# Closing the Gap Between Formal and Digital Libraries of Mathematics

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**Abstract.** The representational gap between formal mathematics and most users of digital mathematics resources is a challenge for any approach to mathematical knowledge management which aims to combine the benefits of formal and informal mathematics. In this chapter we study this gap in the context of a digital library of mathematics based on the Mizar Mathematical Library and make recommendations for improving such formal systems support for MKM.

## 1 Introduction

The aim of mathematical knowledge management (MKM) is to organise a substantial proportion of all mathematical knowledge to make it more easily and widely available [7]. Much of the current effort in this area could be called *formalist*, in that it is based upon formal representations of mathematical content in various logics [6]. The formalist approach has great potential, as mathematical proof can be guaranteed valid and content is far more amenable to automatic processing.

A key problem for formalist MKM is how to account for the *representational gap* between formal and informal mathematics: how can such technology find a role in mainstream mathematics, given that the vast majority of mathematics is carried out in non-logical representations such as specialist natural language, formulae and diagrams? Most users of mathematics are not versed in formal mathematics and, even if they were, it is not yet clear that it could support their activities adequately. To be clear, translation across the representational gap is generally possible *in principle*. In practice, it requires skilled effort for either a person or a machine (e.g. [21, 27]).

In this chapter we suggest that this gap is best closed by a variant of formalist MKM where formal and informal representations are distinct but closely integrated. In order to illustrate the challenges faced by this approach, we present Mizone, a digital library of mathematics based on the Mizar Mathematical Library (MML) [16]. The collection — shown in Figure 1 — demonstrates the expected gap between a formal mathematics library and a digital library system, and we discuss what needs to be done to close the gap in this context.

Our main conclusion is that formal material needs to be supplemented by additional information if it is to be made palatable — and we make some specific recommendations for Mizar, the MML and similar formal systems. We also

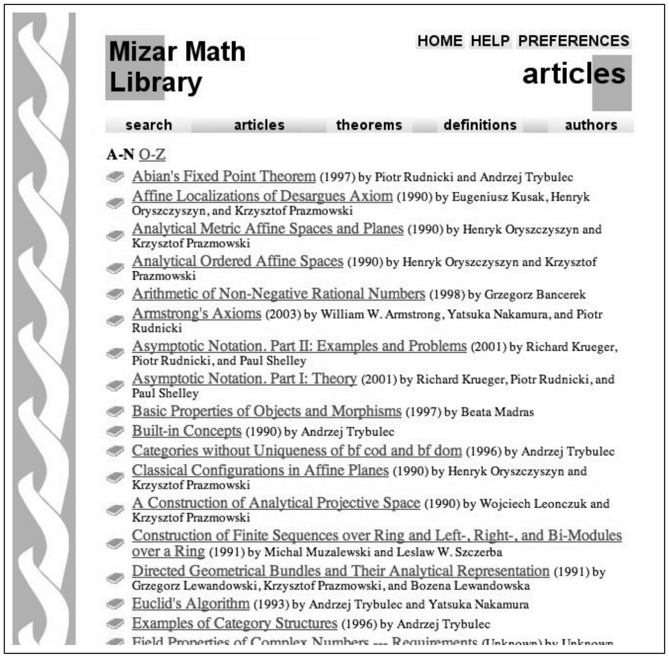


**Fig. 1.** Browsing the article list in Mizone.

discuss how attitudes to formalisation need to change if the integrated approach to MKM is to succeed.

## 2 Minding the Gap

Opinions differ as to if and how the MKM's representational gap should be closed, but we distinguish four fundamental stances here. These caricatures are not necessarily mutually exclusive, and in reality different approaches to MKM might mix them to varying degrees. Note also that this kind of labeling comes with a danger of misrepresentation: in much work on MKM the attitude towards the representational gap is either implicit, agnostic or not present.

The first — *strong formalism* — is that mainstream mathematics should adopt formal mathematics. Mathematicians and other users of mathematics can reap the rewards of the formalist approach by adopting it directly into their working practices. At one extreme, all mathematicians would become users of proof assistants and other systems which, while they may be more advanced than existing systems, require the user to work directly with formal content. MKM would simply become a matter of organising this formal material. Many MKM researchers would reject this strong version of formalism: mathematical practice is unlikely to change this significantly, and formal representations are considered too difficult to work with for widespread adoption. For instance, proofs become much longer when formalised [24].

*Strong informalism* provides MKM solutions using a combination of mainstream mathematical practice and generic knowledge management technology, possibly specialised to the domain but without recourse to formal representations. Currently all mainstream digital mathematical resources fall into this category, such as digital libraries of journal articles. Informalism does not imply that the mathematical content is not rigourous or is unrelated to formal mathematics, but that it has not actually been formalised itself. Again, many MKM researchers would reject the exclusive use of this approach: the widespread adoption of the formalist approach shows that as formal mathematics is viewed as a keystone to MKM. It is both explicit in its content and moreover is provably correct knowledge. Thus, unlike the more informal communications between mathematicians [27], there can be no mistaking the meaning of each theorem.

Both strong formalism and informalism ignores the gap and stay put on one side, and neither currently appeals to the MKM community. We distinguish two further approaches which attempt to bridge the gap: *pluralism* and *integrationism*. Here both formal and informal representations play a role, and it is up to people or A.I. technology to translate between the two. In a pluralist approach formal and informal mathematical activities exist in tandem, but are only weakly connected. For example, a journal article may be formalised and validated by a third-party author. Here MKM involves both formal and informal content, but each is part of separate activities which may involve different people with very different aims. Links between formal and informal content are possible, but generally ad hoc. Following this approach formal mathematics has so far a small impact on MKM via the special cases it has been 'shipped in' — a large scale example of this is the Flyspeck project, which hopes to validate Hales' proof of the Kepler Conjecture using HOL Light [12].

Finally, *integrationism* demands that both formal and informal representations are used and are tightly coupled. In this approach most MKM content has at least one formal and one informal representation which are explicitly related, or at least translation services are available that can provide alternate representations when required. A great deal of MKM research supports or is compatible with this approach. There is work on translation between informal and formal representations [27, 21]. Another key idea is the vision that the same services and databases can provide solutions for a wide range of users and an applications, e.g. dynamically presented content into whatever form is most useful for particular users, using the presentational conventions of different disciplines or cultures. Many MKM researchers seem to favour some kind of intergrated approach, at least in theory.

Of course, real MKM solutions may not conform to these neat positions. For example, Barendregt and Wiedijk's vision of computer mathematics [5] argues for improvements to proof assistants necessary for their adoption by mathematicians, including a more mathematical style of expression and supporting reasoning with gaps. This could be labeled formalist with informalist influence:

a logical representation is made 'more informal' in various ways. Although only a small a representational gap seems to be anticipated, their proposal is also integrationist, in that these representations are bound together within a single system.

We would argue that, in the long term, any approach to MKM needs to decide to what degree their users and systems are formalist or informalist, and if a representational gap remains how strongly integrated the two sides will be. The current trend in MKM is towards some kind of integrationist approach, as opposed to a completely pluralist one.

## 3 An Integrated Approach in Mizar

From the integrationist MKM stance, the rest of the chapter examines some of the problems with an integrated approach, taking the Mizar Mathematical Library (MML) [16] as a exemplar of the formalist approach. Arguably, the MML offers the largest resource of formally proven mathematics. It covers a wide range of topics in from computation through abstract algebra to topology and set theory. Moreover, Mizar uses a rich language that can strongly resemble the natural usage of mathematicians, so the representational gap can be considered slightly narrower than in other systems. With some further processing, this presentation can even be made more natural — as is done for the Journal of Formalised Mathematics [19].

As a mature system Mizar supports its target user very well — one comfortable (or learning to become so) with the Mizar system, language and library and associated tools such as Mizar Emacs mode and MML Query [4]. Such people are actively engaged with developing there own articles, and quite likely in collaboration or contact with Mizar experts or developers. These users have rightly been the focus of Mizar's developers, who are engaged with advancing the state of computer-supported formal mathematics.

However, the audiences and applications for MKM are considerably more diverse than this. For instance, scientists, engineers, economists and the like who need to find the to tackle their particular problems. Here we focus on the dissemination and archiving of mathematical material for professional mathematicians and mathematics students in higher education — a suitable digital library application for a core group of mathematics users. This is a larger and ill-defined than the Mizar user group, but they share a couple of traits. First, such people are interested in proof-centred mathematical content. Secondly, most have little or no understanding or interest in formal mathematics per se [8]. Such people may be step to a wider group of MKM users.

It is clear that there is a significant representational gap between this MKM user group and the MML. To better understand the issues and possible solutions in this context, we used Greenstone [25], a digital library system, to build a simple digital collection based on the MML. Digital libraries are a standard means of delivering collections of (informal) content electronically. They are used widely in publishing diverse collections and are a proven technology that provides valuable resources to knowledge workers in many disciplines [1, 18, 17]. By forcing a library of formal mathematics into a digital library system we hoped to illustrate some aspects of the current representational gap between the MML and MKM users. As one might expect, the result — the *Mizone* collection — is not a particularly usable or useful resource for mainstream MKM. However, the aim here is to highlight the areas in which further effort and research might close the gap.

## 4 Developing the Collection

Greenstone is an open source digital library platform [25, 26]. It was developed as a general research tool for digital library technology and usage, but its flexibility and low entry costs have led to its adoption for numerous real-world projects in a wide range of application areas, such a collections on Maori history, Chopin's music and accounts of the impact of the hurricane Katerina. In 2000, it was adopted by UNESCO as a means to distribute important knowledge on sustainable development to the developing world. This has led to the translation of the Greenstone interface and manuals into forty three different languages. As the MML is a non-standard data set for digital libraries, it made sense to use such a well established and versitile digital library system. The interested reader can find out more about Greenstone, its collections and download the software from `www.greenstone.org`.

The MML consists of a Mizar articles, each made up of a series of items. Each MML article maps to a document within Mizone (see Figure 1), and it was our initial aim to include the entire library. Greenstone allows documents to have a hierarchy of sections and subsections. Each imported Mizar item was mapped to its own section, to allow for theorems and definitions to be individually retrieved and browsed. Not all items were included in Mizone documents. The key concepts in any Mizar article are its definitions and theorems, while we omitted other items which dealt with more the technical details of formalisation and validation within Mizar. The item types which were imported into the Mizone were: `DefinitionBlock`, `NotationBlock`, `JustifiedTheorem`, `Proposition` and `Proof`.

The textual presentation of each item was based on the MML's standard semantic presentation stylesheet [23]. This renders the item as HTML that closely resembles the original Mizar language. It is clear that this presentation could be improved by automatically replacing Mizar keywords with stock natural language phrases, as is done in the Journal of Formalized Mathematics. This is a possible future improvement that will enhance Mizone's readability, but from a theoretical perspective it will probably not tell us much. It was done in a few cases in Mizone as a proof-of-principle, but most of the text is currently in the Mizar style, as shown in Figure 2.

The actual import process automated via scripts, and proceeded as follows: Mizar was run over the MML article, and as a side effect generates an XML representation. A modified version of the semantic presentation stylesheet was

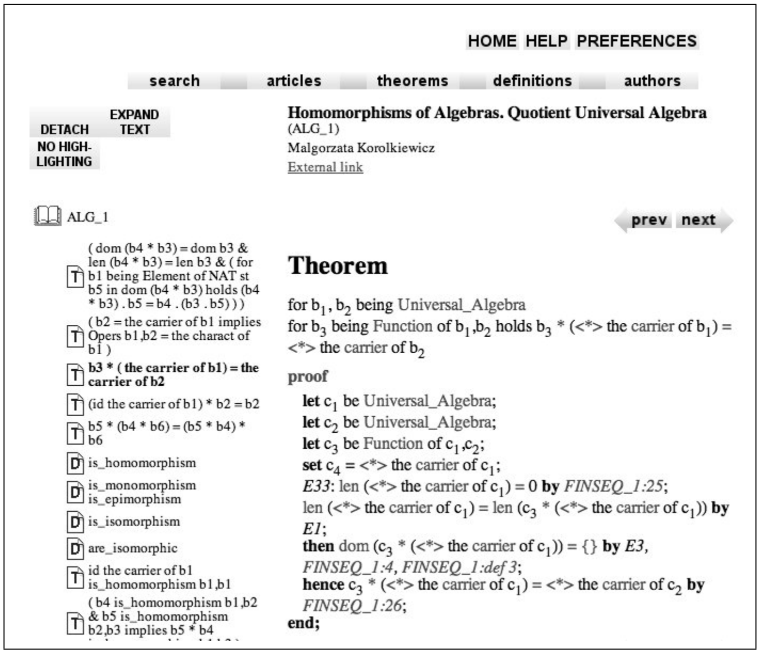


**Fig. 2.** A theorem in Mizone

then used to generate a HTML version of the article, annotated with sectioning information for Greenstone. In a separate process, HTML indexes of the MML and JFM were used to gather metadata about each article, which was merged into a single Greenstone metadata file. The Mizone was then built from the HTML plus metadata using Greenstone. The work was done on a G5 Mac, but a similar process should be possible under Windows or Linux, as both Mizar and Greenstone are support multiple platforms.

Under the default configuration Greenstone is unable to index the entire MML, a problem which can occur with very large collections. This is probably due to the quantity of indvidual terms to be indexed, rather than the actual size of the collection. The current collection is a substantial subset of the MML, but we hope eventually to include the entire library.

## 5 Using the Collection

As with all Greenstone collections, Mizone is accessed by a web interface by default, with the initial page consisting of a simple search box and some instructions for using the system. Greenstone supports browsing of documents by *classifiers*, where a list of values is displayed and selecting a value brings up a sublist of related articles. Mizone currently supports browsing by article title, definitions, theorems and article authors. Browsing modes and search are

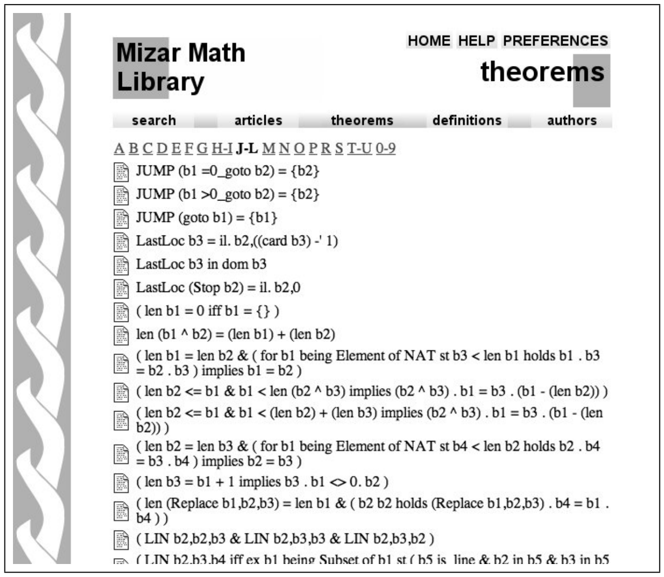


**Fig. 3.** Browsing the theorem list in the Mizone

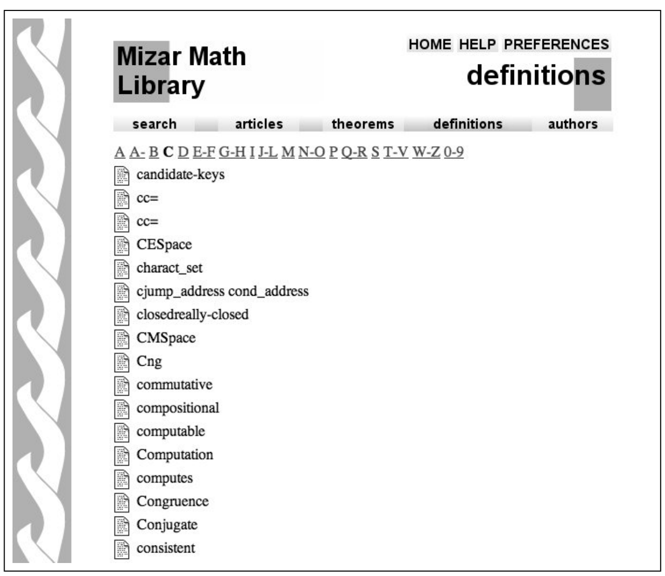


**Fig. 4.** Browsing the definition list in the Mizone

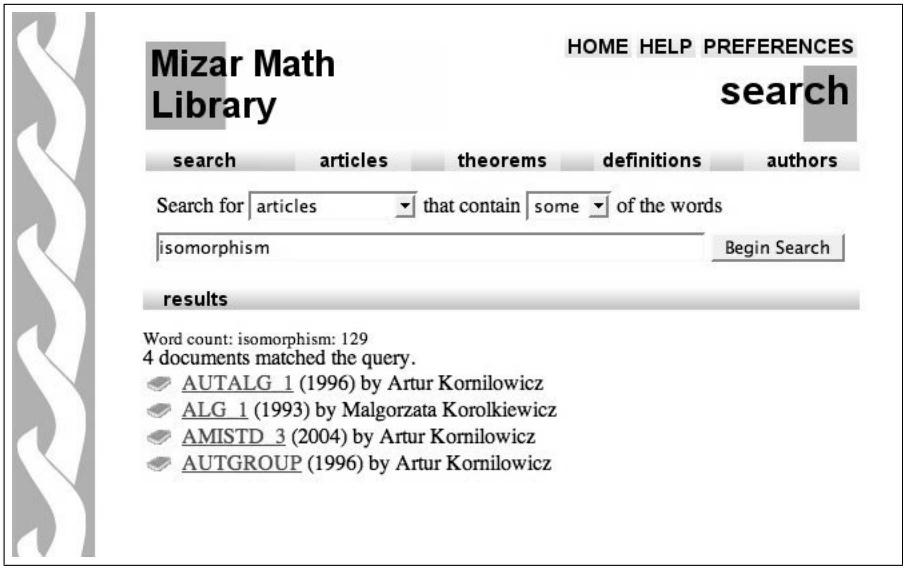


**Fig. 5.** Searching in Mizone

selected via a horizontal classifier/search menu bar that is always visible. Browsing by article title, theorem statement and defined symbol are shown in Figures 1, 4 and 3 respectively. The results of a keyword search are shown in Figure 5.

Selecting an article during browsing or via search results brings up an article page, as shown in Figures 2 and 6. Below the classifier/search bar is a description of the article: title, MML name and authors. Most articles include an external link here to their JFM page. Down the left of the page is a section navigation bar, with rest of the page initially blank and used to display the current section when one is selected. Previous/next arrows are displayed to allow the reader to progress through the sections in order.

The Greenstone import process preserves HTML links between articles, but unfortunately does not currently preserve links to individual items within articles. Instead, the link takes the reader to the top of the item's article.

## 6 Critique

We now evaluate Mizone as an information resource, considering the user group of mathematics professionals. Recall that the goal is not to produce a fully usable system that could be presented to such users but rather a functioning prototype that highlights issues with a integrated approach to MKM. This is not to say that the prototype should be unusable and we would hope that those comfortable with the formal mathematics of the MML would find Mizone useful.

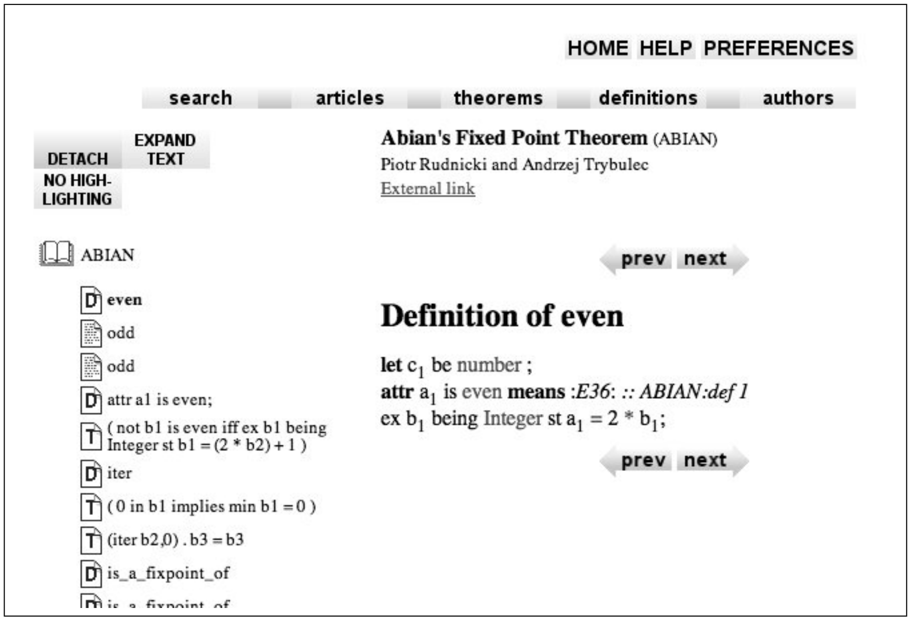


**Fig. 6.** A definition in Mizone.

The questions arising from Mizone are therefore about the relationship between the formal mathematics and the more general requirements of a mathematical knowledge repository: How does it compare to other mathematical resources, formal and informal? How does it compare to other resources based on the MML? What does this tell us about formal mathematics and digital library design? What role can formal mathematics play in digital mathematics libraries? How can the representational gap in Mizone be closed?

We discuss each of our main areas of concern about Mizone below.

### 6.1 Presentation of Syntax

As mentioned in §4, article items are presented in a similar style to the Mizar input syntax, reconstructed from the XML output of Mizar. It is already well-understood that readability can be greatly improved by keyword replacement, i.e. replacing formal syntax with stock phrases in natural language, as is done in the JFM [19]. This does not tell us anything new about the representational gap, but the problem is still severe in Mizone. Although the Mizar language is relatively readable, its target audience would find it quite unnatural to use formal syntax throughout, even though they may be familiar with similar notation in their mathematics. We hope to deal with this in a future version of Mizone.

A more serious problem is the verbosity of the syntax [24]— a problem which keyword replacement would only make worse. Rendering formal mathematics in natural language using more sophisticated techniques has already been identified as a research topic by MKM, and this has the potential to generate reasonable length texts. Such an approach is badly needed here.

Another issue, already discussed in [9], is that the original Mizar article is not faithfully reproduced by Mizar's XML output. It loses the original variable names and precise syntactic structure of terms, as they are irrelevant to its function as a proof checker. Although Mizone is built upon a logically identical XML representation, it is the actual language used that can often provides important cues to the reader about what the mathematics is about [27]. Symbols, syntax and terms are important parts of that.

### 6.2 Informal Commentary

Other than the syntax, the most noticeable feature of Mizone is the lack of any commentary about the mathematics it presents. This kind of material is crucial for our user group, who are used to the more informal 'glue' content that provides examples, motivation and significance that are necessary for the mathematician but not the mathematics. A short-term solution for Mizone would be to import the prose abstracts from the JFM [19], though this would only provide a limited amount of commentary for each article.

Interleaving of such commentary material with formal mathematics is allowed in some systems, such as the Formal Digital Library Navigator [14], but is not practiced in the MML. This is the basic principle behind literate programming [13, 22], which has been applied to formal mathematics in the maze system [10]. This approach places reader-oriented content in comments, to be ignored by Mizar but to be interpreted as commands for a presentation system like Greenstone.

Problems arise when referential links need to cross the formal/informal boundary. For instance, commentary on a theorem should be explicitly linked to the formal theorem object in some way, rather than just collocated. Likewise, discussion of mathematical examples would benefit by being linked to formal example objects whose properties could be formally checked as well as being informally discussed. It is interesting that examples are part of the MML formalised mathematics, but not used to explicitly illustrate other material.

### 6.3 Presentation of Theorems and Definitions

The names of theorems and definitions are used in Mizone as a title for each item's section, which appears as a title on that section's page and in the section menu. These names are currently extracted from the corresponding formal object. In the case of theorems, the basic statement of the theorem without quantification is given. For definitions (and redefinitions), the defined symbol is used.

For theorems, this approach has the disadvantage that the theorem statement may be too lengthy to make a comprehensible and recognisible label. Even better syntactic processing — such as dropping preconditions — will only work in some cases. A problem for both theorems and definitions is that, even when short, the label may still be too obscure for readers not familiar with the formalisation. An obvious fix is to have authors provide additional natural language labels for all formal objects, along with an short version to be used in menus.

We described in §4 how we selected particular kinds of formal items to be included in Mizone, e.g. `DefinitionBlock`. Unfortunately, this meant every one of these items was included, irrespective of its importance to the article, and that other kinds of item were totally inaccessible, preventing the interested reader from finding out the details of the formalisation. It would clearly be preferable if this selection could be made more by the original author, and changed dynamically by the reader.

### 6.4 Article Structure

Articles however have no explicit internal structure other than that inherited from the Mizar language. Mizone reflects the breakdown of articles into theorems and definitions determined by that language, and this fits well with mathematical practice. Notionally, there are sections in the original Mizar articles, but as with syntactic forms (see §6.1) these are omitted from the XML output and hence not in Mizone. A not untypically lengthy article like *Armstrong's Axioms* [2] show the consequences of this very flat hierarchical structure: the side menu that is meant to aid navigation goes on for several screens. It is not clear which items are key, which are not (e.g. lemmas) and what the logical or thematic grouping is between any of them.

This presents a problem for the reader to trying to form a coherent perception of the whole article rather than any particular constituent. Appropriate thematic sections are common practice in mathematics and would alleviate this Problems such as these are not unique to the MML, and have arisen in other Greenstone collections, but they are particularly acute in a mathematical context where users will often want to navigate between sections for reference. In fact, many MML articles do contain appropriate sections in comments — but as explained in §6.1, these are not accessible in Mizar's XML output.

An issue that does seem unique to mathematics is that there is a huge amount of cross-referencing between sections and articles. This can be definitions building on previous definitions or proofs that can employ existing theorems both within a given article and between articles. In our experience, no other Greenstone collection has required such dense cross-referencing. As mentioned in §4, the system does not currently handle direct navigation into subsections when the user follows a linked cross-reference. As discussed in [8], the presence of many cross-reference links does not help the reader identify which links are important for understanding the mathematics. It would be better if authors could include this information in their original articles.

### 6.5 Proof Structure

Mizone uses indenting to show the hierarchical structure of proofs. However, it is clear that long proofs — for instance, Abian:8 [20] — with deep hierarchies present a challenge to the reader in terms of understanding the proof as a coherent whole. In more informal mathematics, it is common to split up long proofs into smaller sections or, where that is not possible or reasonable, to provide indications of where in the progression of the proof the reader is. Such structuring information could be added by the author, though it is not clear what form this could take.

Another possibility is interactively represent the hierarchical structure of the proof as in Lamport's method [15]. It would be possible to add such interactivity to Mizone by customising Greenstone's interface with appropriately scripted HTML. This could help manage the complexity of proofs, and allow proof overviews to be expanded on demand. However, in a previous study we found that interactive Lamport proofs may present problems for some mathematical readers [8] — if so, this would be second-best to more explicit attempt to guide the presentation of the proof. It may be that no matter what informal content is included in an article, the structure of a Mizar proof is always going to jar with the expectations of mathematicians just as the Lamport presentational style may. This issue can only be resolved by implementing and developing systems such as Mizone and then evaluating them with mathematicians.

### 6.6 Search

Perhaps the biggest disadvantage of Mizone is that it has no specialised search tools. Over the past few years, signifcant development effort has gone into producing tools that are valuable to Mizar author, like MMLQuery [3, 4]. This allows users to search the MML in sophisticated ways such as looking for the close cooccurrence of terms in Mizar articles.

Mizone is currently only using a standard search tool that looks for terms occuring in documents. The collection is configured to allow keyword search over articles, article titles, authors, theorem titles and definition titles. Search over multiple fields is not currently supported by the collection. Keyword search combined with a lack of informal labels for items (see 6.3) means the user has to guess the particular syntax, and that search for terms is extremely limited.

It is not clear what would make an appropriate search tool for mathematicians using Mizone. Keyword search may be more effective in this context if informal commentaries (see §6.2) and similar material were available for indexing. Tools like MMLQuery, though effective for formal content, have a complex syntax of their own and are partly dependent on a knowledge of formal mathematics. More informal yet general methods, such as that used in the Alcor system, present an alternative approach [11]. Overall though, good searching presents issues of how dependent a search tool needs to be on the particular language of the mathematics and more importantly what the goals of the users are. In essence then, resolving this is one of the major goals of MKM generally.

Bespoke search interfaces can be added to Greenstone with some effort, as has been done for a music collection with both a musical keyboard and audio input methods. Thus, it should be possible to have Greenstone versions of existing search tools such as Alcor and MMLQuery. This could form the basis for evaluating the value of such tools with a more general audience.

## 7 Recommendations

As well as indicating how Mizone could be further developed, our critique contained several insights into the representational gap between the digital collection and its intended readers. We summarise these here as a set of recommendations that will help close this gap between libraries of formal mathematics and users of mathematical digital libraries. Firstly, we have recommendations for systems and libraries that support formal mathematics, including Mizar and the MML. These are not criticisms of Mizar as a system, but of recommendations for formal mathematics within MKM:

– Documents export formats (e.g. XML) should allow the author's original input article to be fully reconstructed. (§6.1, §6.4)
– Informal commentary material should be included for general readers. (§6.2)
– Formal objects that represent examples should be linked to any other material illustrate. (§6.2)
– Clear natural language labels should be assigned to definitions, theorems etc., with additional short labels to be used for menus. (§6.3)
– Indicate which formal objects should be presented to the general reader, and which are trivial or unnecessary details. (§6.3)
– Group items into thematic sections (§6.4)
– Indicate which cross-references between formal objects are significant for understanding, and which are merely book-keeping. (§6.5)

Secondly, we have recommendations for MKM research in general. Some work has already begun to address these questions, but they are significant challenges for MKM:

– How can formal mathematics be presented in natural language to effectively handle verbosity and clarity. (§6.1)
– Can literate proving support more flexible cross-referencing between informal and formal content? (§6.2)
– How can authors structure or annotate proofs so that proof presentations are more readable? (§6.5)
– Are interactive hierarchical proof suitable for the general reader? (§6.5)
– How can formal mathematics search tools be made more accessible to non-specialists? (§6.6)

## 8 Conclusions

Building and assessing Mizone has been a useful exercise in understanding the representational gap between formal and mainstream mathematics. We argued that this is critical if an integrated approach to MKM is to succeed — that is, one in which formal and informal representations are used and kept tightly coupled. Our critique of Mizone has allowed us to make some concrete recommendations for Mizar-like systems in this context, and to highlight need for more research to understand issues of search and presentation.

The key to the success of these recommendations is the authors of formal mathematics. While automation will always have a role and may improve, we still need authors to annotate, structure and supplement their formal mathematics if it is to find a wider role in an integrated approach to MKM. The problem is similar to the need for programmers to comment their code. However, in MKM we cannot assume the reader will actually understand the code! Rather, they are probably just interested in the ideas it implements. The vision of MKM is that these readers will still be able to interact with the underlying formal objects, and benefit from their formality.

Unfortunately, there is very little motivation for authors to extend their formal mathematics in this way. Some comments may be provided for other specialists, but the process of formalisation is already burdensome and the main goal of the author is that the complete article should be machine checked successfully. The issue here is that articles are written to be checked, not to be widely read. Perhaps if the acceptance process included some element of readability for a mathematical audience then authors would be sufficiently motivated to structure and comment their articles. The success of the JFM is a first step in this direction.

Another solution is to retrospectively add this material to collections like the MML. The expense and time could be prohibitive for large collections, but could perhaps be distributed with a wiki approach, providing a sufficiently motivated group of qualified people existed and quality could be checked before use. However it can be achieved, we cannot avoid the problem of integrating formal and informal mathematical material if we want a integrated approach to MKM to succeed.

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