar way that archaeologists do when uncovering and synthesizing an artefact found at an excavation site. This requires digitally breaking a 3D shape into pieces and physically fabricating them in such a way that the puzzle can be easily re-assembled.

The main technical contribution of this paper is a novel workflow for generating and fabricating the 3D puzzle when the given input is an authentic cultural heritage artefact. The workflow is driven by the requirement of being adaptable to different types of fractures and number of pieces. For instance, less number of pieces might be required if the puzzle is to be easily assembled by a young person or child.

An additional technical contribution of the paper is the experimentation with different fracturing algorithms for generating the puzzle pieces. The paper proposes a new algorithm which mimics the way archaeological artefacts, such as pottery, break into smaller shards.

In order to test the workflow, this is deployed with a late Iron Age burial urn from the area of Sussex (UK) - a significant object from the Brighton Museum and Art Gallery collection. The generated puzzle has been incorporated into the Archaeology Gallery in order to enhance young audiences' visiting experience while engaging them in an educational activity.

The paper is organised as follows. Section 2 discusses relevant work in the field including 3D printing technologies to communicate cultural heritage information and engage audiences. Section 3 introduces the particular artefact which drove the requirements for the development of the puzzle generating workflow and the audience of the application. Section 4 then presents the proposed workflow for the design and fabrication of the puzzle, including the 3D scanning of the artefact, its reconstruction and the algorithms develped for generating the puzzle. Section 4.3 presents the results of the developed algorithms applied to other shapes and types of artefacts in order to demonstrate the generic nature of the workflow. Section 5 discusses the evaluation of the application and the advantages of the adopted approach. Finally, section 6 presents discussions and conclusions.

2 RELATED WORK

2.1 Digital fabrication to communicate CH information

Digital fabrication technologies comprise a combination of programmable digital tools, processes, materials and equipment which allow the creation of physical objects of complexities not achievable by traditional manufacturing processes.

The interest from the CH community in these technologies is high as they offer the ability to manipulate the digital representation of an artefact in creative ways. In addition, these technologies enable a high-level of customisation when producing physical objects in a variety of resolutions, materials, colours and densities. Another important advantage of digital fabrication includes the possibility for multiple replication and/or production in a cost-effective way, while "future-proofing" the information related to the artefact itself. Hence, these technologies are driving new trends for the mass-customisation of CH objects and experiences.

The term "smart" replicas has also become popular over the recent years. This refers to the possibility of combining the physical object with further layers of interpretative multimedia information [6, 16].

Moreover, digital fabrication applications to support the interpretation and communication of CH can be found in many heritage organisations around the world. These examples include applications, such as the full 3D print of the Sarcophagus of the Spouses from the Villa Giulia Etruscan Museum, which can support visitors in having a more holistic approach (by vision and touch) for the interpretation of an artefact [11].

Another example involves audiences in scanning objects and mixing 3D models to produce hybrid artefacts by using digital fabrication. These activities can be oriented to people with knowledge of 3D tools, such as artists participating in 3D scanning and printing Hackathons [19, 21]. However, some institutions (e.g. British Museum and the Art Institute of Chicago) deploy 3D printing in order to involve groups in workshops for non-experts. Such groups include teachers, teenagers and families who engage with the museums' collections through 3D technology [18, 22].

Other examples employ 3D printed artefacts in educational programmes for children. The American Museum of Natural History asked students to capture and replicate dinosaur fossils from the museum's palaeontology collections in order to synthesise a dinosaur and learn to think like palaeontologists [3]. Another application is a megalithic freestanding stone from Wales (UK) that was 3D printed in sliced vertical pieces that slide down a cylindrical pillar [18].

Visually impaired audiences as well as the elderly constitute groups that can also benefit greatly from digital fabrication. [9] propose a system which facilitates the navigation on an architectural 3D printed facade, allowing blind users to listen to audio descriptions. Other research deploys 3D printed reliefs, with complementing interactive applications, to support visually impaired users to feel paintings and natural history exhibits [25, 29].

At the same time, digitally fabricated artefacts can work as engagement vehicles for elder audiences or trauma survivors while experiencing the "healing" properties of object handling and reminiscence [24].

Alternative uses of replicas include the production of edible artefacts, such as the ones created at the MediaLab of The Metropolitan Museum of Art in New York, aiming to support the understanding of artefacts by providing a multisensorial experience to visitors [39]. More "traditional" examples can be found in museums' shops, where replicas are sold as souvenirs or decorative/collection objects [44].

Lastly, replicas have also served purposes related to the repatriation of original artefacts. In these cases, replicas are kept in the possession of the organisation while the original artefact returns to its possessor (the opposite can happen as well) [14].

The breadth and spread of applications demonstrates i) the wide variety of experiential frameworks to provide people with the opportunity to "meet" and "feel" culture in alternative ways, and ii) the potential of digital fabrication technology to support the interpretation of a cultural heritage artefact and engage audiences. Within this context, the development of sustainable workflows to create 3D puzzle experiences for audiences is a novel contribution to the wider efforts in this area.

2.2 Design challenges and relevant cases

The creation of a digitally fabricated puzzle can be achieved by different methods and tools. An important overall requirement is the generation of the puzzle pieces and the mechanism for their assembly. The graphics' community has previously conducted relevant research. For instance, the generation of interlocking parts from a 3D model has been a popular topic over the last years, as it is not currently possible to print a single object that is larger than the working volume of a 3D printer. As such, various systems are proposed which take as an input a 3D model and produce various smaller interlocking pieces for 3D printing [2, 15, 34, 37]. Moreover, [36, 38, 43] present various algorithms for generating puzzles with interlocking pieces, known as burr puzzles, from a 3D model.

Another relevant area of research is the fracturing simulation of 3D models. These fracturing effects are often used to display the breaking or destruction of objects or places in computer games, virtual reality and the film industry. In this area, predefined fracture patterns are often computed, while more novel solutions focus on creating fast approximations which are computational efficient and offer flexible control of fragment generation [12, 20, 30, 45].

Moreover, most of the proposed solutions for creating puzzles which are 3D printable aim to create puzzles consisting of pieces which interlock with each other, without the need to have a fixed installation. In our case, we aim to create a permanent hands-on exhibit for a public space, such as a busy museum gallery. As pottery

(a) Puzzle-pot from the Bristol Museum & Art Gallery (UK), photo courtesy of Andrew Maxted

(b) Puzzle-pot from Rezé Museum (France), photo courtesy of Theophane Nicola

Fig. 2. Examples of pottery puzzles in museums.

is a common archaeological finding at an excavation site, we focus on this type of artefact, and the shape of its fragments or shards when broken. This type of artefact is also interesting as its reconstruction from shards is a problem often faced by archaeologists.

Of relevance to the heritage field is the geometric analysis of a dataset of fracture patterns observed in wall paintings excavated at Akrotiri, Greece [33]. This analysis suggests that when pottery is broken, this fragments in a hierarchical fracture pattern, where fragments break into two pieces recursively along cracks nearly orthogonal to previous ones.

Similar examples of pottery puzzles in other museums (though without deploying a fully digital workflow) are shown in Figure 2. As shown in the images, these puzzles require a static element (the core) that might provide clues about the overall shape of the pot. Moreover, the core helps to secure the pieces in place with the use of magnets or other attachment mechanisms which are placed both on the core's surface and on each puzzle piece. An initial proposal for a digital workflow to generate this type of puzzles is present in [26]. This paper extends the digital workflow to incorporate different fracture patterns as well as presents a full deployment of the workflow on the chosen exhibit of the Brighton Museum and Art Gallery.

3 THE 3D PUZZLE EXPERIENTIAL FRAMEWORK

The motivation to develop the proposed workflow stemmed from the need to design an experiential framework that would engage young audiences with the archaeological collection of the Brighton Museum and Art Gallery. Thus, the requirement was to design a 3D puzzle of a funerary urn, shown in Figure 3. The urn comes from the cliff top at Saltdean, a coastal area near Brighton in Sussex, UK. The pot has curvilinear designs which are usual in Sussex in the two centuries BC, before the arrival of the Romans. The urn is mostly brown and it seems that burnishing had been applied to its surface to give it a "leathery" appearance. The Saltdean funerary urn is a late Iron Age pot (probably 1st century BC) which was thrown on a wheel [42]. It possibly reflects influences from Belgian tribes and people from Brittany who had moved into the area and introduced the use of the potter's wheel in south Britain [1, 7, 8, 13].

The puzzle was designed to be placed as a hands-on activity along with local findings of the Iron Age period and close to the original artefact in the new Archaeology Gallery. The objective of the hands-on activity is to support

Fig. 3. Late Iron Age funerary urn from Saltdean, Sussex (UK).

young audiences, and especially children, in having an interactive experience with a heritage artefact in the form of an educational activity or game. The puzzle also allows wider audiences to experience the challenges linked to archaeological processes, such as reconstructing a shape from a given group of shards or pieces. By assembling the puzzle, audiences will engage with the exhibit, its physicality, function and history, while acquiring new skills and gaining a better understanding about the artefact itself.

3.1 Requirements for the production of the digitally fabricated 3D puzzle

The main design requirements with respect to the 3D puzzle were agreed between the researchers and the exhibition designers taking into account design guidelines about children's puzzles [35]. These requirements included:

(1) to have the urn height scaled-up to around 300 mm (the rest of the dimensions of the artefact were scaled-up proportionally);
(2) to have a thickness of around 10 mm for each individual piece, as this was found suitable for easy handling by small hands;
(3) to have approximately 10 pieces to assemble the puzzle. Thus, pieces should measure at least 50.8 mm across, as 6-8 year olds can handle pieces of this size;
(4) to design a core piece which would be attached to a rotating wooden plate so that the user can easily spin the puzzle core to facilitate interaction (see Figure 4);
(5) to enable attachment of the individual puzzle pieces to the core via magnets. The magnets are inserted in blind holes in the puzzle pieces and in the solid core. The blind holes require to be in predetermined matching positions both in the pieces and core;
(6) to cover each individual piece in a plaster-like finish and paint it to disguise the magnets, provide better texture feeling and an appearance that would be as close as possible to the original one.

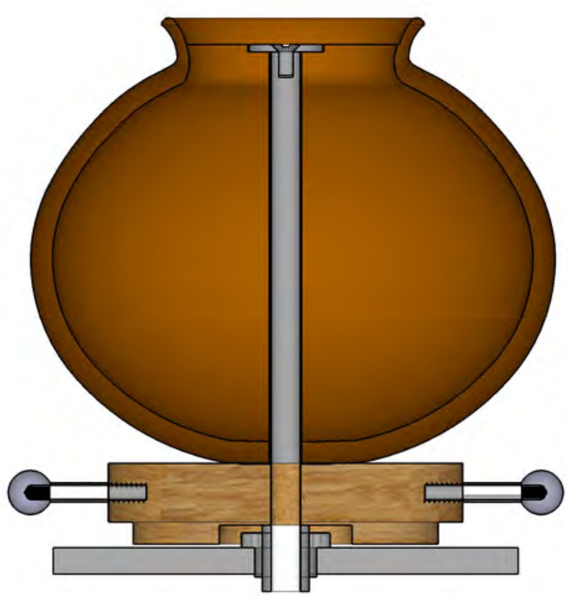


Fig. 4. Design of puzzle core piece on its rotating base, design courtesy of Alex Hawkey.

When discussing with the designer of the museum, it was acknowledged that such requirements could be addressed by using alternative mechanisms to digital fabrication technologies providing similar durability and quality. However, it was deemed that the digital workflow would enable to future-proof such exhibit for replacing parts in a cost-effective manner.

3.2 Audience

The target group for this puzzle activity is young people, in particular children between the age of 6 and 12 years old. This age frame is considered as appropriate in terms of integrating a specific type of interpretation as interpretative means can be different for younger or older children [41].

The selection of this particular group, whether it is families or school children visiting the museum, has been recognised as an important part of most CH organisations' audiences. Children appear to be amongst the people who can benefit the most from CH experiences with the deployment of replicas [5, 18, 22].

Furthermore, official numbers (in the "Overview of data in the Museums, Libraries and Archives Sector" [17]) confirm that most people who visit a museum/CH institution in the UK belong to a family group or a school group. Hence, the Brighton Museum and Art Gallery has a high number of families and school children visiting its premises. Moreover, a survey which recorded visitors' opinions on the potential to exhibit the archaeological collections of the museum revealed that people would be interested in hands-on children's activities [27].

The following section will describe a digital fabrication workflow to produce the 3D puzzle according to the specified requirements, along with a proposed algorithm to semi-automate the design of such 3D puzzles.

4 WORKFLOW FOR GENERATING AND FABRICATING A 3D PUZZLE OF AN ARTEFACT

The proposed workflow involves the following steps:

(1) Digitisation and reconstruction of the digital 3D model of the artefact and its central core piece.
(2) Generation of fracture pattern.
(3) Generation of the individual puzzle pieces.
(4) Generation of attachment mechanisms, such as matching blind holes, both in the core and puzzle pieces.
(5) 3D printing all puzzle pieces and core.
(6) Post-processing of all puzzle pieces and, including adding attachments and painting.
(7) Assembling the puzzle into the final exhibit.

The approach for producing solid surfaces for fabrication, which underlies the proposed workflow, deploys algorithms that combine triangular mesh representations with constructive solid geometry (CSG) operations. CSG is a technique commonly used in solid modelling CAD systems. It allows to create a complex surface by using Boolean operators to combine simpler objects.

The implementation of the workflow uses a mixture of tools and systems including modelling tools, C++ and OpenSCAD. OpenSCAD is a free Computer Aided Design (CAD) software which uses the Computational Geometry Algorithms Library (CGAL) [40] as its constructive solid geometry (CSG) engine. Its script syntax is based upon functional programming philosophy which allows to generate geometry using a functional approach.

The following sub-sections will describe each of the workflow stages in detail using the Saltdean pot as an example artefact.

4.1 Digitisation and reconstruction of the artefact

The acquisition of an artefact can be achieved through different means, including 3D scanning and photogrammetry techniques. In this case, the urn was scanned using the AICON Breuckmann 3D SmartScan scanner. Given the shape of the urn with the narrowed neck above its rounded body, the 3D scanning process captured the external surface of the pot, but it was not possible to acquire the internal surface. The resulting 3D model is shown in Figure 1-a after some small holes were filled in.

In order to reconstruct the internal part of the urn which was not acquired by the scanner, it was considered that the best approach was to solidify the external wall with a 10 mm thickness using the 3D modelling tool Blender. Before doing this, the 3D model was scaled-up to have a 300 mm height according to the design requirements. The resulting 3D model which has been used as input for the puzzle is shown in Figure 5a.

(a) 3D model of the reconstructed urn (b) 3D model of internal core of the puzzle

Fig. 5. 3D models used for puzzle generation.

Afterwards, the central core piece was produced. This can be generated for any type of pottery puzzle by using a Boolean operation. The resulting watertight model is shown in Figure 5b. The core required also a through-hole along its height in order to fit onto the revolving base, as shown in the design in Figure 4.

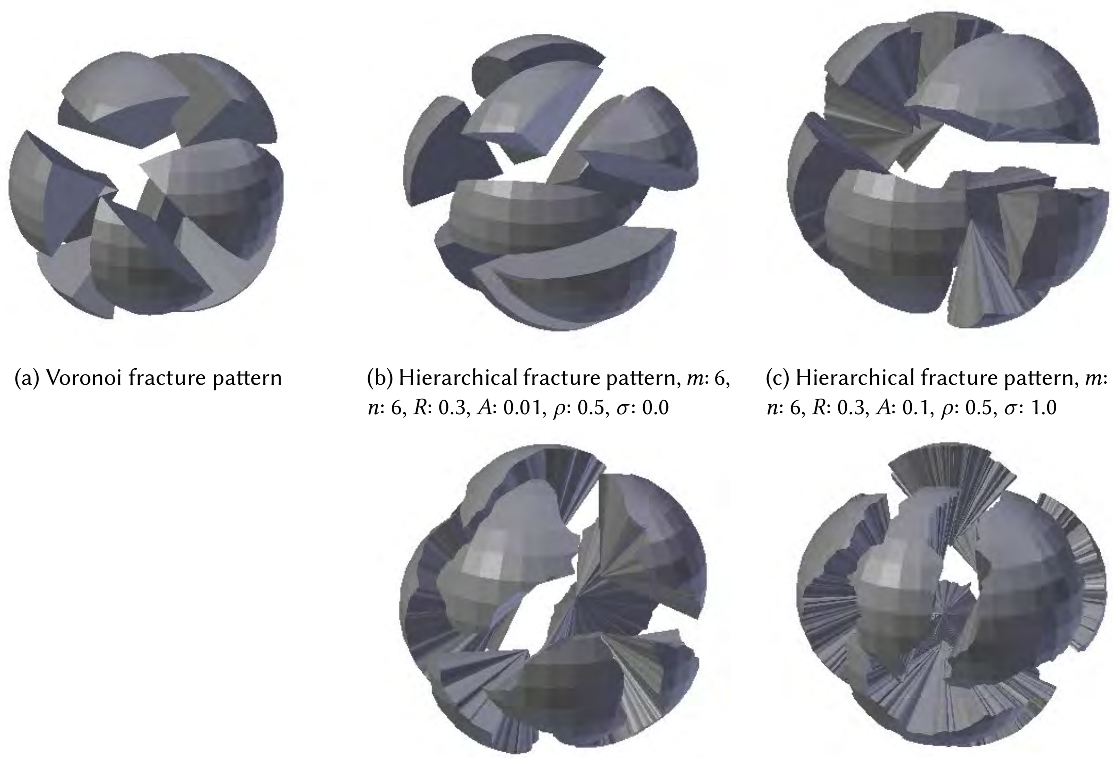
4.2 Generation of fracture patterns

Before creating individual "puzzle pieces" (shards, or *fragments*), we need to determine suitable fragment shapes, that is, hypothetical *fracture lines* along which the ceramic object broke.

We model fracture lines within the spherical domain, by conceptually projecting the artefact's geometry onto a sphere. While this restricts the range of possible fractures, it still spans an expressive-enough range of fragment geometries in our context.

In this work, we investigated two different ways to generate such fractures: *Voronoi tessellation* of the sphere, and *hierarchical fracture* along ragged curves on the sphere.

Voronoi tessellation. In this approach, we generate a random set of n uniformly distributed centroids on the sphere. Fracture lines are then defined as the edges of the corresponding spherical Voronoi diagram. The resulting fragment shapes are generally more uniform than naturally occurring fragments but represent an interesting puzzle challenge due to the higher similarity amongst fragments. Figure 6a shows an example set of spherical Voronoi fragments, visualised as solid spherical sectors.



(a) Voronoi fracture pattern

(b) Hierarchical fracture pattern, m: 6, n: 6, R: 0.3, A: 0.01, ρ: 0.5, σ: 0.0

(c) Hierarchical fracture pattern, m: 6, n: 6, R: 0.3, A: 0.1, ρ: 0.5, σ: 1.0

(d) Hierarchical fracture pattern, m: 6, n: 6, R: 0.3, A: 0.2, ρ: 0.2, σ: 1.0

(e) Hierarchical fracture pattern, m: 6, n: 6, R: 0.1, A: 0.3, ρ: 0.5, σ: 0.3

Fig. 6. Fracture patterns.

Hierarchical fracture. While real-world fracture lines are shaped by complex processes, including impact geometry and volumetric distribution of inhomogeneities in the material, Shin et al. in their study of wall painting fragments [33] found evidence that fracture patterns have similar statistics as cracks created by hierarchical fracture processes [4]. This inspired our second fracture generation method that hierarchically subdivides the spherical domain until a target number of fragments is reached. In each iteration, one fragment is subdivided into two fragments by generating fracture lines at random orientations. In our system we use fractal curves that lend a more realistically looking raggedness to these fracture lines.

See Appendix A for more details on the algorithm. Its parameters affecting the fragmentation are shown in Table 1. Figure 6b–6e shows resulting fracture line characteristics for different parameter values. Figure 6b demonstrates a fragmentation with a low amplitude A (random lateral displacement of the curve) and medium fractal decay ρ (attenuation of displacement toward finer scales) to produce fragments with straight edges. Moreover, Figure 6e demonstrates a fragmentation with a higher amplitude A and a lower random jitter of polygon vertices R to produce fragments with higher raggedness.

Hierarchical Splitting	
N	Number of fragments to be produced.
r	Maximum area ratio between larger and smaller half of a split fragment outline ($r = 4$ in all of our experiments).

Fractal Fracture Line Generator	
m	Initialises fracture line with m-sided spherical polygon along a great circle.
n	Number of recursive bisections of polygon edges.
R	Random jitter of polygon vertices along the polygon, as multiples of edge length.
A	Amplitude (as multiples of edge length) by which that polygon is tangentially displaced away from the polygon.
ρ	Fractal decay: $\rho = 1$ creates fractal curves that are self-similar across all scales; values of $0 < \rho < 1$ attenuate both R and A toward finer scales.
σ	$\sigma \in [0, 1]$ denotes strength of additional smoothing of the result at its finest scale.

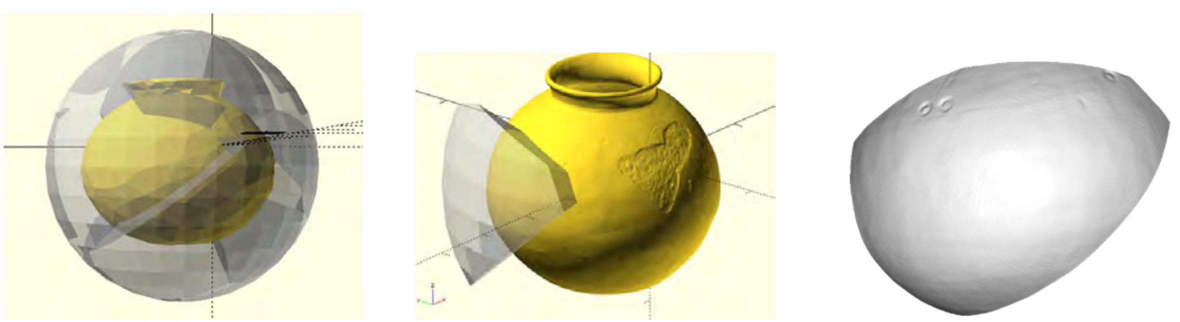
Table 1. Parameters of our hierarchical fracture algorithm. See Appendix A for algorithmic details.

The output of this process is a set of fragment shapes which can later be applied to various 3D geometries. Given the fact that the computation time for producing the patterns is of few milliseconds, it is possible to generate various patterns which can later be used to create alternative puzzles of the same geometry.

4.3 Generating the individual puzzle pieces

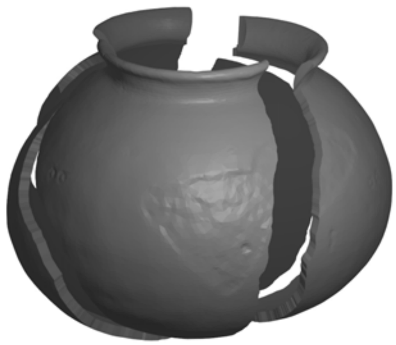
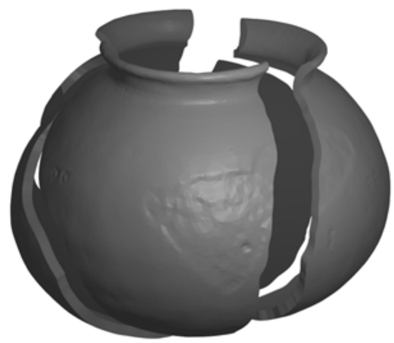
Once the fracture pattern has been generated, it is used to produce the puzzle pieces. For this, CSG operations are used to generate all the printable puzzle pieces. Firstly, the 3D model is translated to the centre of the fractured sphere (see Figure 7a). Thereafter, as shown in Figure 7b, for each fragment, the 3D model of the artefact is intersected with a spherical sector spanned by the fragment outline. As a result, a puzzle piece is produced as shown in Figure 7c. The algorithm iterates over all sections of the fractured sphere to automatically produce all puzzle pieces. This process is repeated to generate puzzle pieces at two different levels of detail so that they can be used in subsequent operations.

Figure 8 illustrate the generation of alternative versions of puzzles pieces for the Saltdean urn using various levels of raggedness for similarly-sized fragments. For the museum gallery, we generated 16 individual puzzle pieces using straight edges based on the voronoi approach as this was deemed to be the simpler version for

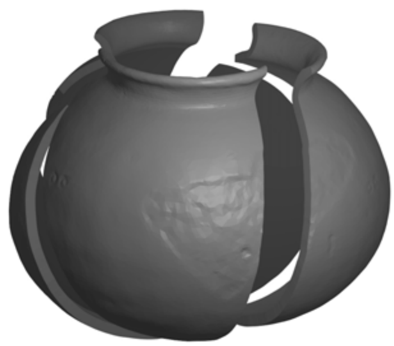
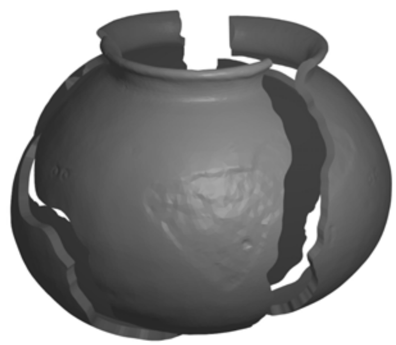


(a) 3D model overlaid with all fragments' sphere sectors (b) 3D model of reconstructed urn with one fragment's sphere sector (c) Resulting 3D model of puzzle piece

Fig. 7. Puzzle fragment generation through CSG operations between a 3D model and sphere sectors spanned by the fragment outlines.

(a) m: 6, n: 6, R: 0.3, A: 0.1, ρ: 0.5, σ: 0.0 (b) m: 6, n: 6, R: 0.3, A: 0.1, ρ: 0.5, σ: 1.0

(c) m: 6, n: 6, R: 0.3, A: 0.01, ρ: 0.5, σ: 0.0 (d) m: 6, n: 6, R: 0.3, A: 0.2, ρ: 0.2, σ: 1.0

Fig. 8. Examples of puzzles with different parameters.

children to complete. Seven of these pieces were retained to be used for the base of the puzzle. The user would be able to use this base as a reference when assembling the rest of the puzzle.

Moreover, Figure 9 demonstrates the generation of alternative versions of puzzles of various shapes from cultural heritage collections. The other two shapes have been digitised in a similar workflow than the Saltdean urn and underwent a similar process to generate the digital puzzle pieces.

As shown in Figure 9, it is possible to create alternative versions of the puzzle from the same geometry. This ensures the user has some control over the best version of the puzzle for a given 3D mesh. To achieve this, the best approach is to test the patterns with a simplified version of the 3D mesh. Once the user has selected the best pattern, this can be applied to a high-resolution version of the geometry. The reason behind this is that, as expected, the computation time increases with the number of faces and the complexity of the 3D model. Table 2 presents the computation times for various 3D models with different number of faces in a laptop with Intel Core i9 processor and 32 GB DDR4.

Users might require various attempts before finding a suitable puzzle configuration. Although fragment shapes vary across the different runs, they tend to be of consistent characteristics. Figure 10 shows various examples generated for the Saltdean pot archaeological artefact. The puzzle pieces have been generated with 16 random seeds using the same settings of m: 6, n: 6, R: 0.3, A: 0.1, ρ: 0.5, σ: 0. As illustrated in the figure, some configurations work better than others to produce feasible puzzles which are manufacturable and that work well in the physical domain. This includes avoiding very small puzzle pieces and cuts which might be infeasible for manufacturing

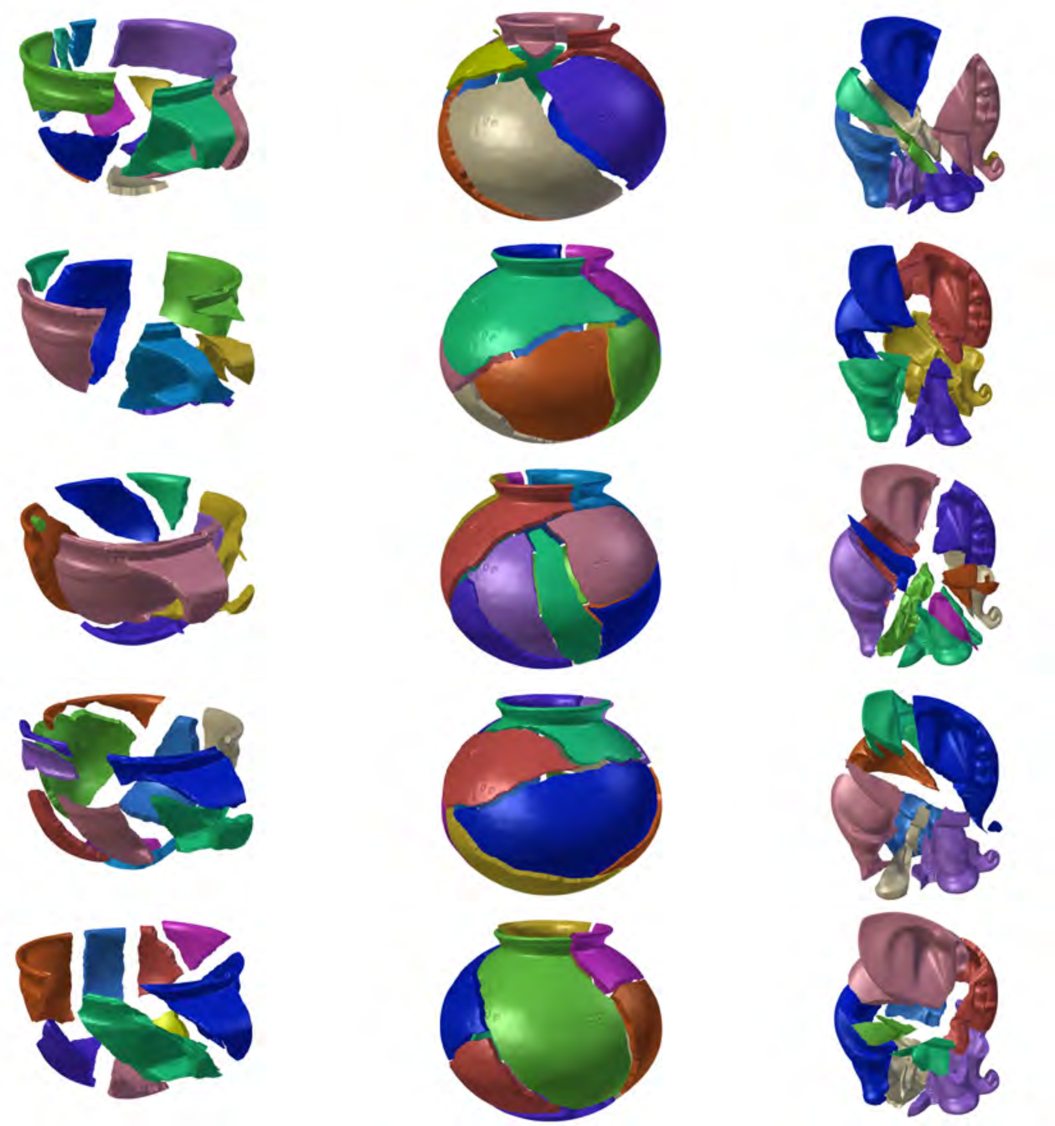


Fig. 9. Examples of algorithm applied to scans of an amber cup, Saltdean pot and ceramic elephant figurine.

or cuts that can cause the physical piece to easily break. It is possible to mitigate these effects to a certain extent by rotating the resulting fracture pattern before creating the intersection with the 3D model in order to see if there is a more suitable solution. However, this is mostly a manual process and in most situations it is faster to generate a large number of solutions automatically and then pick up the best. The bottom row of Figure 10 illustrates a good set of puzzle pieces that have a suitable size and can be manufactured.

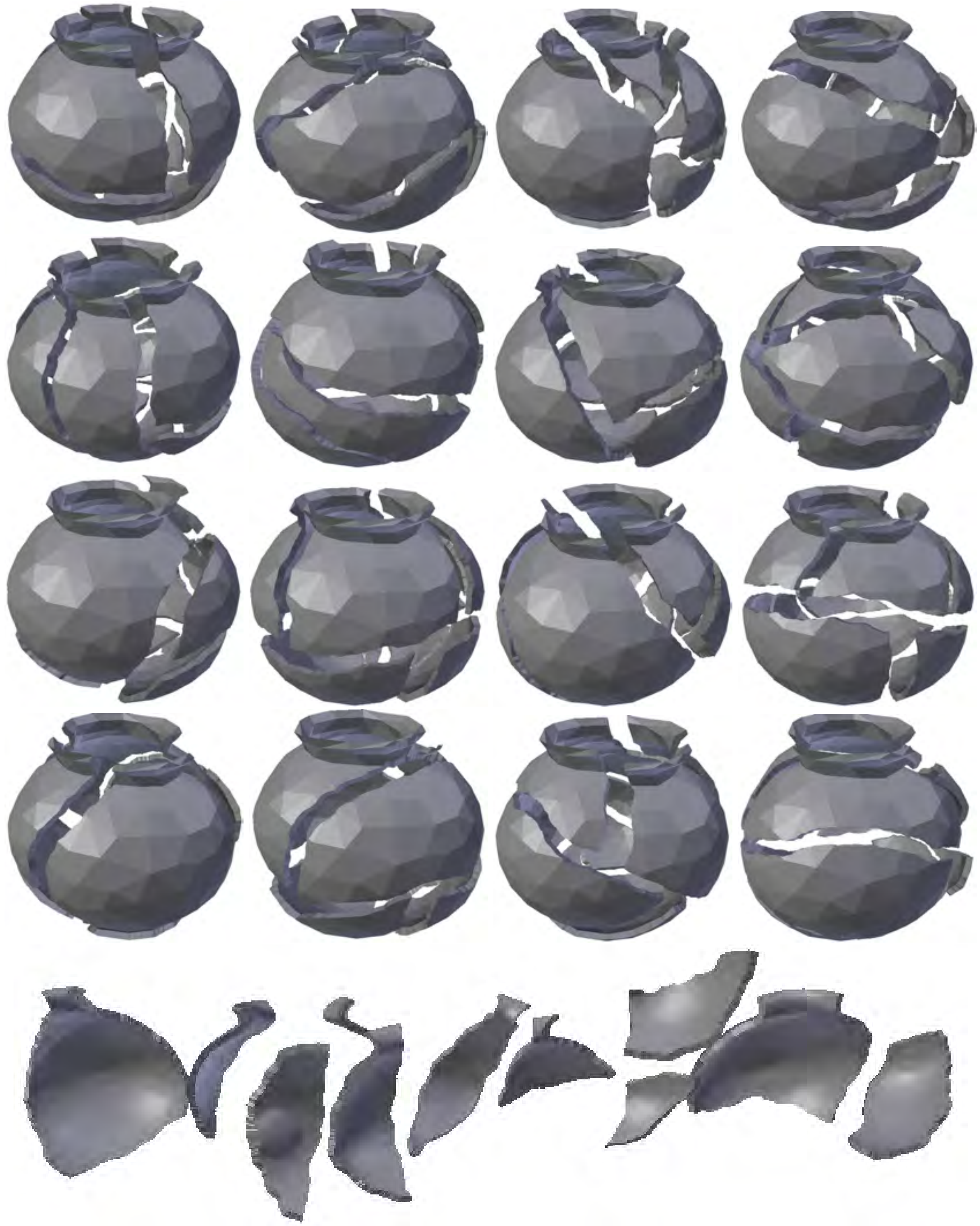


Fig. 10. Examples of puzzles produced for a simplified mesh of the Saltdean pot scan (800 faces) with a fracture pattern generated with settings m: 6, n: 6, R: 0.3, A: 0.1, ρ: 0.5, σ: 0. The results represent consecutive runs of the algorithm for twelve randomly chosen random seeds, previewed on low-resolition geometry. The bottom row shows the pieces generated with one of these fracture patterns (top, third-column), applied to a higher-resolution model of the pot (30,000 faces).

Mesh	Number of faces on mesh	Average processing time for one puzzle piece
3D scan of amber cup	800	8 milliseconds
Sphere	42,710	18 seconds
3D scan of Saltdean pot	57,050	10 minutes
3D scan of amber cup	100,000	36 minutes
3D scan of ceramic elephant	183,168	55 minutes
3D scan of Saltdean pot	1,269,160	3 hours 28 minutes

Table 2. Processing times to compute fragments for a sample of 3D meshes.

4.4 Generating attachments both in the core and puzzle pieces

Once all puzzle pieces were produced, it was necessary to generate attachments to secure the puzzle pieces onto the core. Magnets are suitable attachments as they can be buried within the puzzle pieces and core. However, other female/male attachments are possible as well. In this step, the attachments for the Saltdean urn are generated both for the individual puzzle pieces and the central core to fit the magnets in. The blind-holes have consistent width and depth which should be enough to hide the magnets in.

To generate the blind-holes across the surface, a set of points in 3D space is given as an input. This set of points should offer full coverage across the surface. The set can be randomly generated as random points on a sphere. However, given the requirement to have a specific number of holes for each piece, the positions were manually determined to ensure an even distribution. Each point is then used to generate a cylinder whose origin is the centre of the 3D model, as illustrated in Figure 11a.

The algorithm to create the blind-holes is based on the CSG operations of intersection and difference. Hence, the algorithm produces an intersection between the cylinder and the puzzle piece as shown in figure 12a. It then translates the resulting geometry of the interaction towards the origin by taking into account a thickness value. This code generates a geometry, which will later become the hole, for each cylinder as shown in Figure 12b.

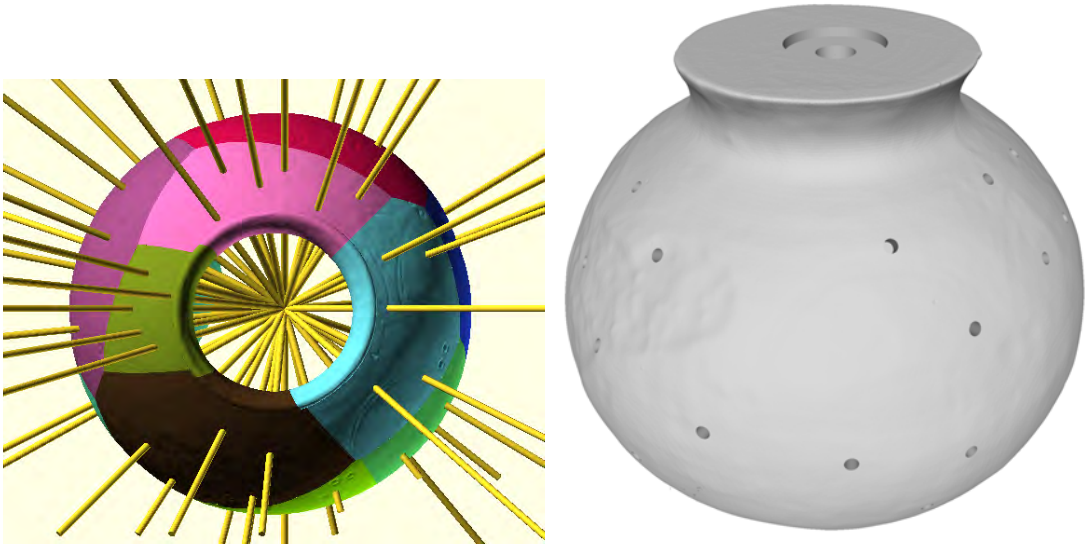
The generated geometries are then used to produce the blind-holes. This is achieved by using the difference operation between the 3D puzzle piece and the generated geometry (see Figure 12c). The same process is repeated for all puzzle pieces to produce the required blind holes.

Furthermore, a similar process is repeated for generating the blind-holes in the central core piece using the same 3D points. However, this time the direction in which the intersected geometry is translated is reversed. The resulting geometry is shown in Figure 11b.

4.5 3D printing puzzle pieces and core

Before proceeding to fabricate the whole 3D puzzle, we undertook a prototyping phase. This has proven to be crucial in the overall design and fabrication process. Hence, a sample set of pieces were 3D printed to validate the dimensions of the design as well as to check whether the overall measurements, infill density, material strength and weight were suitable for the puzzle pot activity (see Figure 13a). Colours and textures were also tested (see Figure 13b). Various adjustments were made to the dimensions of the design to take into account tolerances caused by attaching printed pieces together and the layer thickness of the print. This thickness usually depends on the nozzle size and the machine and varies for different printing technologies.

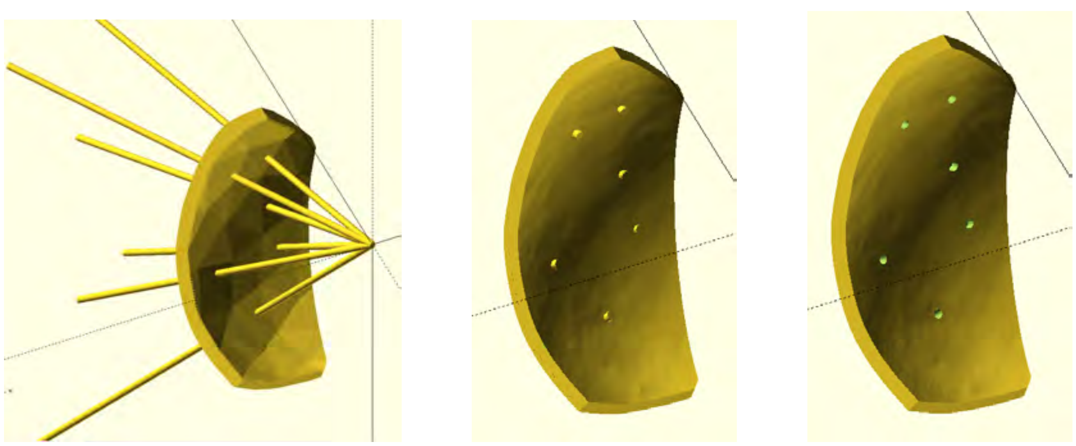
Finally, all the puzzle pieces were 3D printed, as shown in Figure 14a. Although the core could be printed all at once, it was split into four sections to achieve better printing quality and less supporting material by allowing each section to be positioned flat on the printer's bed.



(a) Cylinders generated using as input a set of points in 3D space

(b) Core with blind-holes across the surface

Fig. 11. Generating matching holes for the magnets which constitute the attachment mechanism for the Saltdean urn puzzle.



(a) Puzzle piece with cylinders for blind-holes

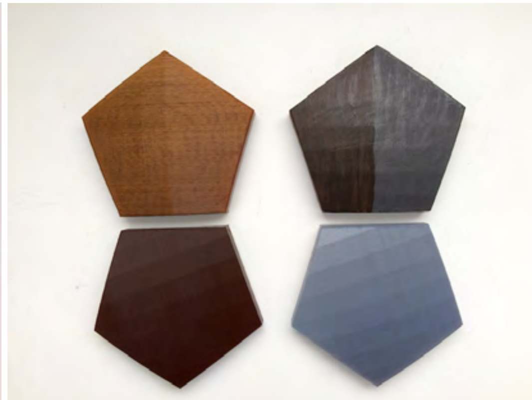
(b) Puzzle piece with geometry generated for the creation of each blind-hole

(c) Puzzle piece with blind-holes

Fig. 12. Generating blind holes for each puzzle piece.

All pieces were printed in PLA (Polylactic Acid) filament on a FDM (Fused Deposition Modelling) 3D printer at a 0.2mm layer thickness with an infill value of 12%. The core piece was printed at a 0.4 mm layer thickness.

(a) 3D printed prototypes for validation



(b) Colour and texture for the puzzle pieces

Fig. 13. Prototyping was performed before full fabrication of puzzle.

4.6 Post-processing all puzzle pieces and core

Post-processing of the puzzle pieces and the core included removing the supporting material around the pieces and sanding any rough surfaces. Then, the magnets were inserted in holes at the back of each puzzle piece and on the core. The magnets that were considered as most suitable, after testing various types, were ø 6mm × 4mm N52 high grade neodymium disk magnets with 1798g pull strength. After placing the magnets in the core and puzzle pieces, the holes were covered with plaster and sanded accordingly.

When the plaster dried, a coating using a mixture of PVA (Polyvinyl acetate) glue, marble powder, white acrylic colour which worked as a primer and water was applied on the puzzle pieces and core in order to provide a ceramic-like texture. Figure 14b demonstrates the 3D puzzle assembled before the final application of paint. Finally, an artist painted the pieces to resemble the original pot and the core (see Figure 14c). Lines to provide clues about the shape of the puzzle pieces and facilitate assembly were also painted on the core. The final interactive exhibit was then placed at the museum gallery (see Figure 14d).

5 PRELIMINARY USER EVALUATION

The puzzle, its design and performance were initially tested with the design team and the curators of the museum. Functional testing has been an iterative process throughout the digital fabrication workflow to see whether the requirements of the activity/exhibit were met. The feedback from this process informed design decisions at subsequent steps of the workflow.

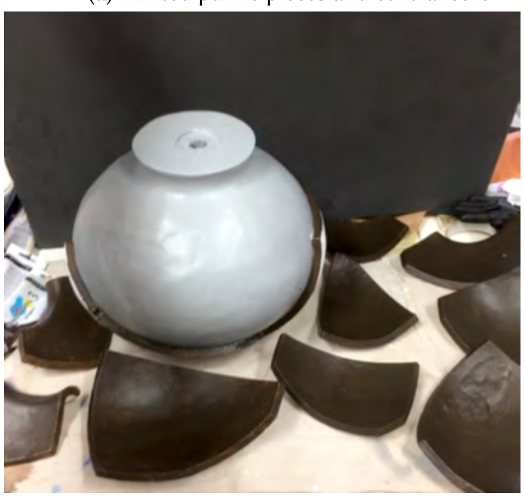
The evaluation of the puzzle pot activity and its performance in terms of enhancing the visiting experience for families of the Archaeology Gallery has already taken place and the detailed analysis of data will be presented in the future as part of a larger research project on digitally fabricated interpretation material. The method for the evaluation of the overall experience is similar to the one tested with another artefact and museum [28, 29] and deploys both quantitative and qualitative data collection.

A preliminary analysis of data with respect to the puzzle pot highlights the importance that hands-on activities have for families visiting the Archaeology Gallery as well as the excellent potential that 3D printing and its applications can have in this domain.

Observational data collected during the evaluation sessions demonstrate that the activity successfully engages families and especially children even beyond the targeted age (6-12 year old). All the families that participated

(a) Printed puzzle pieces and central core

(b) Post-processed puzzle pieces and central core



(c) Painting puzzle pieces and central core, photo courtesy of Russell Webb

(d) Painted and assembled puzzle at the museum gallery

Fig. 14. Fabricated puzzle pieces and central core generated by proposed workflow and final hands-on exhibit.

in the research were able to assemble the puzzle in times that range between less than 1 minute and 3 minutes depending on the age and skills of the child/children and the provision of adult support in the assembly process. None of the participating families quit. Also, the majority of users positively ranked the features of the replica and found the puzzle appropriate for children of the targeted age to assemble. Many users highlighted the significance of helpful clues, such as the outline of each piece on the puzzle core and the colour/pattern details on the pieces.

Some further remarks, resulting from observational and interview data, demonstrate the importance of the overall setup for the activity. Hence, we have found that the rotation of the core and base facilitates assembling the puzzle. Placing the activity at an appropriate height is also important as younger members of the family might want to assemble the puzzle too. As for the space where the puzzle pieces rest, after some observation the museum included a physical barrier in order to store the pieces and prevent them from falling down and possibly braking.

Lastly, it has been noticed that visitors are more willing to try the activity when they find the puzzle unassembled and its pieces lying around, instead of seeing the fragmented pot already put together. An explicit invitation to assemble the puzzle is essential to overcome some people's reluctance to touch an object inside the exhibition setting.

6 DISCUSSION AND CONCLUSIONS

This paper presented a novel digital workflow for the generation of 3D puzzles for museum galleries. The workflow was deployed with a particular artefact, the Saltdean urn. The 3D puzzle activity is part of the Archaeology Gallery at the Brighton Museum and Art Gallery.

The proposed workflow's input is an artefact which is digitised and converted into a digital format. A series of steps are then employed in order to generate printable puzzle fragments or shards and the attachments, such as the blind-holes for the magnets on the pieces and core. An additional contribution of the paper includes an algorithm for generating alternative fracture patterns in order to generate different types of puzzle pieces.

Some of the challenges that we experienced and could be faced when creating similar puzzle activities are related both to the design and fabrication steps. Firstly, although the generation of the digital puzzle pieces can be mostly automated; the selection of the right fragmentation pattern might require of various iterations in order to comply with the design requirements. In addition, it should not be underestimated the time required for fabrication as well as post-processing steps (e.g. pasting pieces together, using certain materials), which might greatly affect the final appearance and functionality of the exhibit. That is why prototyping and iterative testing is crucial, in order to understand how measurements, settings and processes might need to change in order to successfully produce a satisfying result.

A final consideration is given to how the proposed solution compares to other fabrication methods of archaeological puzzles. For instance, a similar puzzle to the one produced could be made using a more traditional approach: by modelling a pot from a hard wearing clay, firing and glazing it. This pot could then be smashed, reassembled and a core could be made (even though producing a core should be a rather demanding task). The whole process would not be very expensive, as it would probably cost one third of the cost of our proposed approach, and also requires only access to clay and a kiln. However, the puzzle pieces cannot be fragmented in a controlled manner. For instance, the pieces would not necessarily break with the dimensions required and in equal sizes, the magnets would be difficult to fit neatly. Moreover, if a single piece disappeared from the gallery, the process to replace the piece would be costly and complex.

Alternatively, it is possible to make a pot using clay and produce a mould or negative using plaster of Paris or silicone. Tin would then be used to make the shard shapes in the mould. The puzzle pieces and core would then be casted in jesmonite. Holes for the magnets would have to be drilled in the casted material. Although this process is slightly more expensive than the previous one, it would be possible to replace a piece if this was lost. However, the mould would require to be carefully guarded so it would not get lost, and it would suffer of inevitable wear out. The latter issue would affect the reproduction of subsequent copies of puzzle pieces.

The proposed approach has the following innovations in relation to the previous approaches: i) it uses an authentic artefact of the collection, ii) it is far simpler and more cost-effective making multiple copies or replacements once the digital design and testing is done, iii) it allows the generation of puzzles of other shapes and sizes in a more cost-effective way, and iv) the digital model is a valuable outcome by itself, for instance it can be used in interactive puzzle-making applications on the web and can be shared with other museums in case they wanted to replicate the physical experience.

In addition, the innovative aspects of the proposed hierarchical algorithm with respect to other approaches is that it mimics the way cracks are created in pottery objects by hierarchical fracture processes [33]. The proposed algorithm also enables the user to manipulate various parameters which can affect the number and raggedness of the resulting puzzle pieces. Although there is an element of randomness in the fractures generation process; the two-step approach for generating puzzle pieces has its advantages as it allows the user to further manipulate the 3D geometry and fracture patterns to generate different versions of puzzles.

To conclude, we argue that the significance of the proposed workflow is that it can provide a CH organisation with a cost-effective "future-proof" solution. Hence, the process can be easily repeated either to replace lost pieces of the puzzle or replicate the whole exhibit with minor changes. Moreover, the presented process is relatively low cost in comparison to other traditional design and production methods and can be deployed to enhance the interpretation of artefacts in heritage environments.

Future work will present the effects that such an object and activity have in engaging young audiences as well as analysing the audience's opinion about the physical characteristics of the puzzle.

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A FRACTAL HIERARCHICAL FRACTURE

In the remainder, we provide a more formal description of the hierarchical fracture algorithm used in Section 4.2.

Fracture lines (along which larger fragments are split) and fragment outlines are represented as *spherical polygons* on the unit sphere. Once each final fragment's outline is determined in that spherical domain, 3D fragments are created by Boolean intersection of the artefact's geometry with sphere sector polytopes spanned by each fragment outline (see Section 4.3).

Spherical polygons are represented as an (ordered) sequence of points on the unit sphere, represented as vectors $p_i \in \mathbb{R}^3$, $\|p_i\| = 1$. Their edges are defined as great-circle segments connecting consecutive P_i and P_{i+1}, and the geodesic distance between two points is the arc length of that segment. The area of a simple (non-selfintersecting) m-sided spherical polygon is $(\sum_{i=1}^{m} A_i) - (m-2)\pi$, with A_i its ith interior angle.

Many geometric algorithms, such as inside/out tests for polygons, have straight-forward correspondences on the spherical domain; however, edge cases exist due to the antipodal ambiguity in spherical geometry. We eliminate these cases by discarding fracture attempts that would lead to polygons straddling more than one hemisphere of the domain.

A pseudo-code summary of our hierarchical fracture approach can be found in Algorithm 1 and 2.

Algorithm 1: Hierarchical fracture.

Input:

N: target number of fragments.

r: maximum area ratio between larger and smaller half of a split fragment outline.

Result:

$\mathcal{F}$: set of N spherical polygons denoting a fragment outline for each puzzle piece.

begin

 // Initialise active set of fragment outlines.

 $P := \mathit{randSphericalPolygon}(\,)$

 $\mathcal{F} := \{P, \overline{P}\}$,

  where $\overline{P}$ denotes the reversed spherical polygon P, with inside and outside regions on the sphere swapped.

 // Loop until N fragment outlines have been reached.

 while $\|\mathcal{F}\| < N$ **do**

  // Pick largest fragment outline and generate random fracture line:

  $P := \operatorname{argmax}_{P \in \mathcal{F}} \mathit{area}(P)$ *// pick largest, for even distribution of fragment sizes*

  $C := \mathit{randSphericalPolygon}(\,)$ *// ragged fracture line*

  // Split P with C:

  $P_+ := P \cap C$

  $P_- := P \cap \overline{C}$

  // Test whether resulting halves are well-formed:

  if $P_+ \neq \emptyset \wedge P_- \neq \emptyset \wedge \mathit{isSimple}(P_+) \wedge \mathit{isSimple}(P_-)$

   $\wedge \max(\mathit{area}(P_+)/\mathit{area}(P_-), \mathit{area}(P_-)/\mathit{area}(P_+)) < r$ **then**

   // Replace P in $\mathcal{F}$ by its two halves:

   $\mathcal{F} := (\mathcal{F} \setminus P) \cup \{P_+, P_-\}$

  end

 end

end

Algorithm 2: Random fracture generator *randSphericalPolygon*( ), based on the 1-D random midpoint displacement method by Fournier et al. [10].

Input:
m: order of initialisation polygon
n: number of recursive bisections of polygon edges
R: amount of random midpoint jitter along the curve
A: amplitude of lateral random displacement
ρ: fractal decay
$\sigma \in [0, 1]$: strength of additional smoothing of the result
Result:
P: random spherical polygon, as a sequence of $p_i \in \mathbb{R}^3, \|p_i\| = 1$

```
begin
    P := regular m-polygon along great circle C_0
    Q := random rotation ∈ SO(3)
    P := QP
    depth := 0
    𝒩 := 1 . . . m        // indices of new points
    while true do
        // Perturb new points.
        a_i := average arc length of edges adjacent to p_i
        t_i := sphere tangent in average direction of edges adjacent to p_i
        t'_i := sphere tangent orthogonal to t_i
        ∀i ∈ 𝒩 : p_i := normalise(p_i + (a_i · R · randSigned( ))t_i + (a_i · A · randSigned( ))t'_i)
          where   randSigned( ) := random value ∈ [−1, 1], uniformly distributed
        // Exit if desired depth is reached.
        if depth = n then
            break
        end
        // Continue with bisection scheme.
        after each p_i, insert bisection point (p_i + p_{1+(i mod ||P||)})/2
        𝒩 := indices of newly inserted points
        // Attenuate amount of randomness.
        R := ρR
        A := ρA
        depth := depth + 1
    end
    P_smooth := circularConvolution(P, 5-tap-sigma-1-Gaussian)
    P := (1 − σ)P + σP_smooth
end
```