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Content-Aware Surface Parameterization for Interactive Restoration of Historical Documents

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Abstract

We present an interactive method to restore severely damaged historical parchments. When damaged by heat in a fire, such manuscripts undergo a complex deformation and contain various geometric distortions such as wrinkling, buckling, and shrinking, rendering them nearly illegible. They cannot be physically flattened due to the risk of further damage. We propose a virtual restoration framework to estimate the non-rigid deformation the parchment underwent and to revert it, making reading the text significantly easier whilst maintaining the veracity of the textual content. We estimate the deformation by combining automatically extracted constraints with user-provided hints informed by domain knowledge. We demonstrate that our method successfully flattens and straightens the text on a variety of pages scanned from a 17th century document which fell victim to fire damage.

1 Introduction

The objective of mesh parametrization is to compute a map that flattens a 3D surface into the 2D plane while introducing as little distortion as possible. In this paper, we study a related but more general problem, where an existing surface has been deformed by an unknown, arbitrary, and complex deformation which we wish to revert. We are motivated by the problem of historical document restoration. A common medium for historical documents is parchment, which is made from limed animal hide. This material is sensitive to the environment and can shrink, swell, and buckle if exposed to heat or humidity, introducing dramatic distortions to its shape. Physically flattening the parchment using conventional conservation methods is impossible since they are very delicate and brittle, and great care must be taken when handling them to minimize the risk of further damage.

The goal of this work is to estimate the deformation that the original document underwent and revert it to virtually restore the document. While the parchments (such as the one shown in Figure 1) are severely deformed, we know that they originally were indeed flat and rectangular with a regular text structure. Such priors allow us to estimate the deformation of a parchment by analyzing its content. Inverting the estimated deformation flattens the geometry of the document, restoring it and its associated texture to its original state. The output of our algorithm is an image of the document, virtually recovered to an extent not possible with physical restoration methods. We use our algorithm to restore

Figure 1: *A distorted document damaged by fire and water (top), is virtually restored to its original state (bottom). Some text lines, used as constraints in our algorithm, are highlighted in blue.*

a variety of pages of the Great Parchment Book (Figure 2), a 17th century property survey of the Ulster estates managed by the City of London, commissioned by Charles I, which has been severely damaged by a fire, leaving its pages extremely fragile. Its content is of great interest to historians studying the history of this region.

Figure 2: *Photograph of several pages of the Great Parchment Book. (Reproduced with the permission of The Honourable The Irish Society and the City of London Corporation, London Metropolitan Archives)*

2 Related Work

Virtual restoration and analysis. Previous works on document flattening often either consider a specific type of deformation [ZT05] or assume high text contrast, facilitating the extraction of features that are used to estimate the underlying deformation [SBR07, TN11]. Fewer assumptions are being made in [BS01] and [SYY*05], where a mass-spring system is used to flatten documents with high levels of physical distortions, but possibly introducing unwanted self-overlaps. A global conformal mapping is used in [BP05] to unfold a document, however, this approach still makes strong assumptions about the type of deformations present. [PCCS11] flattens user-selected rectangular surface strips of a 3D surface, using a parametrization based on an underlying cross field, enabling the analysis of the high-frequency details of the corresponding texture. Similarly, the problem is tackled in [PTW13b] with an interactive local approach. A text direction field is automatically computed and is used to drive an interactive document browser in which local regions of a document can be flattened individually. These methods work well for small regions with locally homogeneous distortion, but cannot be used to globally recover the original undistorted shape.

Mesh parametrization. In mesh deformation [SB09] and parametrization [FH05, SPR06] various energies have been proposed to preserve different intrinsic properties of the mesh geometry as much as possible. These include conformality [LPRM02, SLMB05, MTAD08] and isometry [TSL00, SA07, LZX*08]. Other methods do not rely solely on the metric of the surface but optimize for specific properties of the corresponding texture [SGSH02, BTB02]. Several recent methods [RLL*06, KNP07, BZK09, BLP*13] formulate an energy based on the local alignment of the gradient of the parametrization to a precomputed direction field; [WMZ12, Lip12, SKPSH13, AL13] show how to extend those methods to create locally injective mappings. Such mappings have the property that a surface is guaranteed to have no inversions when flattened, which is clearly a desirable property for a document restoration algorithm.

3 Problem Analysis

Recovering the original flat shape of distorted documents is a difficult task due to the nature of the material and the damage it underwent. The uneven exposure to heat and moisture, the irregular fibre structure, the variations in thickness, and other additional factors such as the presence of scar-tissue make the deformation very irregular. Existing approaches apply surface parametrization methods to tackle this problem. Computing a global, conformal or isometric parametrization may be sufficient for simpler deformations, but is not an adequate solution in general. The conformality and isometry metrics are computed from the 3D geometry which also contains the distortion we wish to undo. In some sense, respecting these metrics would preserve the distortion while flattening the document. This effect can be seen in Figure 3. Local methods

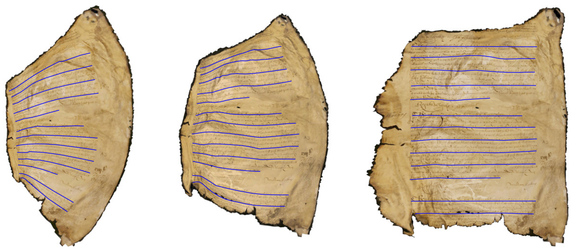


Figure 3: *Global parametrization methods like Least Squares Conformal Maps [LPRM02] (left) or As Rigid As Possible [LZX*08] (middle) try to preserve the characteristics of the original surface, producing unwanted distortion in the text. Our method (right) reverts the deformation caused by fire and moisture damage, straightening the text and increasing its legibility.*

[PTW13b] are successful in flattening individual regions, but are not able to restore the entire document. The rationale for applying a local parametrization strategy is that, for small regions of the surface, it is always possible to find a low distortion parametrization that makes the analysis of the text easier. However, these local, low-distortian reconstructions do not stitch up to a globally consistent solution. That is, the output of this approach is fundamentally different from our global method, and a direct comparison is not possible. Our aim is to estimate the global deformation undergone by each page and to obtain the restored, flat version of the page by reversing the deformation.

Therefore, we define a new metric based on the texture associated with the surface, instead of using the geometry itself. In the case of historical documents, we can make use of domain-specific knowledge. The fact that the parchments originally contained equally spaced, horizontal lines of text, regularly sized letters and a rectangular page margin can be used to estimate the true deformation. The level of damage to the pages makes a completely automatic computation of the deformation unreliable. We hence adopt an interactive approach in which an automatic analysis of the parchment is followed by a semi-automatic phase in which a user refines the estimate of the deformation until a satisfactory solution is found.

4 Method

We represent a scanned historical document as a triangle mesh $\mathcal{S} = \{\mathcal{V}, \mathcal{T}\}$, $\mathcal{V} \subset \mathbb{R}^3$ with a high-resolution texture containing the script. The texture atlas is a 80 megapixels image that is automatically generated, together with the UV mapping, during the acquisition process [PTW13a]. We assume that the original document was flat, and we denote by $f : \mathbb{R}^2 \to \mathcal{S}$ the deformation of the parchment. Our objective is to virtually flatten the document by computing the inverse of f and recover the document's original state.

Our algorithm operates in two steps. We first estimate the Jacobian $\mathbf{J}_f$ of f by analyzing the deformed texture. The text, even if handwritten, has a regular structure that we exploit (Section 4.1). At run-time, we then use $\mathbf{J}_f$, coupled with additional user-constraints, to compute f^{-1} (Section 4.2). The entire pipeline is designed to provide interactive feedback and to enable a domain expert to define additional constraints, and see their effect, in real-time.

As a final optional step we use an image-based technique to remove the shading and discolouration from the parchment, improving legibility while maintaining the character of the text.

4.1 Estimation of $\mathbf{J}_f$

We employ specific domain knowledge on the content of the texture to automatically estimate the Jacobian $\mathbf{J}_f$. The majority of the text was originally of equal scale (with the exception of initials and marginalia) and written in parallel text lines inside a rectangular region of the page. These simple observations allow us to define a sparse set of constraints (based on the scale and orientation of the text) which are sufficient to obtain a good estimate of $\mathbf{J}_f$.

Uniform scaling. An initial estimate of the scaling field is automatically computed by detecting a sparse set of single, "square" characters (letters without ascenders or descenders) on the parchment surface using optical character recognition (OCR). The bounding box of these characters should correspond to the x-height (corpus size) of the text. To detect characters, we apply a pattern-matching approach using templates of a small selection of characters. The templates are generated by manually extracting a number of examples of such characters from the parchment data set and aligning and blending them together. On a new parchment, characters are detected by automatically extracting roughly aligned patches of text at various locations on the parchment surface, using an approach similar to [PTW13b]. For each patch and character template, we compute the normalized cross correlation response image, identifying peaks as matches. To account for the scale variation across the parchment, the matching is performed with a range of scales for each template.

We find that the letters *a* and *n* work best as they typically have a more distinctive shape in the script of the parchments than other letters. Because they are very common, we can always detect a sufficient number in each document. The scale, at a detected character location, is defined as the distance between its top and bottom bounding edges. Figure 4 shows a sparse set of detected characters and a visualization of the isotropic scaling field computed by interpolating the scales from this set.

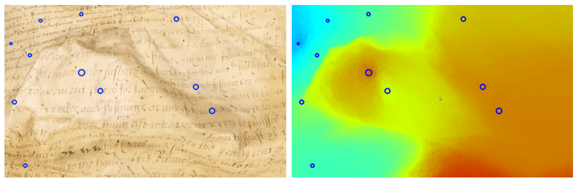
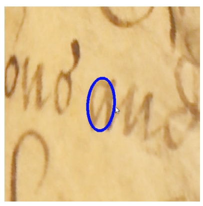


Figure 4: *OCR detects characters in the texture. Their bounding boxes are used to estimate the uniform scaling.*

Anisotropic scaling. In regions with high anisotropic distortion, the OCR approach can fail to detect characters. In these cases, we employ additional user input to specify the anisotropic scale by annotating the flattened page with ellipses, as shown in the inset. The anisotropic scale is defined as the ratio between the lengths of the axes of the ellipses.



Text lines and page margins. Line detection is complicated by the distortions, discolorations, faded text, and pronounced ascending/descending characters which are found in the parchments. These factors can cause the line tracing method to jump between different text lines. Therefore, we use a semi-automatic approach.

Once a user begins to manually trace a line on the flattened page, the system continuously proposes an estimate of the next line section. The user can either accept the suggestion or manually define the next point of the continuing line, forcing the system to adjust its estimation as illustrated in Figure 5. However, the margins of the page tend to be extremely faded or even not visible at all and must be marked manually. Line tracing is performed using the observation that the location of a text baseline corresponds to a minimum in the vertical gradient of the rendered image in the current view. We compute the vertical gradient by convolving it with a Gaussian derivative kernel. Pixels with a steep negative gradient are likely to correspond to a text baseline, and so a baseline can be found by tracing a minimum-cost path through the image which follows these pixels. For each pixel we define a unary cost as the vertical gradient of the normalized gradient image. The binary cost for transitioning from a pixel in column c_i to a pixel in column c_{i+1} is zero if they are in the same row or a constant otherwise (0.75 in our experiments). The minimum-cost path through the image is then computed using dynamic programming. By sampling points uniformly along this path and mapping them back onto the surface, we generate the suggestion for the line section. The user can either accept the suggestion entirely, or instead manually define the next point on the line with a mouse click.

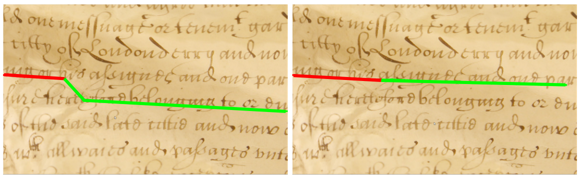


Figure 5: Left*: The automatic line tracing (green) is misguided by the presence of ascenders and descenders.* Right*: After after a single user correction, the suggestion moves back to the correct baseline.*

Parametrizing the Jacobian matrix. Let us represent f using the canonical axes of $\mathbb{R}^2$, u and v. On each triangle $t \in \mathcal{T}$ of the mesh, f is affine; assuming we are given some planar basis for the image of the triangle, then $f(u,v) = (f_1(u,v),\ f_2(u,v))$. The user-assisted character recognition and text line detection give us a sparse set of constraints on $\mathbf{J}_f$. Note that $\mathbf{J}_f$ is a constant 2×2 matrix on t, since f is affine there. To efficiently interpolate the constraints over $\mathcal{S}$, we parametrize the Jacobian $\mathbf{J}_f$ inside each triangle t using three parameters α, β, and γ:

$$\mathbf{J}_f(t) = \begin{pmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} \end{pmatrix} = \begin{pmatrix} \alpha\ \cos(\gamma) & \alpha\ \sin(\gamma) \\ \beta\ \cos(\gamma+\frac{\pi}{2}) & \beta\ \sin(\gamma+\frac{\pi}{2}) \end{pmatrix}.$$

This parametrization of the Jacobian has an intuitive explanation. First of all, we assume that the two rows of $\mathbf{J}_f$, which correspond to the gradients of each component of f, are orthogonal. This stems from the fact that the flow of text gives us a good estimate of the derivative w.r.t. u, and we assume the other derivative to be orthogonal. The parametrization could be extended to encode shear using non-orthogonal components. However, since the text is handwritten by a number of scribes and varies from page to page, as well has within pages, we cannot make useful assumptions about the correct angle between vertical strokes and the baseline, and so any attempt to annotate shear onto the page would contain a significant amount of guesswork. The factors α and β directly represent the anisotropic scaling, while γ is the angle between the first row of $\mathbf{J}_f$ and an arbitrarily chosen edge of the triangle.

The constraints extracted from the texture are directly mapped to constraints in $\mathbf{J}_f$. The length of the axes of each ellipse fixes α and β for the Jacobian of the triangle that contains it. Similarly, each text line fixes the γ for the Jacobian of all triangles that intersect the line. This sparse set of constraints is interpolated to the entire surface by solving three linear systems, as detailed in the following.

Interpolation of the anisotropic scale. We observed that the scale variation is more pronounced on the creases of the parchment, while being considerably lower on flat parts. We believe that this is caused by the irregular fiber structure of the limed animal hide. We model this behavior by approximating the scaling as an harmonic function that interpolates the sparse set of constraints, with a special weighting scheme (i.e., with a special metric). The interpolant for α is computed by solving the following linear system:

$$\mathbf{L}\ \mathbf{h} = \mathbf{0},\ \ \mathbf{h}(p_i) = \mathbf{h}_0(p_i)\ \ \ \forall p_i \in \mathcal{C} \tag{1}$$

where $\mathcal{C} \subset \mathcal{V}$ is the set of vertices containing an estimate for α and $\mathbf{h}_0$ is their estimated value. $\mathbf{L}$ is a discrete Laplacian matrix defined as:

$$\mathbf{L}_{i,j} = \sum_{j\in N(i)} \left(\frac{1}{2}(\mathbf{n}_i^T\mathbf{n}_j+1)\right)^4 \tag{2}$$

for off-diagonal entries ($i \neq j$), and on the diagonal $\mathbf{L}$ has minus the sum of all entries in the same row; $\mathbf{n}_i$ and $\mathbf{n}_j$ are the normals of the vertex i and j respectively; $N(i)$ is the set of vertices that are connected to i by an edge. The factor β is interpolated in the same way. With this weighting scheme, the edges in flat regions receive a high weight, favoring the diffusion of the scale where the curvature is low.

Interpolation of the text flow. The angle γ is interpolated by generating a smooth vector field over $\mathcal{S}$ that interpolates the given directional constraints using [RVLL08, CDS10].

Integrability. Interactivity in our application is guaranteed, since the three interpolations are very efficient. They only require a linear system solve of the size of the number of vertices of $\mathcal{S}$, which in our datasets is always lower than 50k. However, there is no guarantee that the interpolated Jacobian $\mathbf{J}_f$ will be integrable, i.e., that a function f, whose Jacobian is $\mathbf{J}_f$, exists. In the next Section, we will look for an approximation of this function, or equivalently, we will modify $\mathbf{J}_f$ to make it integrable by solving a non-linear variational problem.

4.2 Computation of f^{-1}

Given the Jacobian of the unknown deformation f, we can compute the Jacobian of its inverse using the inverse function theorem:

$$\mathbf{J}_{f^{-1}}(t) = \mathbf{J}_f^{-1}(t). \tag{3}$$

We then define f^{-1} to be the function whose Jacobian matches $\mathbf{J}_{f^{-1}}$ in the least square sense. We parametrize f^{-1} as a set of 2D coordinates for the vertices of $\mathcal{S}$, denoted as $(\mathbf{u}, \mathbf{v})$, which can be found by solving the following Poisson problem:

$$f^{-1}(\mathcal{S}) = \operatorname*{argmin}_{\mathbf{u},\mathbf{v}} \sum_{t \in \mathcal{S}} \left\| \begin{pmatrix} \nabla\, \mathbf{u}(t) \\ \nabla\, \mathbf{v}(t) \end{pmatrix} - \mathbf{J}_{f^{-1}}(t) \right\|_F^2 . \tag{4}$$

Straight-line constraints. In Section 4.1, we extracted lines of text and used them to estimate $\mathbf{J}_f$. To guarantee that the text in these lines will be perfectly horizontal when flattened, we introduce additional hard constraints for each pair of points $p_i, p_j \in \mathcal{S}$ that lie on the same line of text on the deformed parchment:

$$f_u^{-1}(p_i) = f_u^{-1}(p_j) \tag{5}$$

Note that in general p_i might not correspond to a vertex of $\mathcal{S}$; in this case p_i will be expressed as a linear combination of the three vertices of the triangle that contains it. A similar constraint is also used for the page borders.

Fold-over avoidance. Solving Equation 4 subject to the constraints in Equation 5 is not sufficient when the parchment is heavily distorted. Large regions of the parchment might flip their orientation when flattened, causing self-overlaps and hiding some portions of the document (see Figure 6). We introduce additional constraints to prevent this:

$$\det\left(\mathbf{J}_f^{-1}(t)\right) > 0, \quad \forall t \in \mathcal{S} \tag{6}$$

Solver. Solving the minimization problem in Equation 4 subject to (5) and (6) is numerically challenging. General purpose nonlinear solvers could be used, but they would be too inefficient to allow dynamic updates of the constraints. We use instead the solver proposed in [SKPSH13] that is specifically designed to interactively minimize a deformation energy while keeping the determinant of the Jacobian positive. We use Tutte's parametrization to generate a bijective map by fixing the parchment's boundary to a rectangle. This map is used as the starting point of the optimization. Straight-line constraints are added to the energy as a penalty term (Equation (1) in [SKPSH13]), and fold-over avoidance is guaranteed by using barrier terms (Equation (2) in [SKPSH13]).

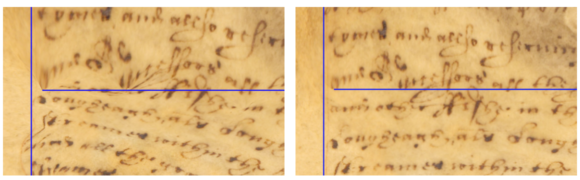


Figure 6: *A large region of the parchment is folded over, hiding many words of the script (left). By constraining the determinant of the Jacobian to be positive, the fold-overs are eliminated (right), revealing the original script.*

5 Results

We present the results of our system applied to a variety of pages from The Great Parchment Book (Figure 2), which were scanned using a multiview stereo based pipeline [PTW13a]. Our system is applicable to a wide variety of documents and is independent of the particular 3D acquisition method. The entire pipeline of the system is summarized in Figure 7.

Figure 8 shows the effect of different constraints in the virtual restoration process.

We successfully applied our algorithm on six pages of the Great Parchment Book, as demonstrated in Figure 9. The parchments exhibit different levels of damage; in all cases our algorithm restores the documents only requiring a small number of user-provided constraints. In the additional material, we provide high-resolution images for all our results.

Note that even after successful rectification, the color texture still exhibits intensity and color variations that convey the false impression the parchment was still wrinkled. These variations exhibit the baked-in shading present at the time of acquisition, but also show genuine discoloration taken place in the course of the damage. We find preserving these observed appearance variations a useful feature to study the rectified text in the context of the original damage, mitigating the risk of misinterpreting potential artifacts introduced by over-processing the content [Ter11]. On a more general level, preserving the original texture also serves preservation of veracity, an aspect of vital importance for historians [BK13].

In many cases, however, a reader will prefer a cleaned-up color appearance in addition to the unwarped geometry. We hence optionally remove color variations by normalizing the parchment texture's appearance by independently scaling each color channel by a spatially-varying factor, so that all ink-free regions of the parchment roughly match the same color. To determine this factor for a given surface point, we estimate its observed parchment color by sampling the 95-percentile of pixel intensities in a small, disk-shaped neighborhood. The percentile-based sampling introduces robustness against fine-scale intensity variations (most importantly, those caused by ink), and the resulting smooth scaling field preserves subtle texture variations of the parchment despite the color normalization while removing coarse-scale color variations; Figure 10 shows the results of applying our system on a variety of folios with various types and levels of damage.

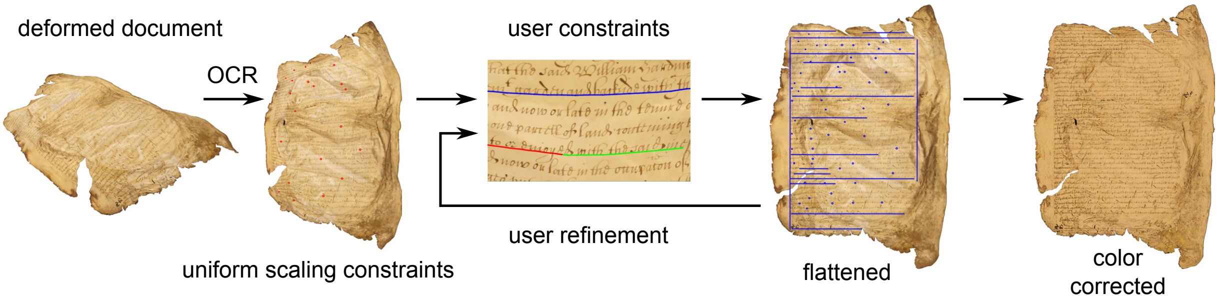


Figure 7: *All steps of our algorithm. From left to right: The original distorted document is flattened using the Jacobian estimated from the OCR analysis. An interactive refinement of the constraints improves the restoration, and is iterated until the desiderata of the domain expert are satisfied. Ultimately, we remove the intensity and color variations from the texture.*

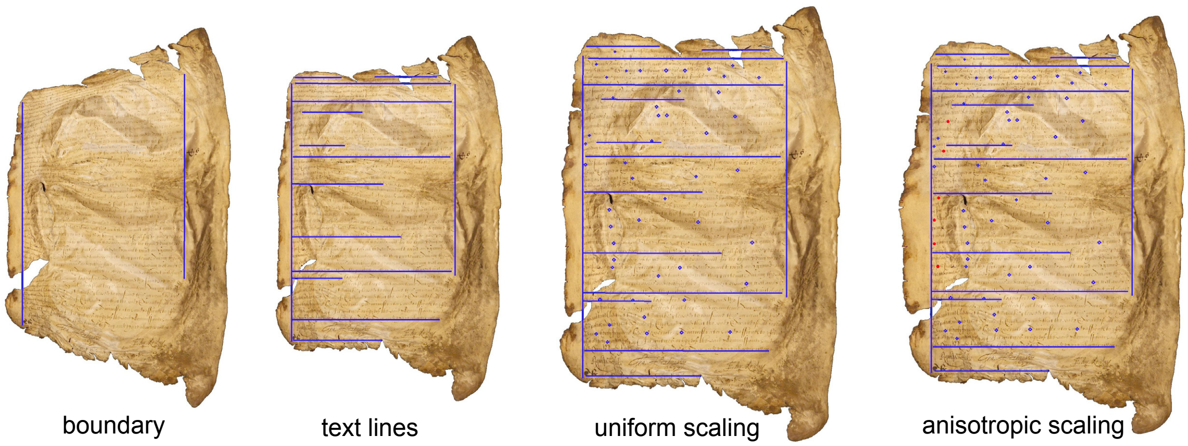


Figure 8: *Effect of different constraints in the virtual restoration process. By constraining only the boundary of the page, a first flattening can be obtained. However, this result still contains pronounced shrinking (visible along its left side), and the text bends around the creases. With additional line constraints, the bending is reduced, and the tear in the bottom left sealed, greatly improving readability. The text shrinking is only solved by introducing the isotropic constraints. The user-provided anisotropic constraints are used to fine tune the left part of the page (highlighted in red).*

Limitations. Our optimization is guaranteed to generate an inversion-free map. However, it is not guaranteed to find a global minimum of the energy due to the nonlinear term which [SKPSH13] uses to prevent inversions. In our current implementation the user can provide unsatisfiable constraints, such as intersecting text lines. In this case, our system will try to satisfy them in a least-squares sense. In all our experiments, the solver satisfied the constraints and converged in less than 10 iterations.

6 Future work and concluding remarks

We presented an interactive method to restore severely damaged and distorted documents. We have shown how other parametrization approaches are unsuitable for this problem because they are designed to maintain the original surface metric, which in our setting is the undesired distortion present in the documents. Instead, we directly estimate the distortion caused by the damage using a sparse set of constraints based on the texture of the documents, and then invert this distortion.

We have adopted an interactive, semi-automatic approach to this problem to enable an expert user, such as a conservator or paleographer, to efficiently guide the estimation of the distortion.

In this work, the extracted features are based on the high-resolution texture associated with the parchments. Combining our analysis with a physical simulation of the deformation of the parchment material could increase the quality of the

Figure 9: *Within each row: The distorted parchments are annotated with the constraints shown in blue (top). The documents are virtually restored, flattening the pages and removing the distortion (bottom).*

results and consequently reduce the required user input. However, due to the unknown, complex and inhomogeneous structure of the material, this is a very challenging problem that we leave as a direction for future work.

We have demonstrated our algorithm on a variety of pages from The Great Parchment Book, and plan to use it to restore all of the remaining pages.

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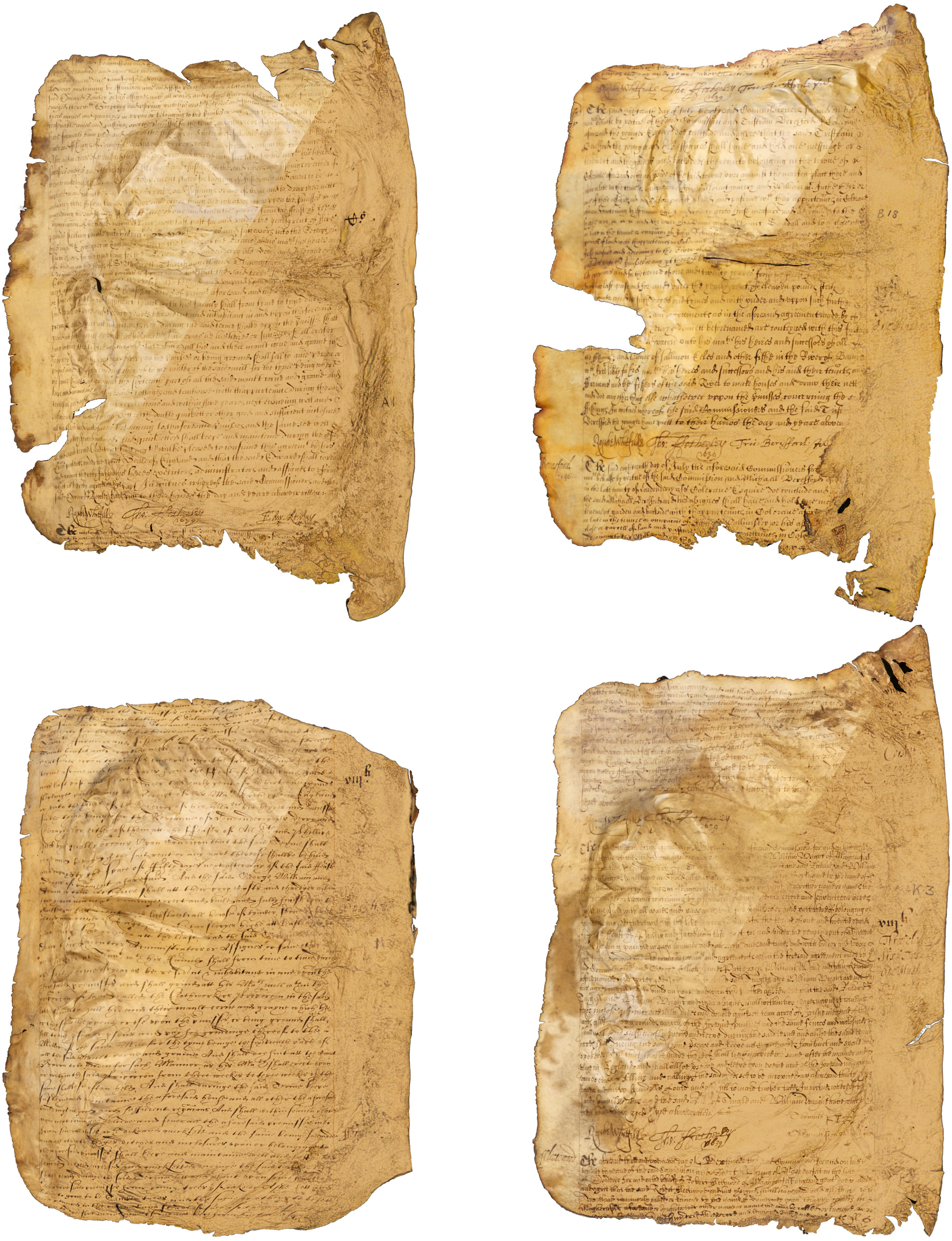

Figure 10: *We show four pages of the Great Parchment Book restored by our algorithm, comparing the result obtained with and without removal of shading and decoloration.*

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