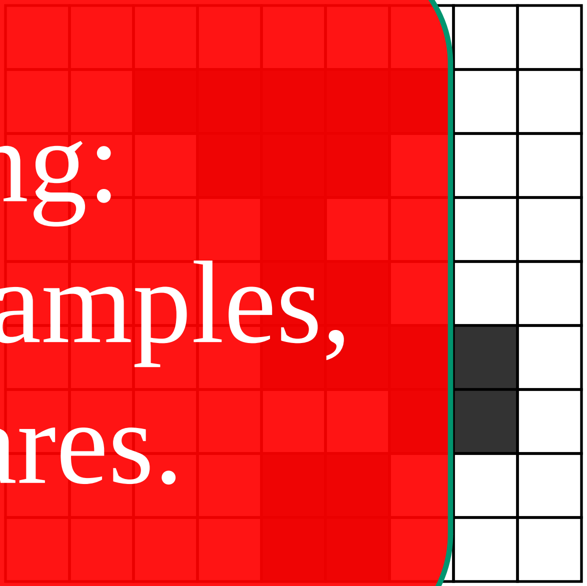
Morphological Operations

Outline

- Basic concepts:
  - Erode and dilate
  - Open and close.
- Granulometry
- Hit and miss transform
- Thinning and thickening
- Skeletonization and the medial axis transform
- Introduction to gray level morphology.

Pixel connectivity

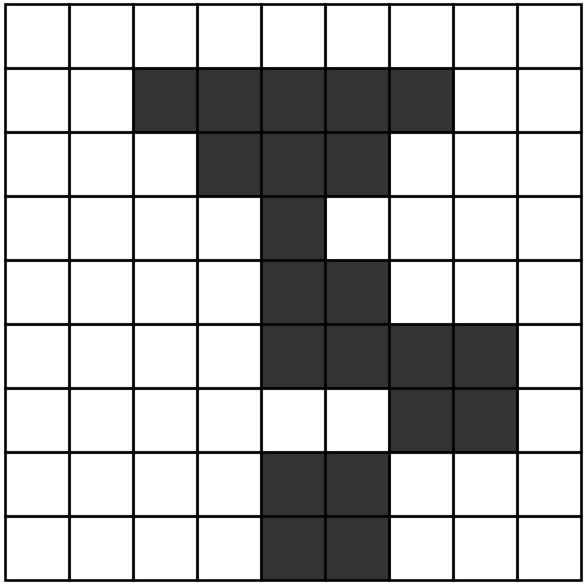
- We need to define which pixels are neighbors.
- Are the dark pixels in this array connected?



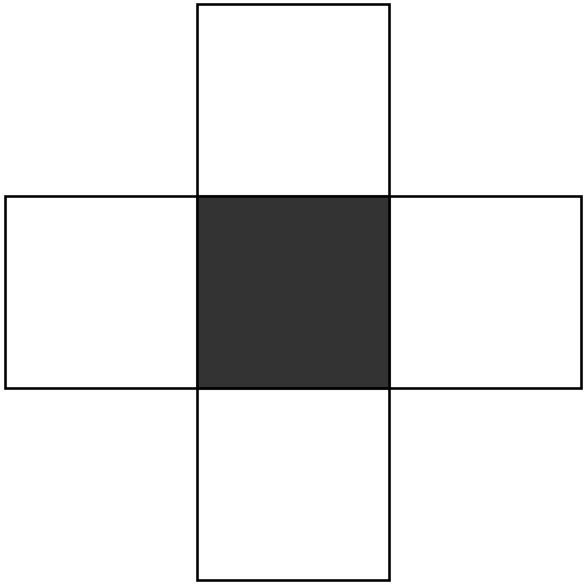
Warning:
Pixels are samples,
not squares.

Pixel connectivity

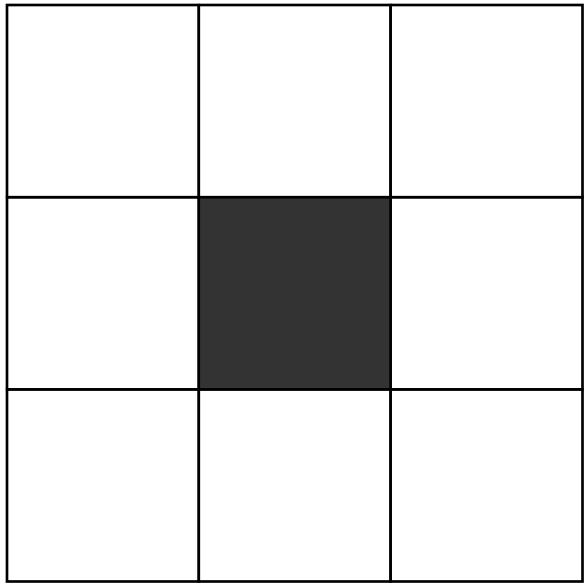
- We need to define which pixels are neighbors.
- Are the dark pixels in this array connected?



Pixel Neighborhoods



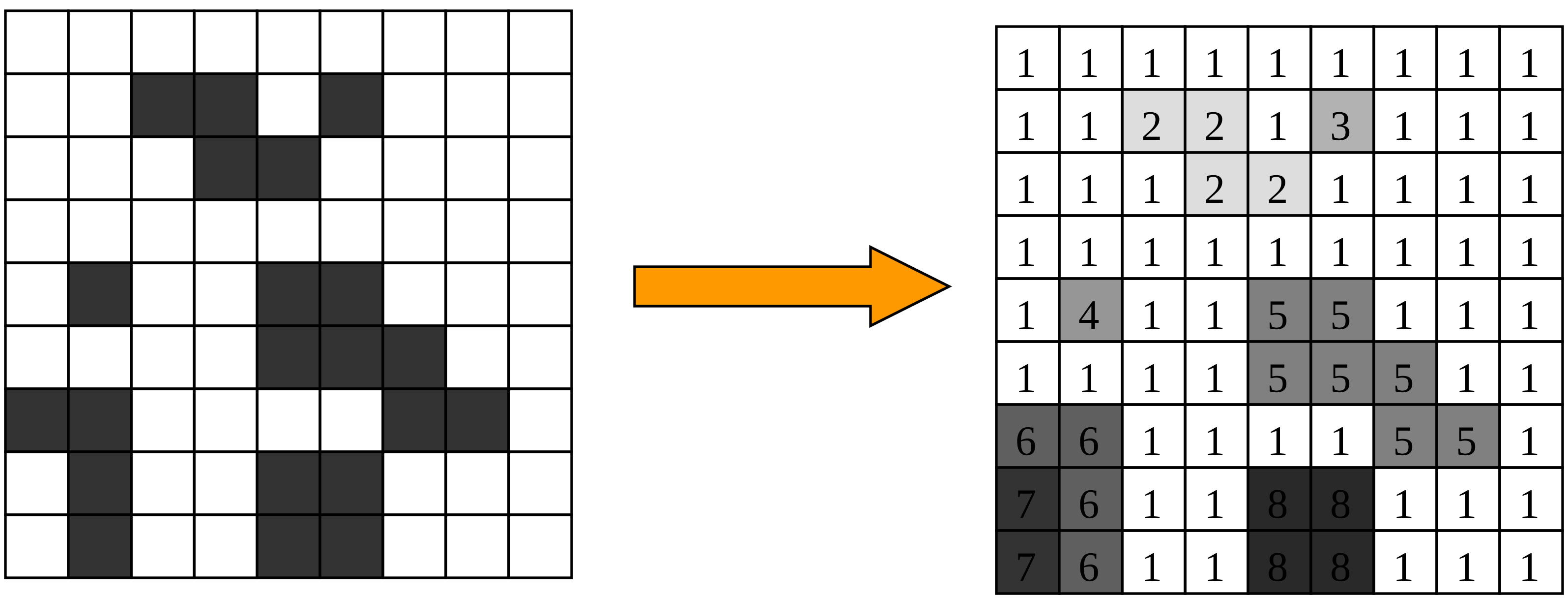
4-neighborhood



8-neighborhood

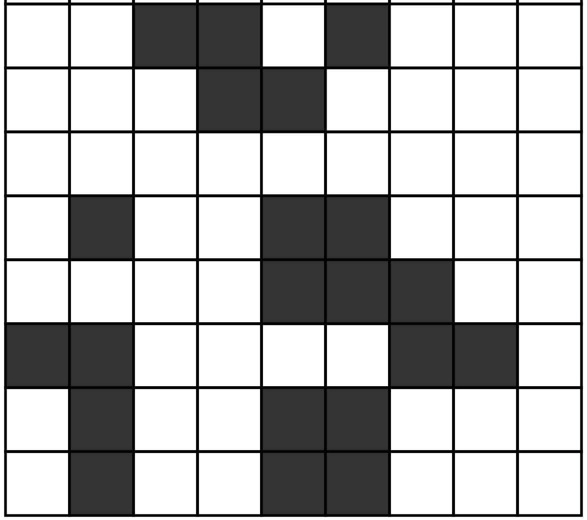
Connected components labelling

- Labels each connected component of a binary image with a separate number.



Foreground labelling

- Only extract the connected components of the foreground



1	1	1	1	1	1	1	1	1
1	1	0	0	1	0	1	1	1
1	1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1	1
1	0	1	1	0	0	1	1	1
1	1	1	1	0	0	0	1	1
0	0	1	1	1	1	0	0	1
2	0	1	1	0	0	1	1	1
2	0	1	1	0	0	1	1	1

Roundabout Example

- Input
- Output

What Are Morphological Operators?

- Local pixel transformations for processing region shapes
- Most often used on binary images

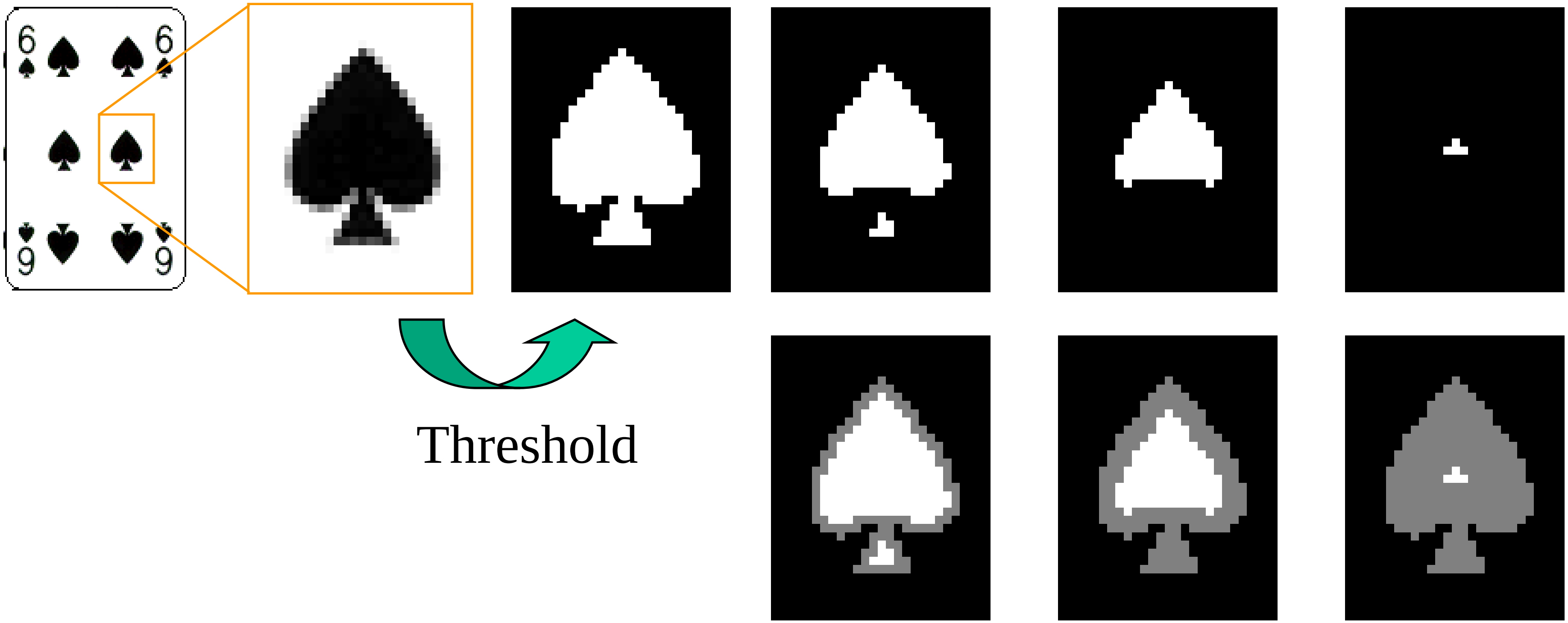
- Logical transformations based on comparison of pixel neighbourhoods with a pattern.

Simple Operations - Examples

- Eight-neighbour erode
  - a.k.a. Minkowsky subtraction

- Erase any foreground pixel that has one eight-connected neighbour that is background.

8-neighbour erode



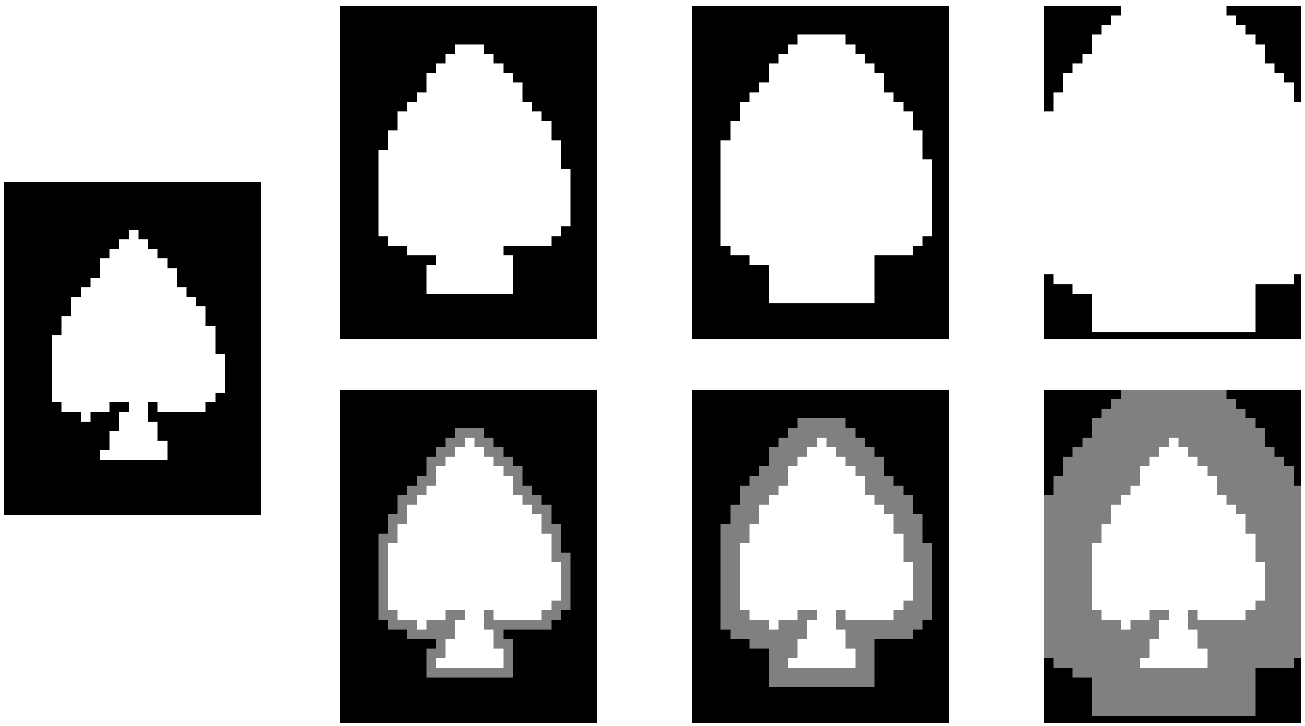
Erode ×1 Erode ×2 Erode ×5

8-neighbour dilate

- Eight-neighbour dilate
  - a.k.a. Minkowsky addition

- Paint any background pixel that has one eight-connected neighbour that is foreground.

8-neighbour dilate



Dilate ×1 Dilate ×2 Dilate ×5

Why?

- Smooth region boundaries for shape analysis.
- Remove noise and artefacts from an imperfect segmentation.
- Match particular pixel configurations in an image for simple object recognition.

Structuring Elements

- Morphological operations take two arguments:
    - A binary image
    - A *structuring element.*
- Compare the structuring element to the neighbourhood of each pixel.
- This determines the output of the morphological operation.

Structuring elements

- The structuring element is also a binary array.
- A structuring element has an origin.

1	1	1
1	1	1
1	1	1

0	1	0
1	1	1
0	1	0

0	1	0	1
1	0	1	1
0	1	0	0

Binary images as sets

- We can think of the binary image and the structuring element as sets containing the pixels with value 1.

1	1	1	0	0
0	1	1	0	0
0	0	0	0	0
0	0	0	1	0
0	0	0	0	0

$I = \{(1,1), (2,1), (3,1), (2,2), (3,2), (4,4)\}$

```
(Matlab)
S = SPARSE(X) converts a sparse or
full matrix to sparse form by
squeezing out any zero elements
```

Some sets notation

- Union and intersection:

$$I_1 \cup I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ or } \underline{x} \in I_2\}$$

$$I_1 \cap I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ and } \underline{x} \in I_2\}$$

- Complement

$$I^C = \{\underline{x} : \underline{x} \notin I\}$$

- Difference

$$I_1 \setminus I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ and } \underline{x} \notin I_2\}$$

- We use ϕ for the empty set .

More sets notation

- The *symmetrical set* of S with respect to point $\underline{o}$ is

$$\breve{S} = \{\underline{o} - \underline{x} : \underline{x} \in S\}$$

$$\begin{array}{|c|c|}\hline 1 & 1 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|}\hline 1 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 1 & 1 & 0 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|}\hline 0 & 1 & 1 & 1 & 0 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 0 & 1 & 1 & 1 & 0 \\ \hline \end{array}$$

$$\downarrow$$

$$\begin{array}{|c|c|c|c|c|}\hline 0 & 1 & 1 & 1 & 0 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 0 & 1 & 1 & 1 & 0 \\ \hline \end{array}$$

Fitting, Hitting and Missing

- S fits I at $\underline{x}$ if

  $\{\underline{y} : \underline{y} = \underline{x} + \underline{s}, \underline{s} \in S\} \subset I$

- S hits I at $\underline{x}$ if

  $\{\underline{y} : \underline{y} = \underline{x} - \underline{s}, \underline{s} \in S\} \cap I \neq \phi$

- S misses I at $\underline{x}$ if

  $\{\underline{y} : \underline{y} = \underline{x} - \underline{s}, \underline{s} \in S\} \cap I = \phi$

Fitting, Hitting and Missing

Image

0	1	0	1	1	1	1	1
1	0	1	1	1	1	1	1
0	1	1	1	0	0	0	0
0	1	1	1	0	0	0	0
0	1	1	1	1	0	0	0
0	1	0	1	0	1	0	0
0	1	0	0	0	0	1	0
0	1	0	0	0	0	0	0

Structuring element

0	1	0
1	1	1
0	1	0

Erosion

- The image $E = I \ominus S$ is the *erosion* of image I by structuring element S.

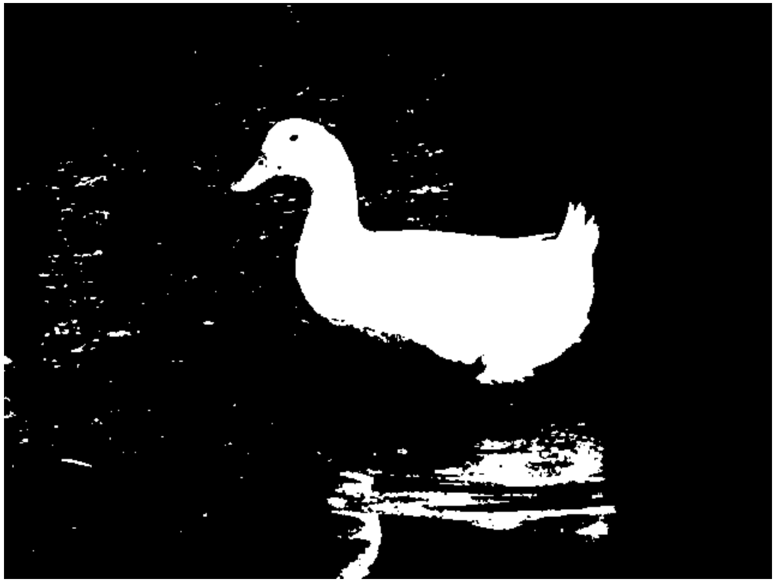
$$E(\underline{x}) = \begin{cases} 1 & \text{if } S \text{ fits } I \text{ at } \underline{x} \\ 0 & \text{otherwise} \end{cases}$$

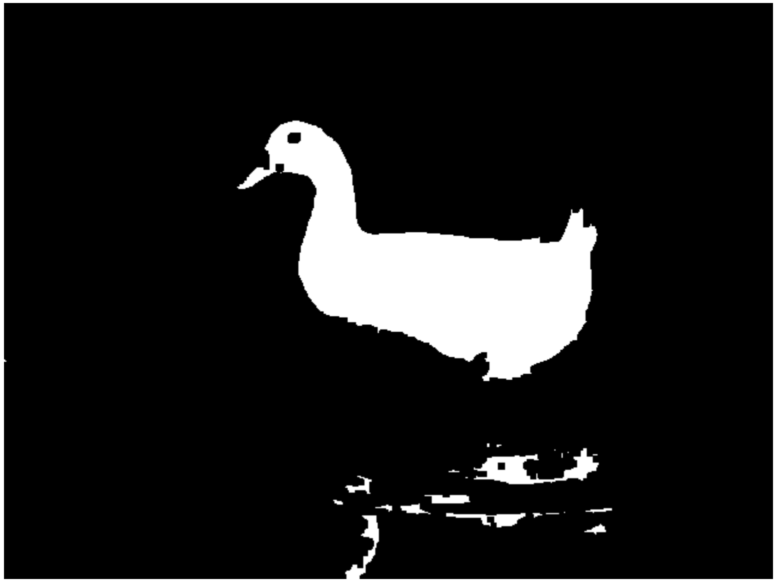
$$E = \{\underline{x} : \underline{x} + \underline{s} \in I \text{ for every } s \in S\}$$

Implementation (naïve)

```
% I is the input image, S is a structuring element
% with origin (ox, oy).  E is the output image.
function E = erode(I, S, ox, oy)
[X,Y] = size(I);
[SX, SY] = size(S);
E = ones(X, Y);
for x=1:X; for y=1:Y
    for i=1:SX; for j=1:SY
        if(S(i,j))
            E(x,y) = E(x,y) & ImageEntry(I, x+i-ox, y+j-oy);
        end
    end; end
end; end
```

Example

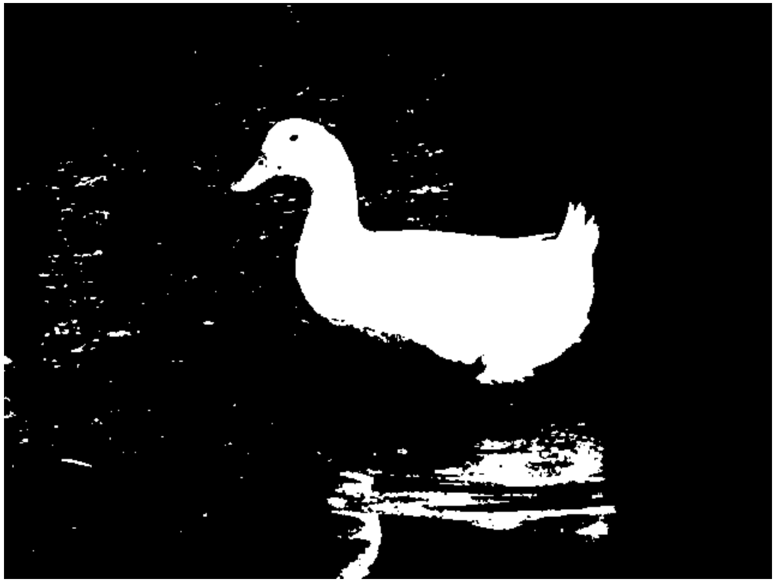


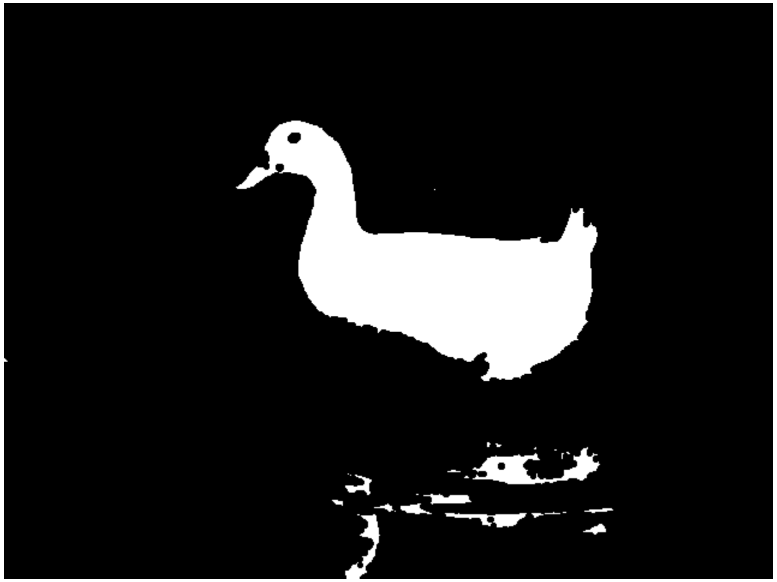


Structuring element

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Example

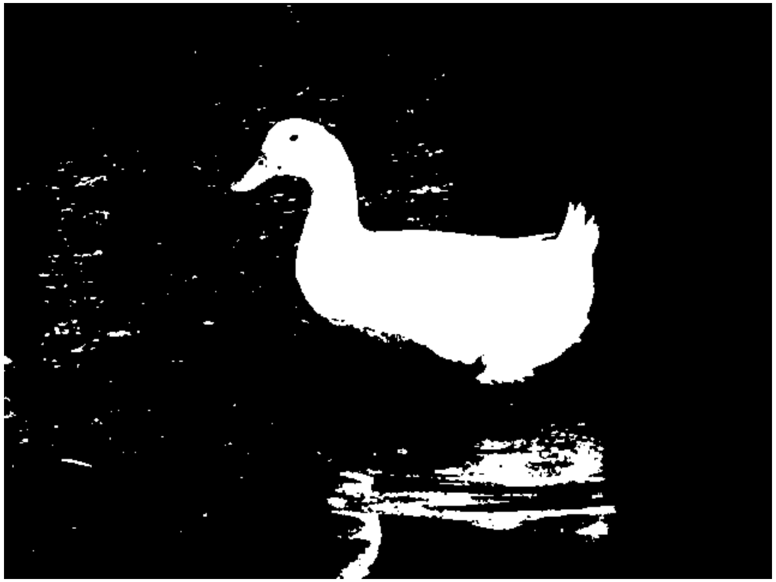


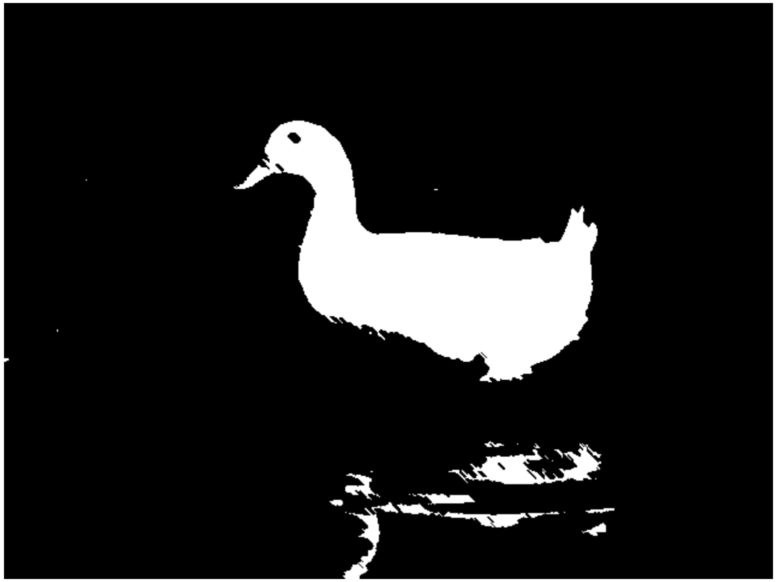


Structuring element

0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

Example





Structuring element

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

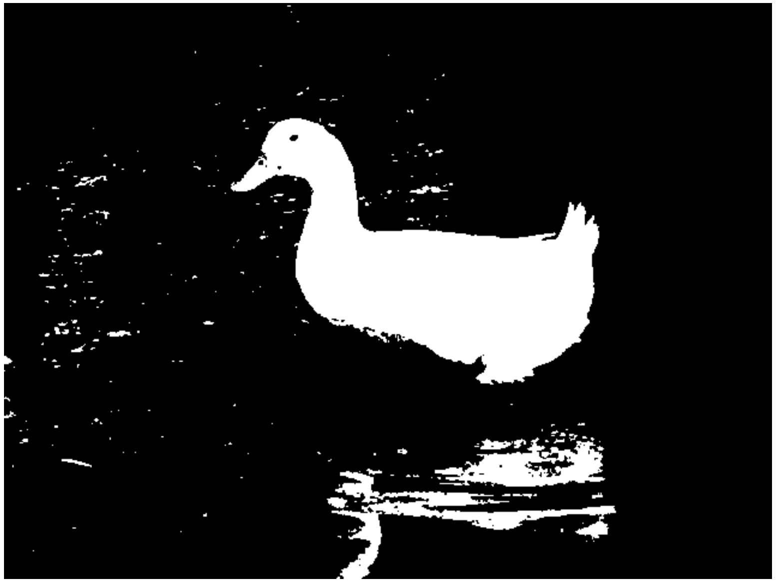
Dilation

- The image $D = I \oplus S$ is the *dilation* of image I by structuring element S.

$$D(\underline{x}) = \begin{cases} 1 & \text{if } S \text{ hits } I \text{ at } \underline{x} \\ 0 & \text{otherwise} \end{cases}$$

$$D = \{\underline{x} : \underline{x} = \underline{s} + \underline{y},\ \underline{y} \in I \text{ and } \underline{s} \in S\}$$

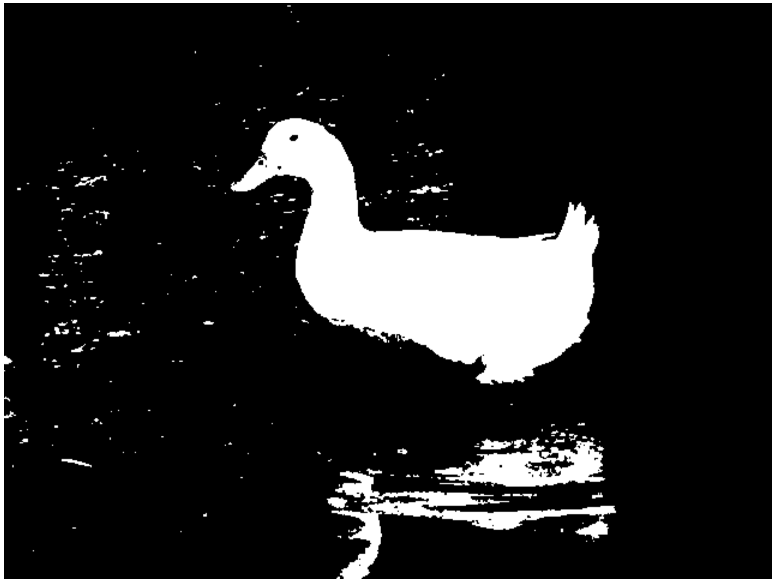
Example



Structuring element

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

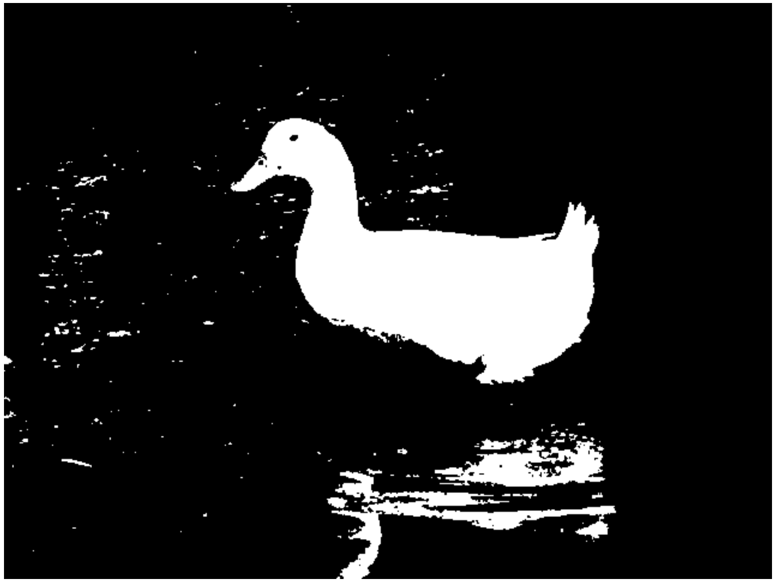
Example

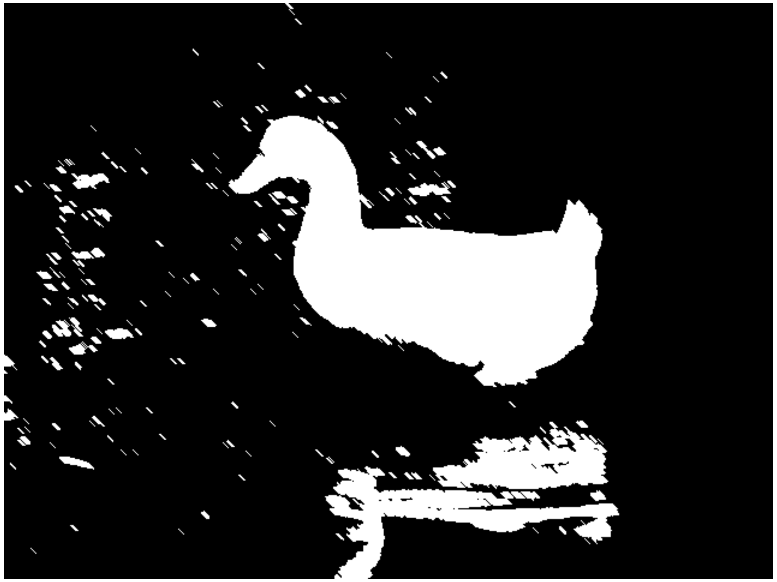


Structuring element

0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

Example





Structuring element

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Erosion and dilation

- Erosion and dilation are dual operations:

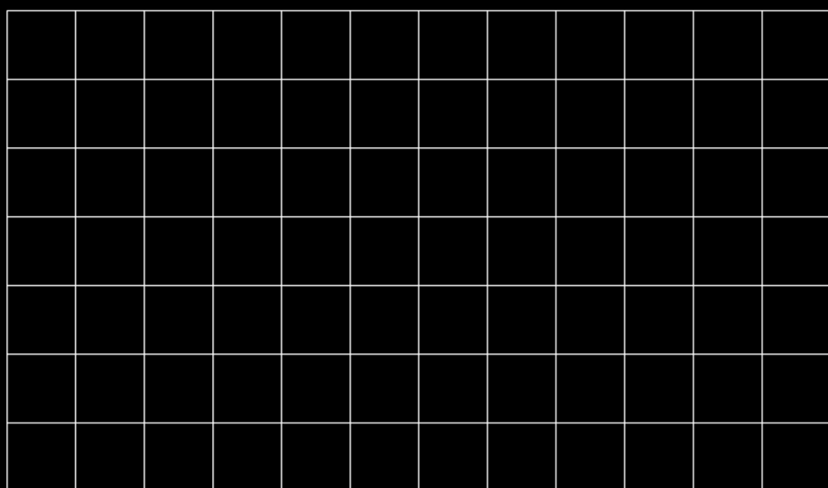
$$(I \ominus S)^C = I^C \oplus \breve{S}$$

- Commutativity and associativity

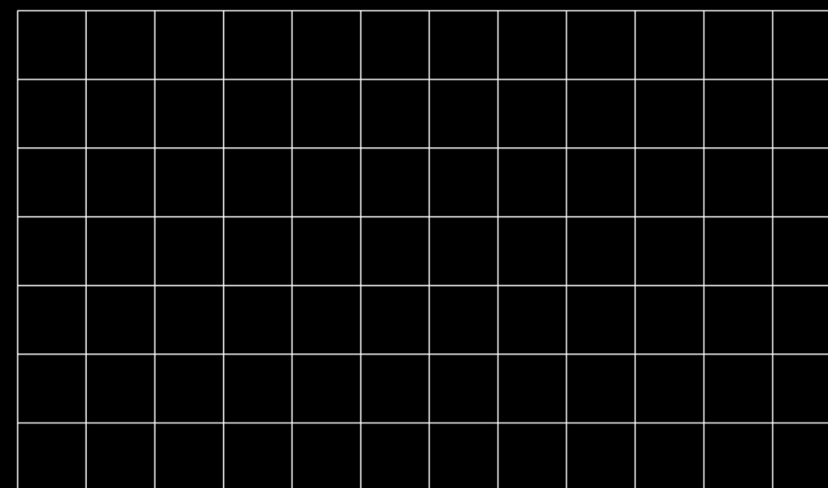
$$I \oplus S = S \oplus I \qquad (I \oplus S) \oplus T = I \oplus (S \oplus T)$$

$$I \ominus S \neq S \ominus I \qquad (I \ominus S) \ominus T = I \ominus (S \oplus T)$$

Input



Output



Structuring Element:

Opening and Closing

- The *opening* of I by S is

$$I \circ S = (I \ominus S) \oplus S$$

- The *closing* of I by S is

$$I \bullet S = (I \oplus S) \ominus S$$

Opening and Closing

- Opening and closing are dual transformations:

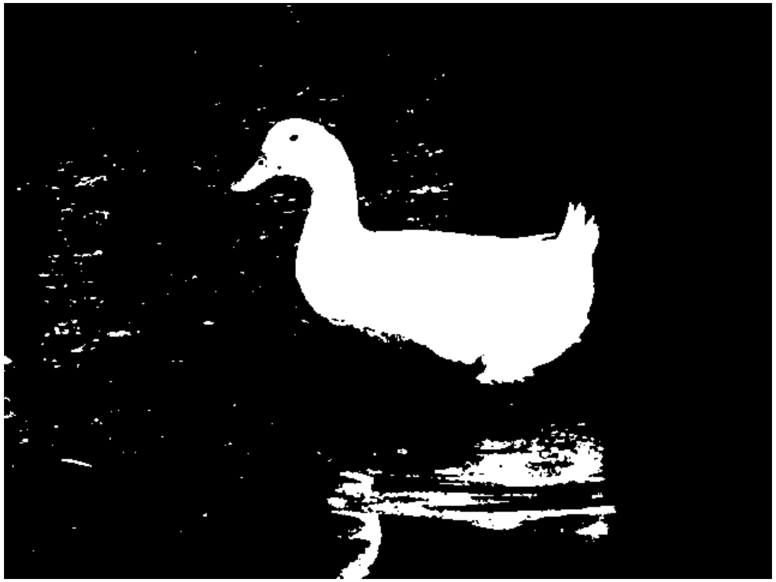
$$(I \bullet S)^C = I^C \circ \breve{S}$$

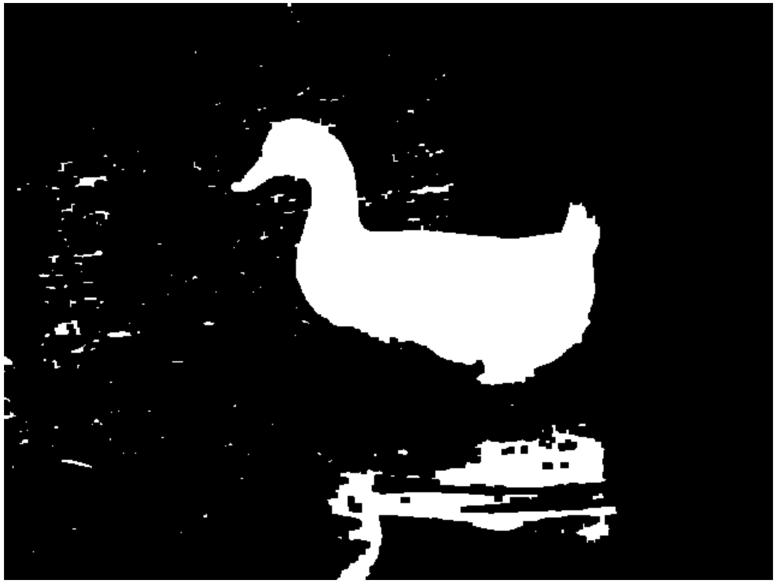
- Opening and closing are *idempotent* operations:

$$I \circ S = (I \circ S) \circ S$$

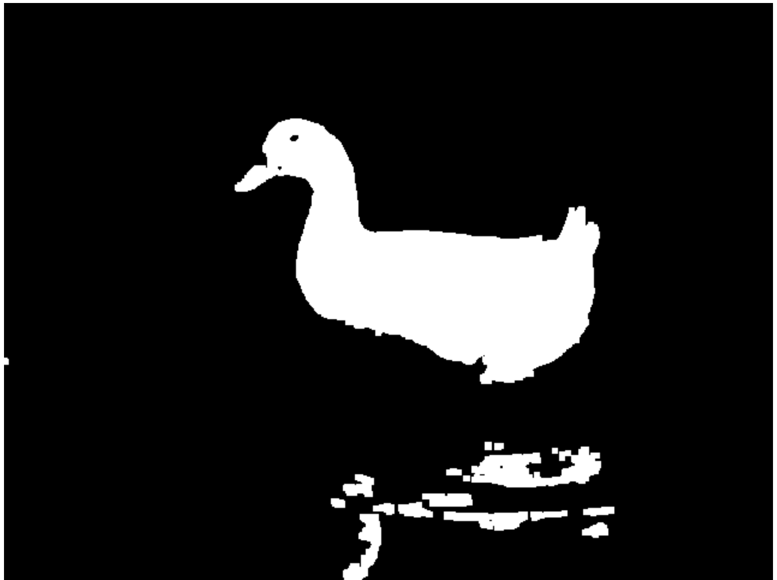
$$I \bullet S = (I \bullet S) \bullet S$$

Example





close

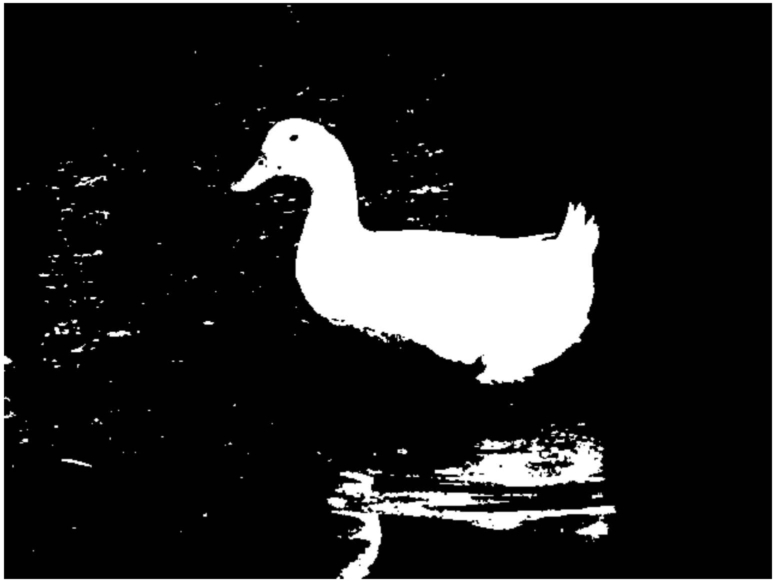


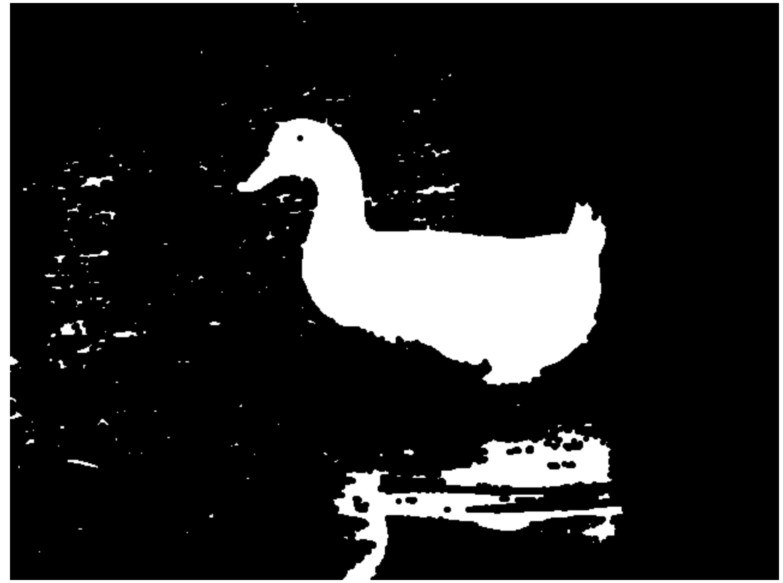
open

Structuring element

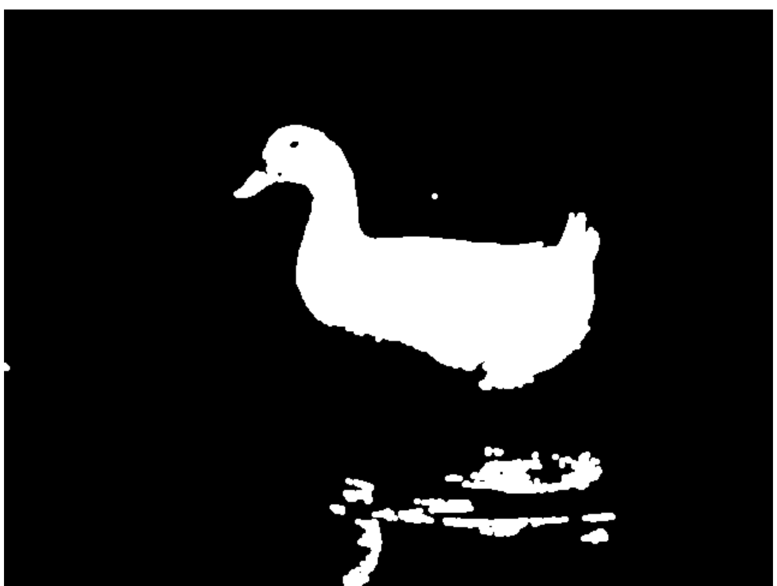
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Example





close

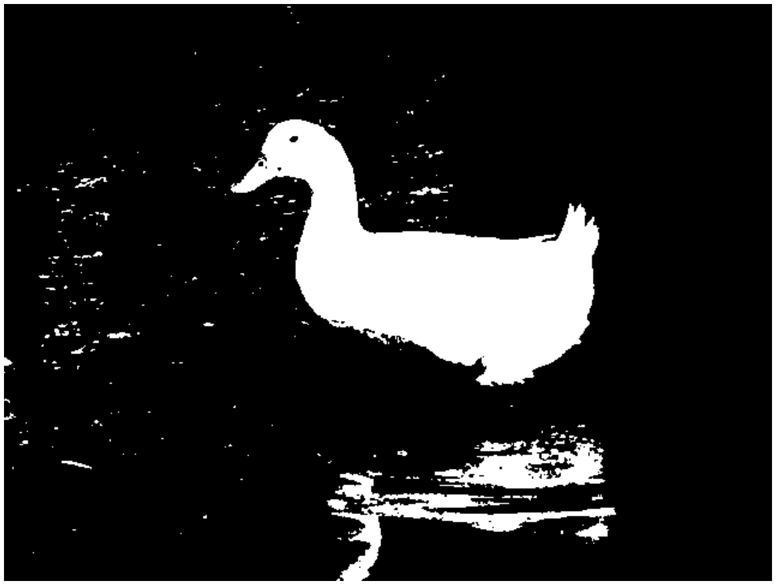


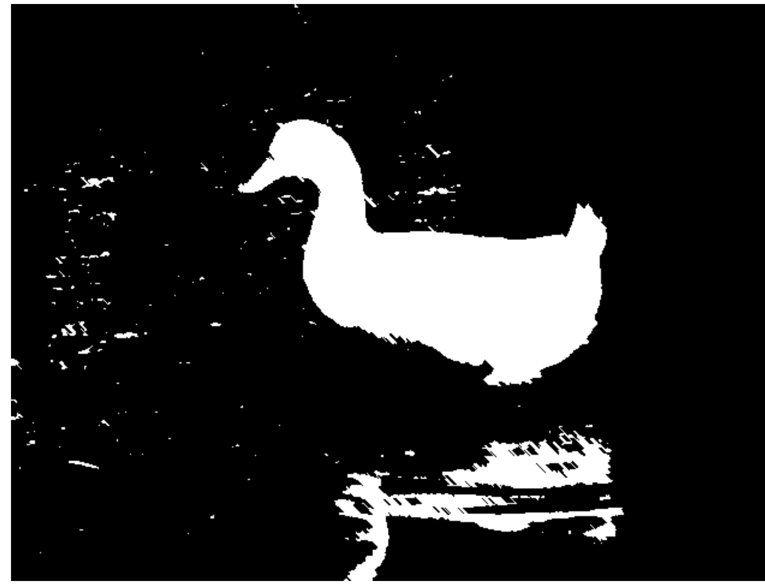
open

Structuring element

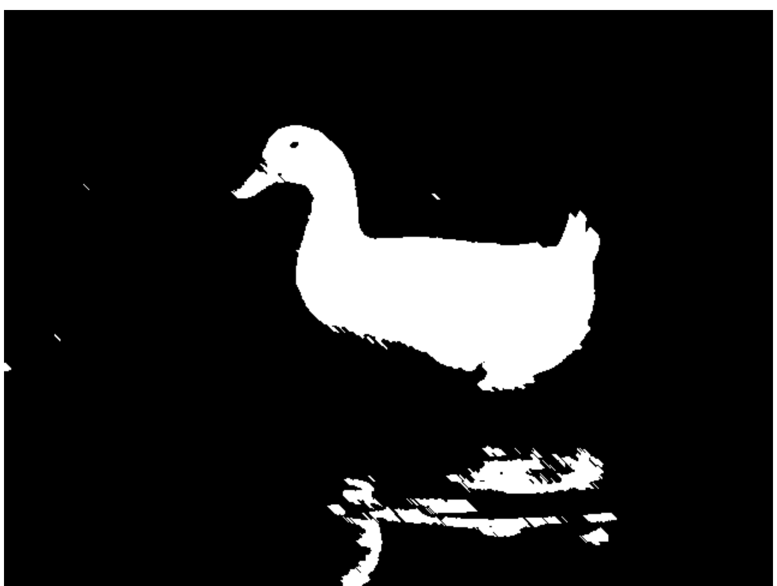
0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

Example





close



open

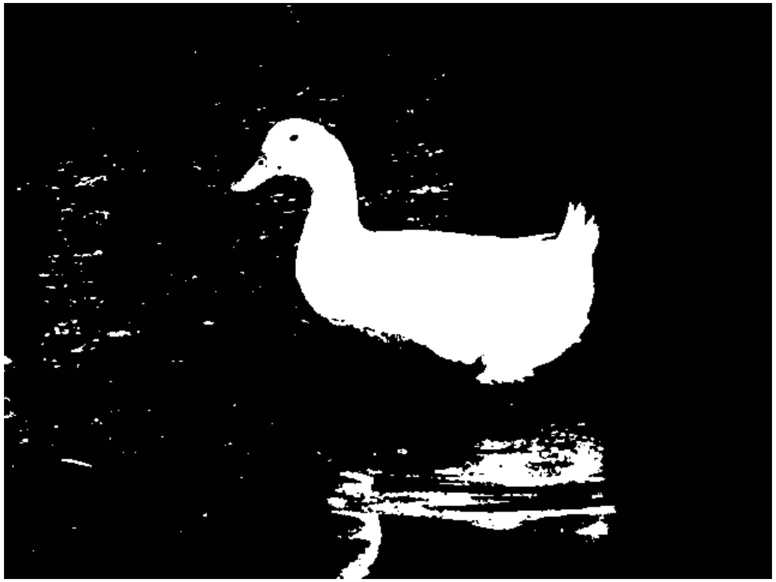
Structuring element

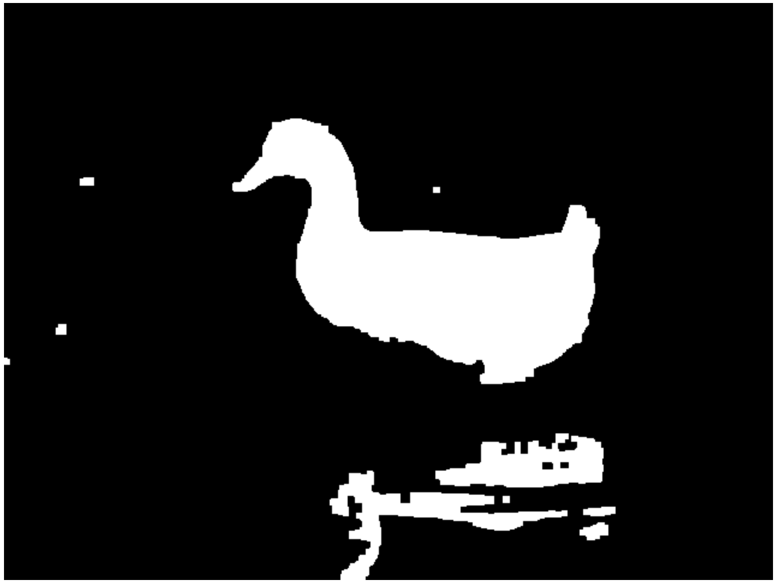
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Morphological filtering

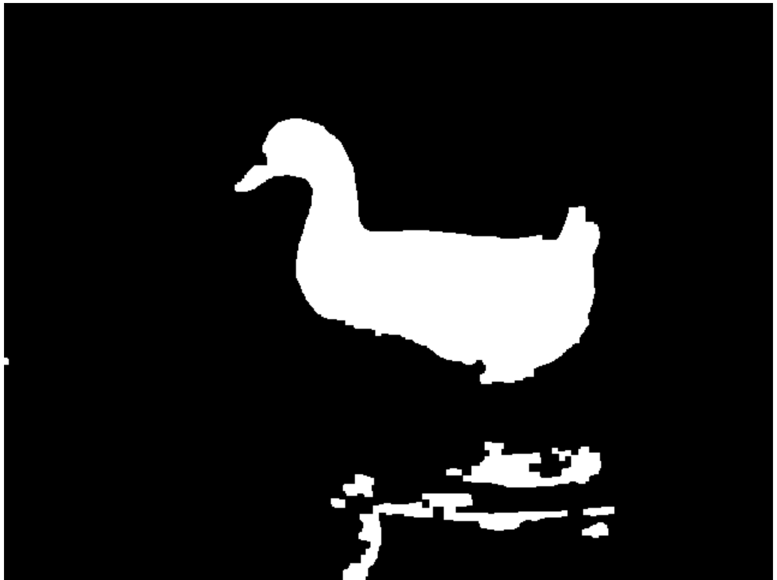
- To remove holes in the foreground and islands in the background, do both opening and closing.
- The size and shape of the structuring element determine which features survive.
- In the absence of knowledge about the shape of features to remove, use a circular structuring element.

Example





Close then open

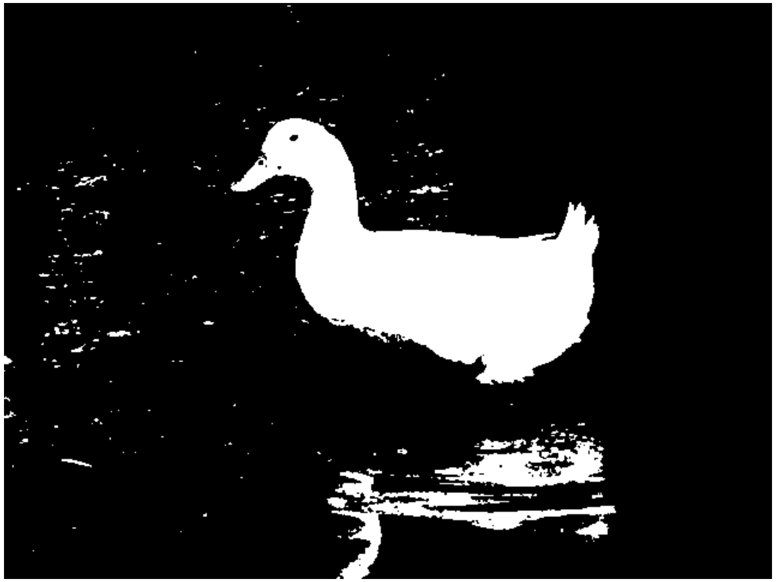


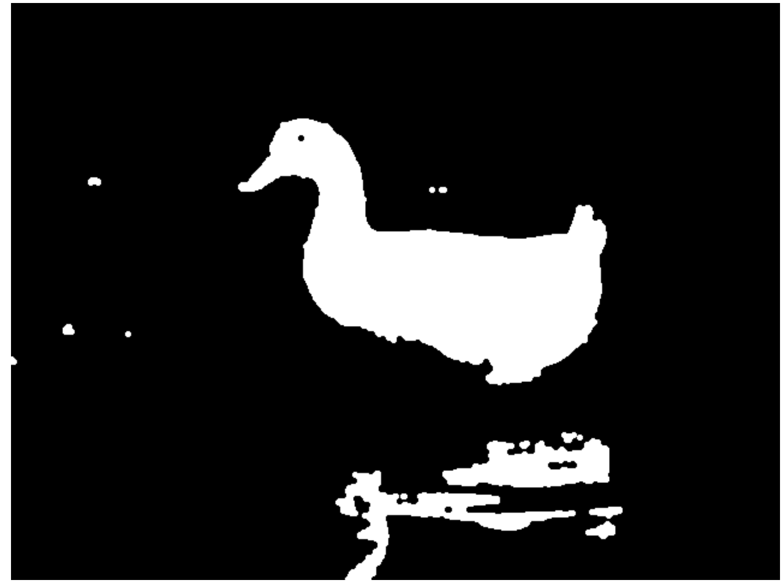
Open then close

Structuring element

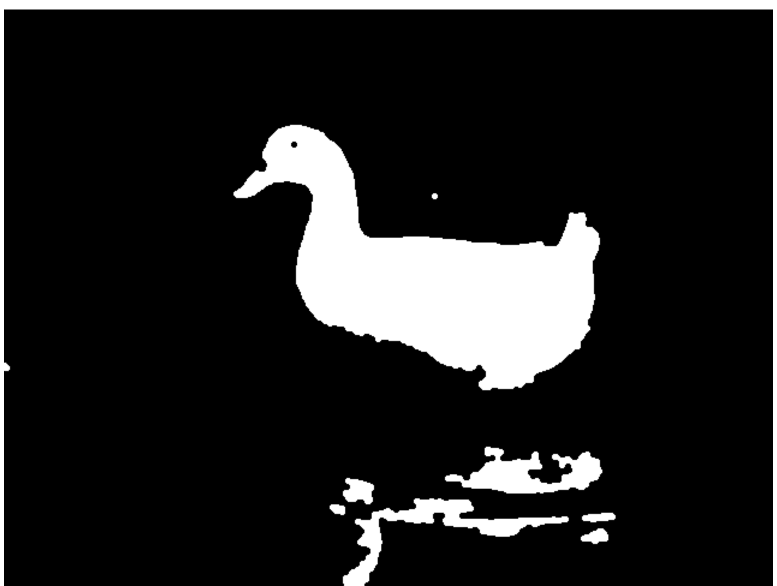
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Example





Close then open

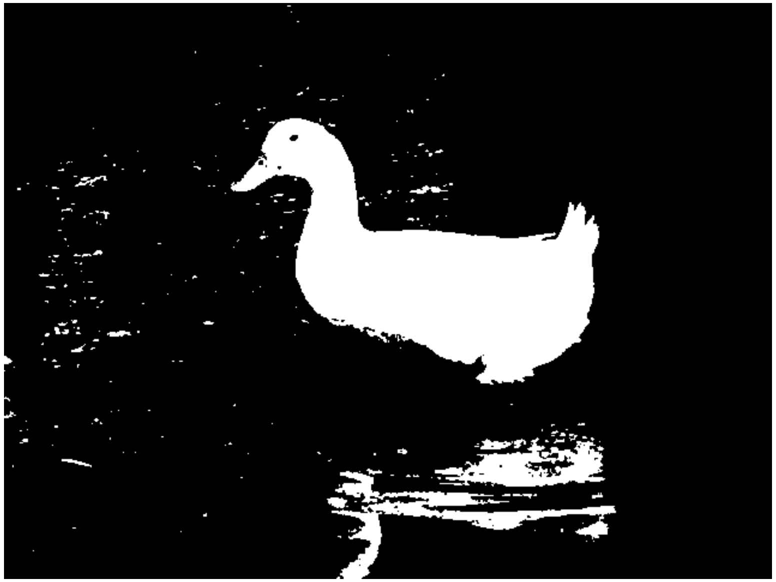


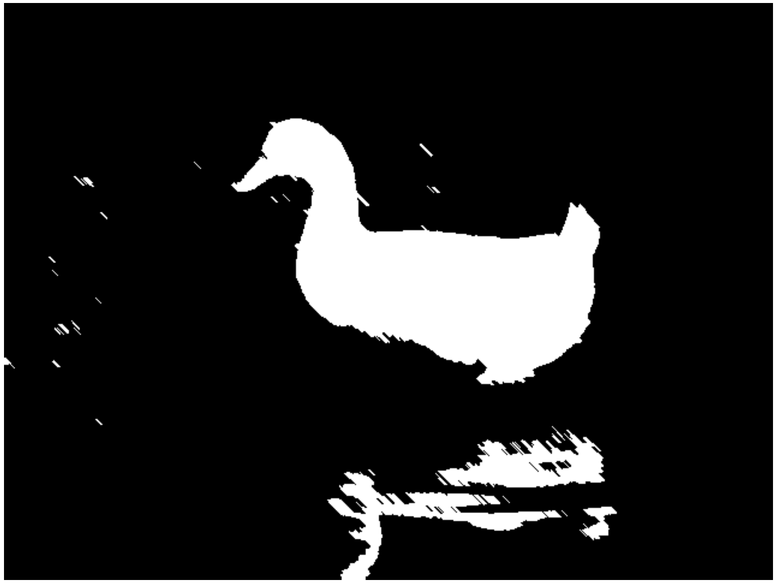
Open then close

Structuring element

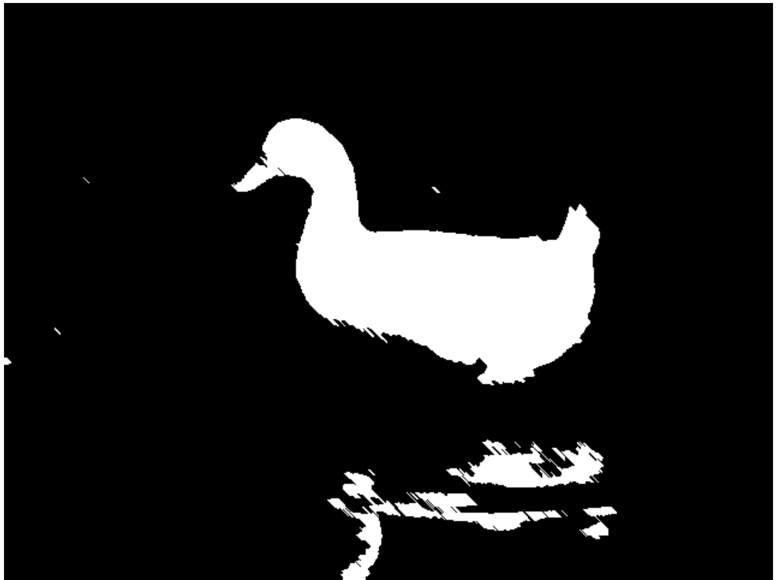
0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

Example





Close then open

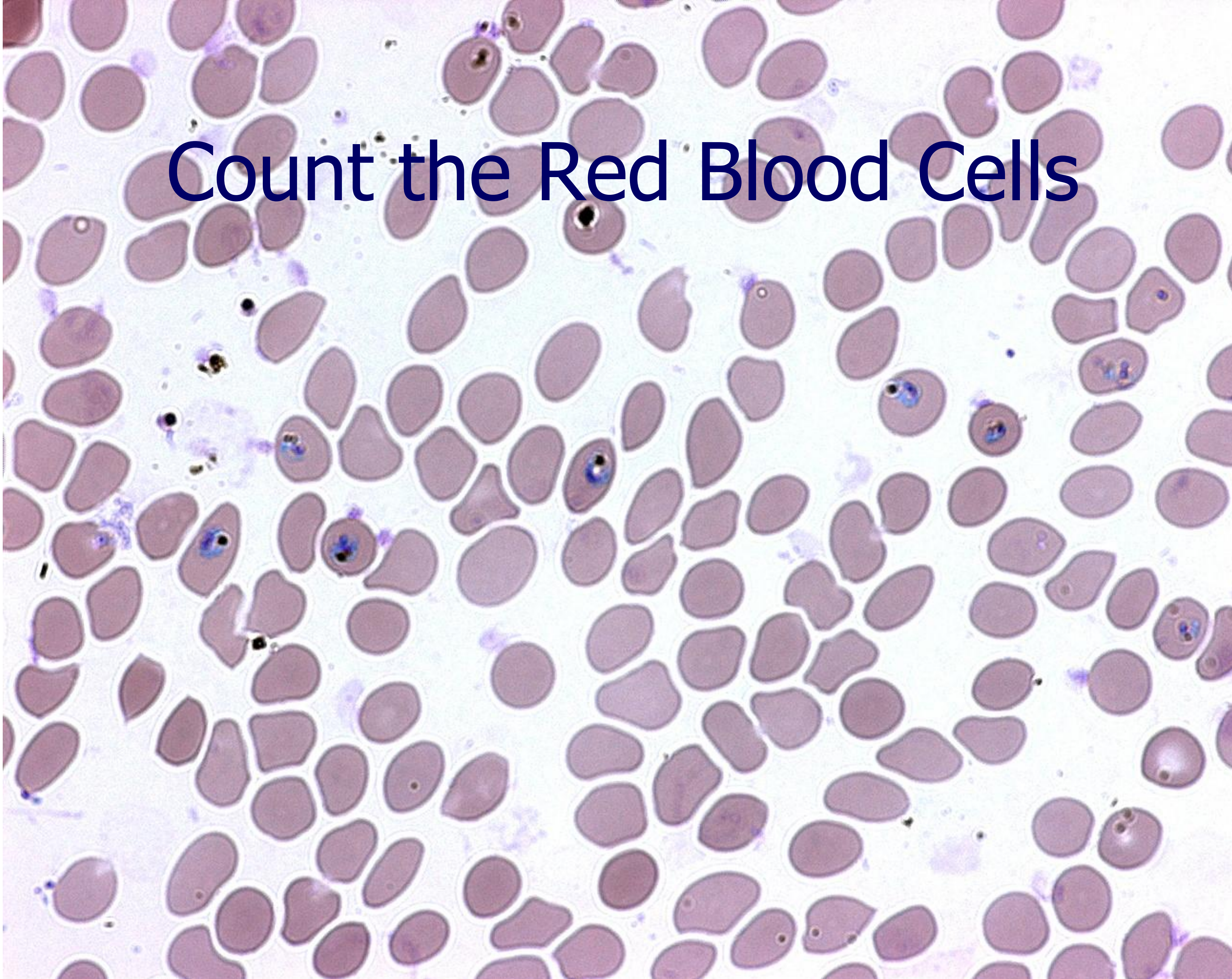


Open then close

Structuring element

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Count the Red Blood Cells



Granulometry

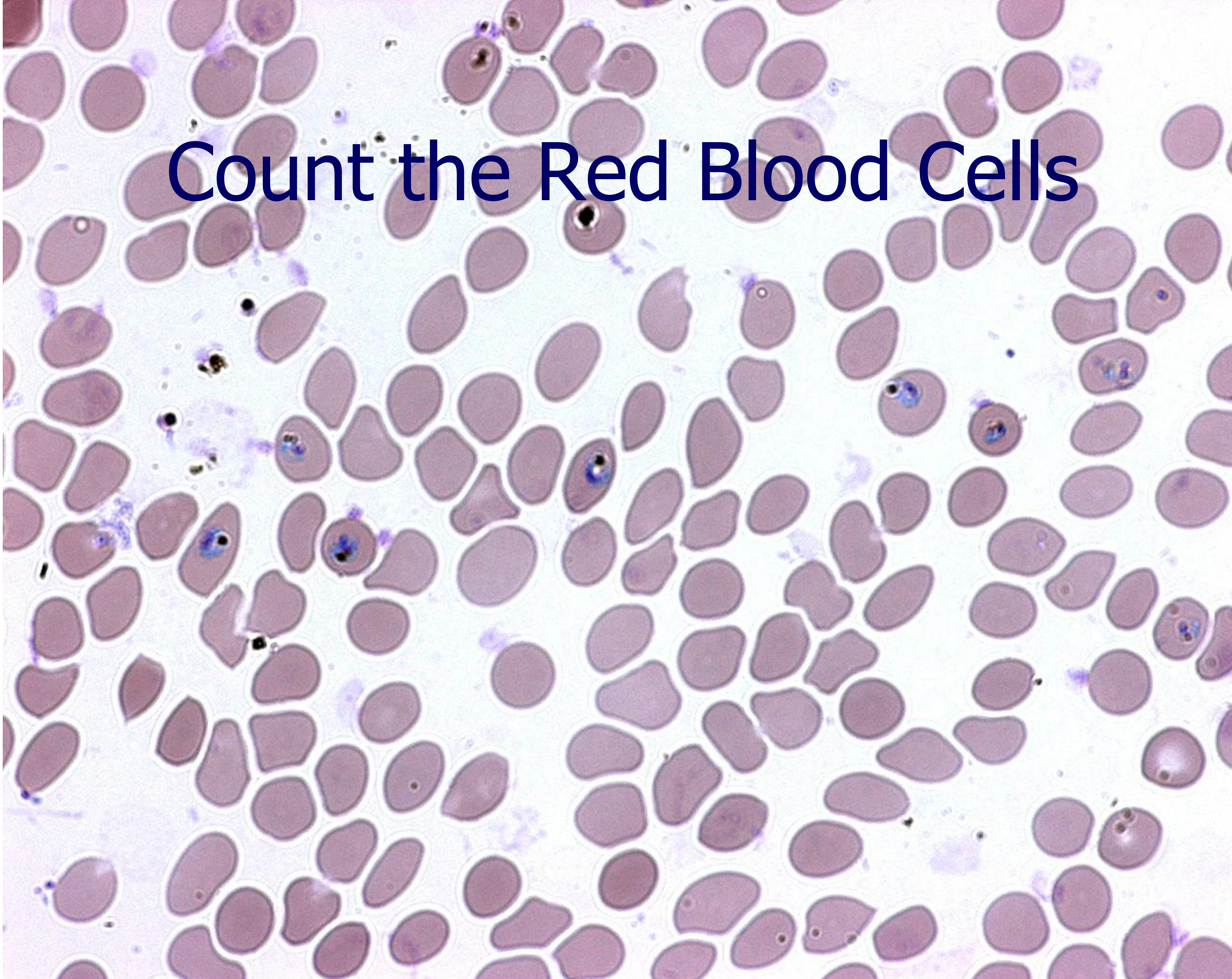
- Provides a size distribution of distinct regions or “granules” in the image.
- We open the image with increasing structuring element size and count the number of regions after each operation.

- Creates a “morphological sieve”.

Granulometry

```
function gSpec = granulo(I, T, maxRad)
% Segment the image I
B = (I>T);
% Open the image at each structuring element size up to a
% maximum and count the remaining regions.
for x=1:maxRad
    O = imopen(B,strel('disk',x));
    numRegions(x) = max(max(connectedComponents(O)));
end
gSpec = diff(numRegions);
```

Count the Red Blood Cells



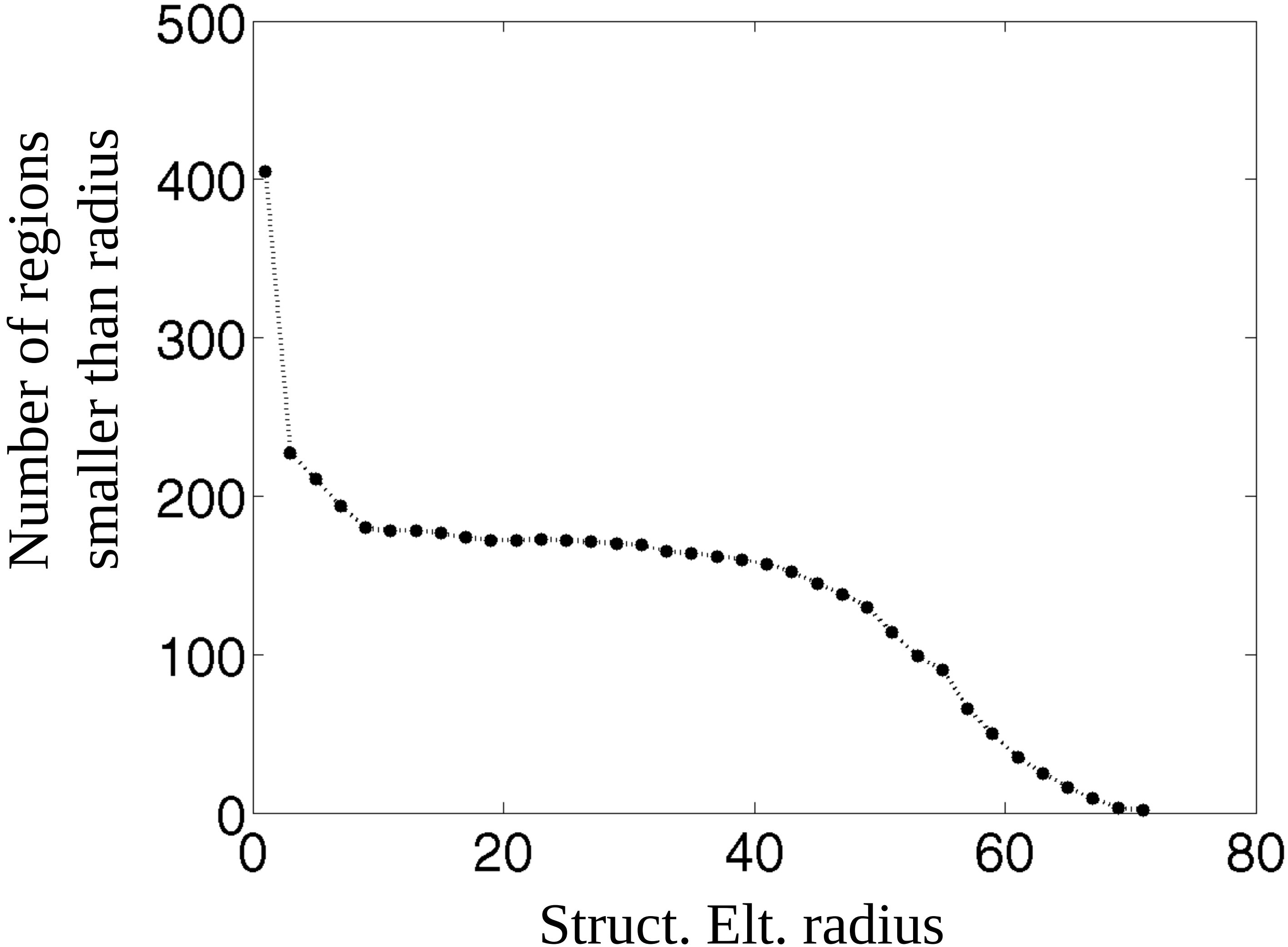
Threshold and Label


Disc(11)

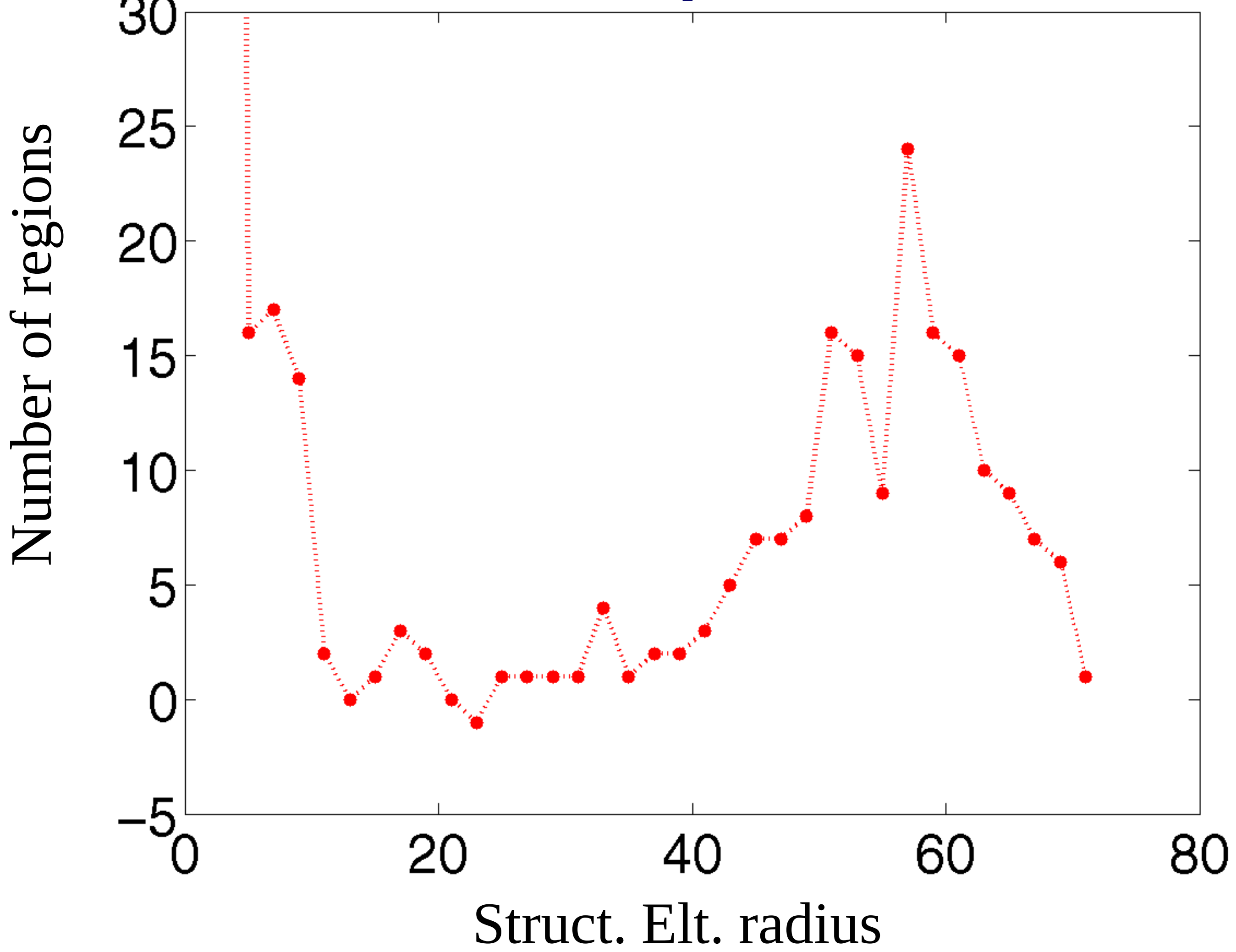

Disc(19)


Disc(59)

Number of Regions



Granulometric *Pattern Spectrum*



Hit-and-miss transform

- Searches for an exact match of the structuring element.
- $H = I \otimes S$ is the hit-and-miss transform of image I by structuring element S.
- Simple form of template matching.

Hit-and-miss transform

$$\begin{array}{|c|c|c|c|c|}\hline 1 & 0 & 1 & 1 & 1 \\ \hline 1 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 & 1 \\ \hline 1 & 1 & 0 & 1 & 1 \\ \hline 1 & 0 & 1 & 0 & 1 \\ \hline \end{array} \otimes \begin{array}{|c|c|c|}\hline 1 & 0 & 1 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|}\hline 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 0 & 1 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|}\hline 1 & 0 & 1 & 1 & 1 \\ \hline 1 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 & 1 \\ \hline 1 & 1 & 0 & 1 & 1 \\ \hline 1 & 0 & 1 & 0 & 1 \\ \hline \end{array} \otimes \begin{array}{|c|c|}\hline 1 & 0 \\ \hline * & 1 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|}\hline 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

Upper-Right Corner Detector

x	0	0
1	1	0
x	1	x

0	0	0
1	1	0
0	1	0

J

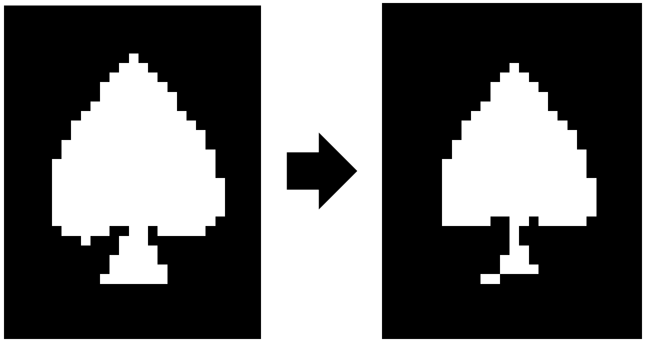
0	1	1
0	0	1
0	0	0

K

```
bwhitmiss(BW1,SE1,SE2) ==
imerode(BW,SE1) & imerode(~BW,SE2)
```

Thinning and Thickening

- Defined in terms of the hit-and-miss transform:



- The *thinning* of *I* by *S* is

$$I \oslash S = I \setminus (I \otimes S)$$

- The *thickening* of *I* by *S* is

$$I \odot S = I \cup (I \otimes S)$$

- Dual operations: $(I \odot S)^C = I^C \oslash S$

(Note) Difference: $I_1 \setminus I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ and } \underline{x} \notin I_2\}$

Sequential Thinning/Thickening

- These operations are often performed in sequence with a selection of structuring elements $S_1, S_2, \ldots, S_n$.
- Sequential thinning:

$$I \oslash \{S_i : i = 1,\ldots,n\} = (((I \oslash S_1) \oslash S_2) \ldots \oslash S_n)$$

- Sequential thickening:

$$I \odot \{S_i : i = 1,\ldots,n\} = (((I \odot S_1) \odot S_2) \ldots \odot S_n)$$

Sequential Thinning/Thickening

- Several sequences of structuring elements are useful in practice
- These are usually the set of rotations of a single structuring element.

- Sometimes called the *Golay alphabet*.

Golay element L

L_1

0	0	0
*	1	*
1	1	1

L_2

*	0	0
1	1	0
*	1	*

L_3

1	*	0
1	1	0
1	*	0

L_4

*	1	*
1	1	0
*	0	0

L_5

1	1	1
*	1	*
0	0	0

L_6

*	1	*
0	1	1
0	0	*

L_7

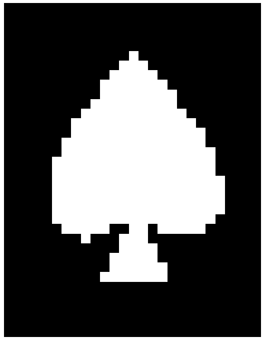
0	*	1
0	1	1
0	*	1

L_8

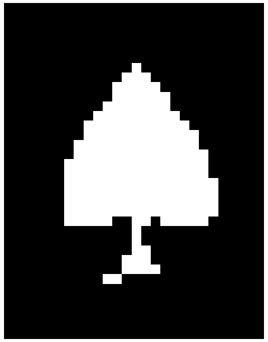
0	0	*
0	1	1
*	1	*

Sequential Thinning

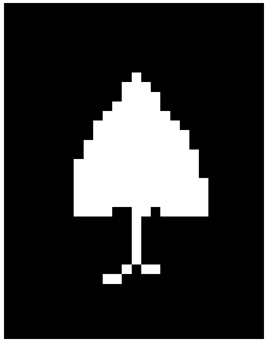
- See `bwmorph` in matlab.



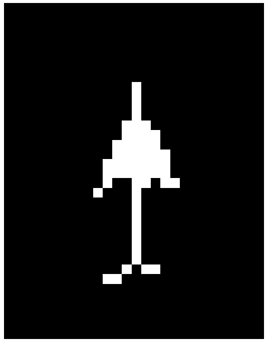
0 iterations



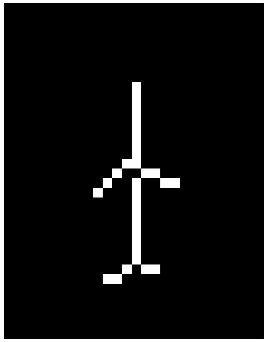
1 iteration



2 iterations

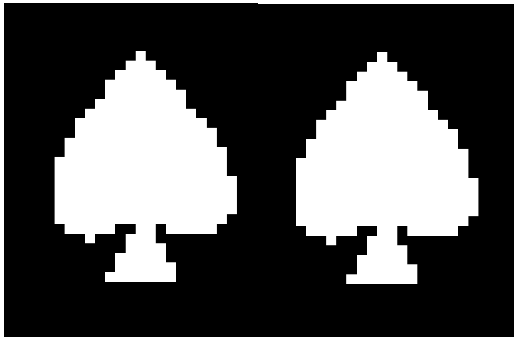


5 iterations

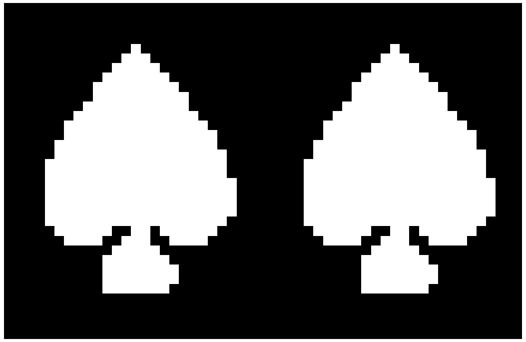


Inf iterations

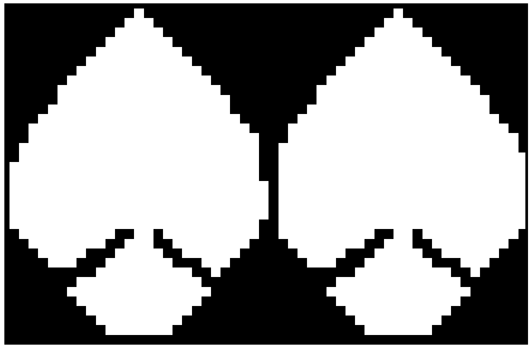
Sequential Thickening



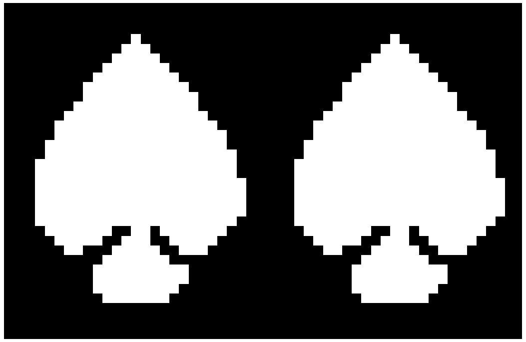
1 iteration



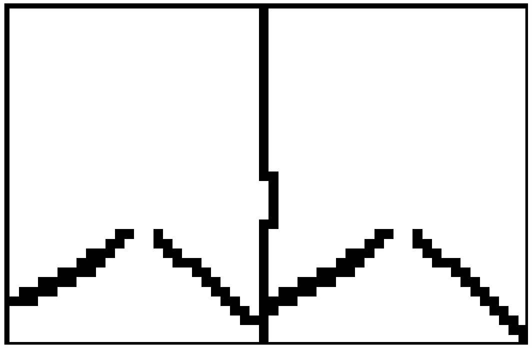
5 iterations



2 iterations



Inf iterations



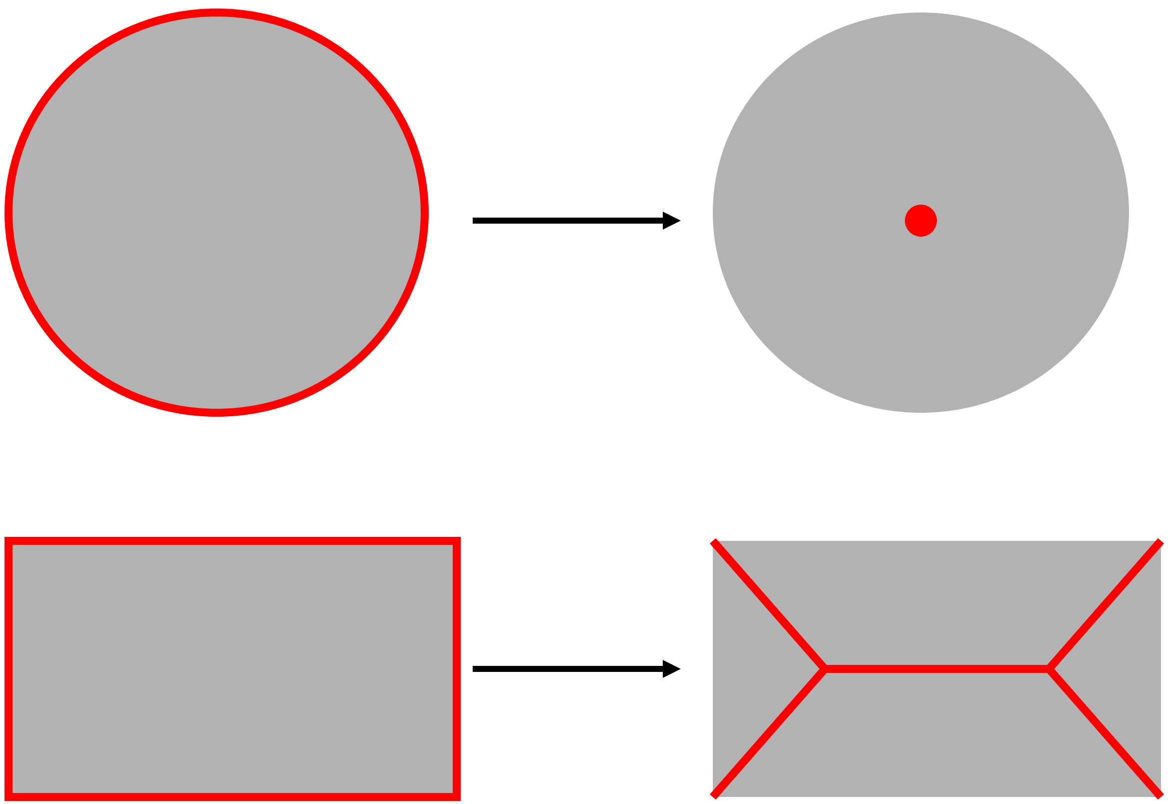
Skeletonization and the Medial Axis Transform

- The skeleton and *medial axis transform* (MAT) are stick-figure representations of a region $X \subset \Re^2$.

- Start a grassfire at the boundary of the region.

- The skeleton is the set of points at which two fire fronts meet.

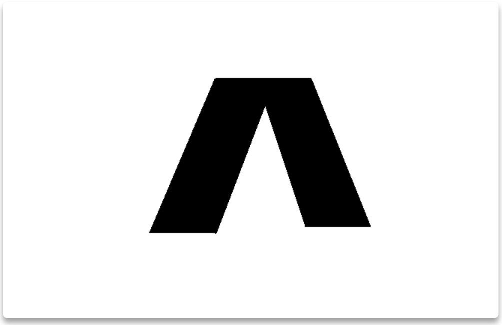
Skeletons



Matlab Example

```
I = imread('im1.bmp');
image(bwdist(I)); colormap(gray)

Idist = bwdist(I);
IdistNormed = 255* (Idist / max(max(Idist)));
IdistNormedThreshed = IdistNormed;
IdistNormedThreshed(IdistNormed<1) = NaN;
surf(IdistNormedThreshed)
```

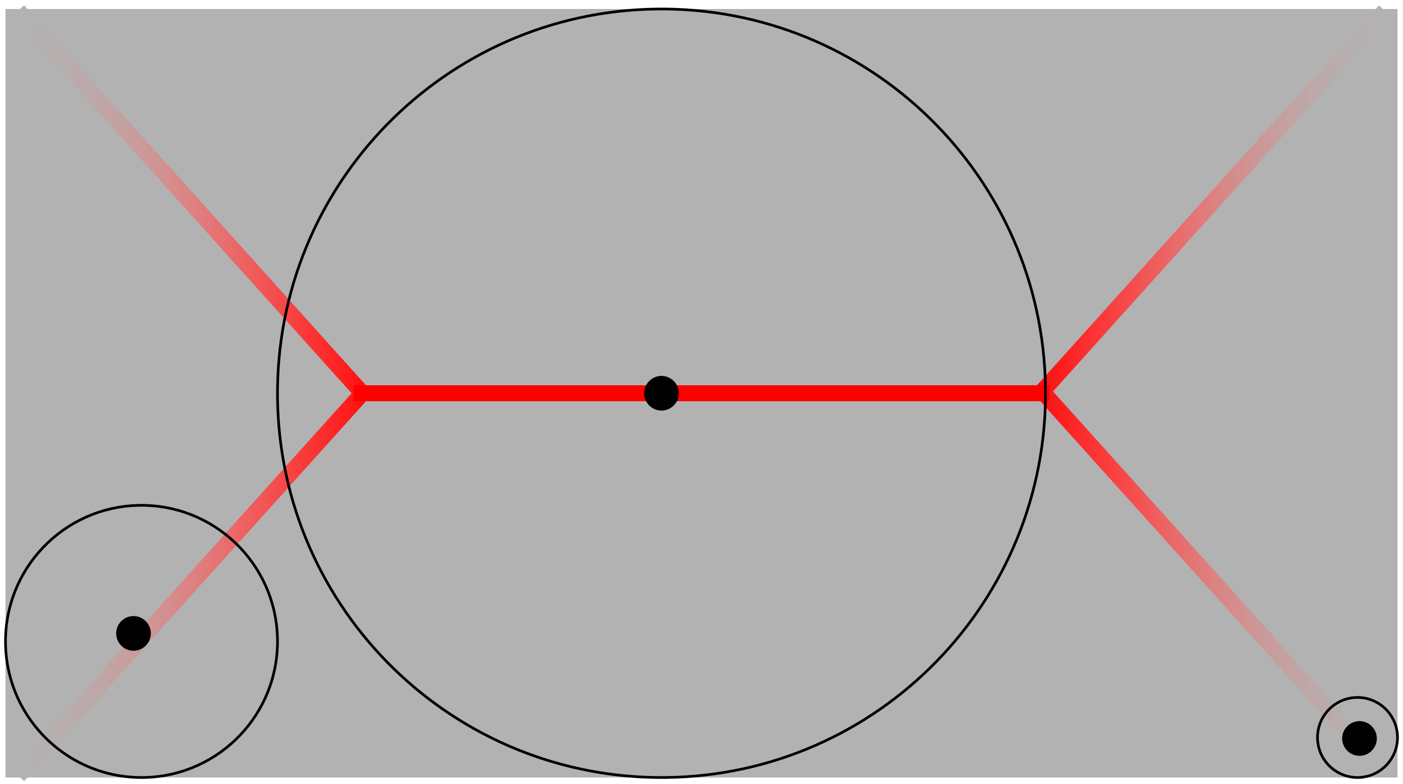


Medial axis transform

- Alternative skeleton definition:
  - The skeleton is the union of centres of maximal discs within *X*.
  - A *maximal* disc is a circular subset of *X* that touches the boundary in at least two places.

- The MAT is the skeleton with the maximal disc radius retained at each point.

Medial axis transform



Skeletonization using morphology

- Use structuring element $B = \begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & 1 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$

- The *n*-th skeleton subset is

$$S_n(X) = (X \ominus_n B) \setminus [(X \ominus_n B) \circ B]$$

$\ominus_n$ denotes n successive erosions.

- The skeleton is the union of all the skeleton subsets:

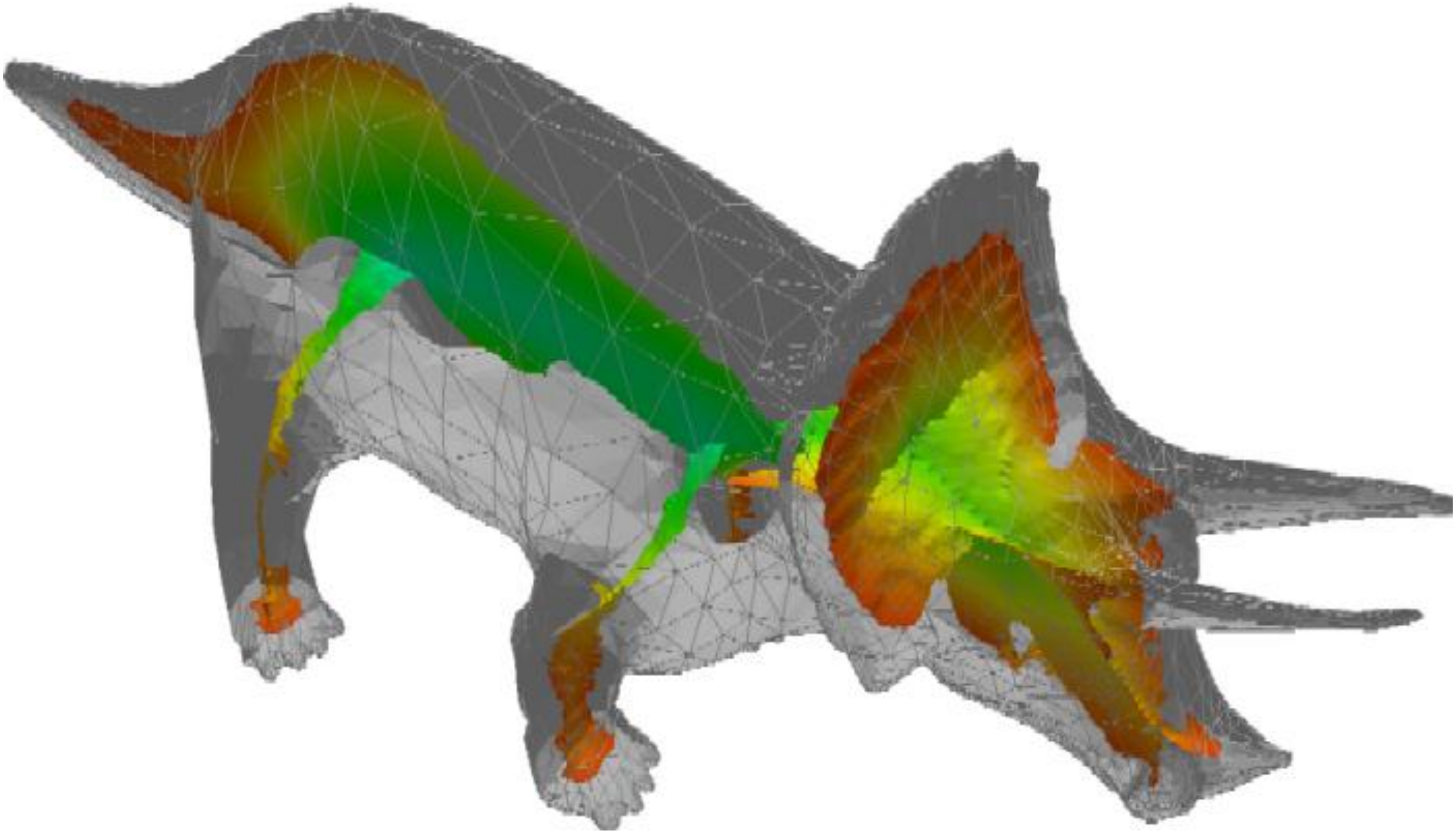
$$S(X) = \bigcup_{n=1}^{\infty} S_n(X)$$

Reconstruction

- We can reconstruct region X from its skeleton subsets:

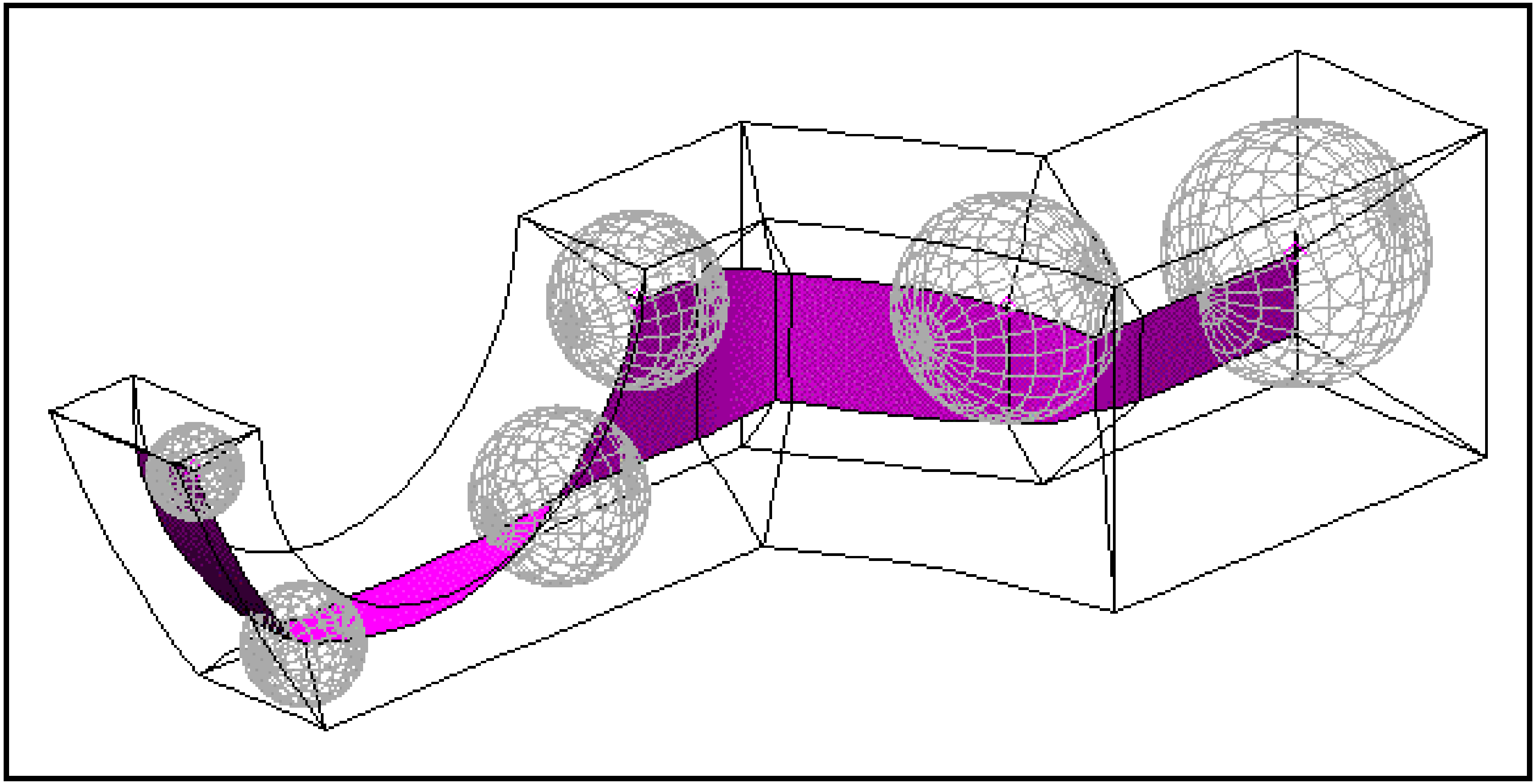
$$X = \bigcup_{n=0}^{\infty} S_n(X) \oplus_n B$$

- We can reconstruct X from the MAT.
- We cannot reconstruct X from $S(X)$.



DiFi: Fast 3D Distance Field Computation Using Graphics Hardware
Sud, Otaduy, Manocha, Eurographics 2004

MAT in 3D



from Transcendata Europe Medial Object
Price, Stops, Butlin Transcendata Europe Ltd

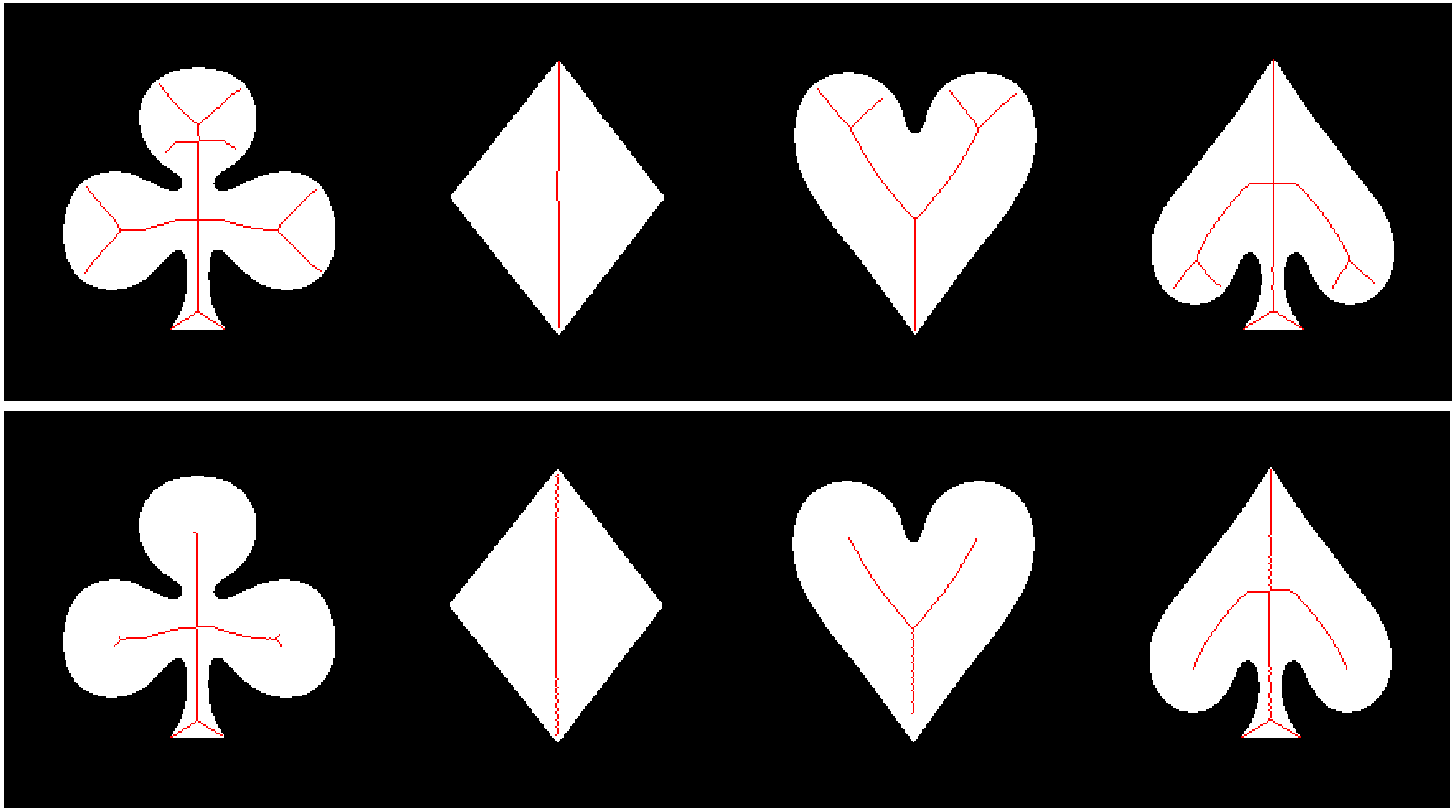
Applications and Problems

- The skeleton/MAT provides a stick figure representing the region shape
- Used in object recognition, in particular, character recognition.
- Problems:
    - Definition of a maximal disc is poorly defined on a digital grid.
    - Sensitive to noise on the boundary.
- Sequential thinning output sometimes preferred to skeleton/MAT.

Example

Skeletons:

Thinned:



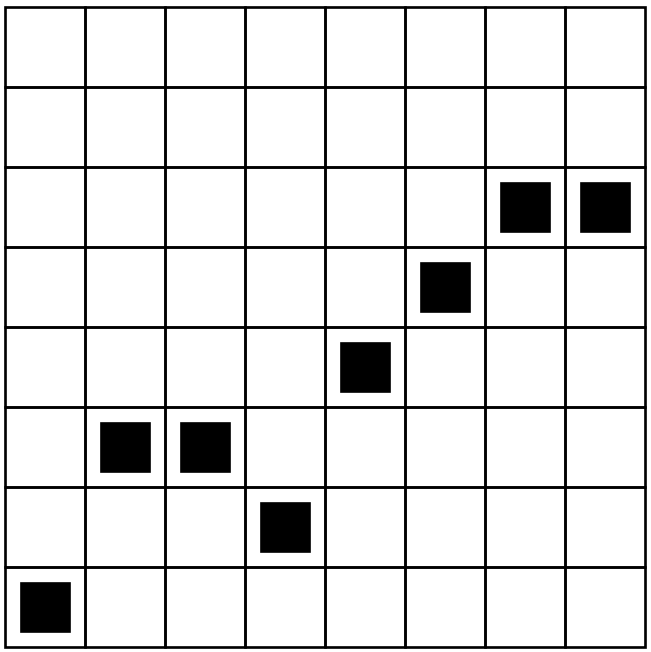
Gray-level Morphology

- Erosion, dilation
- Opening and closing
- The image and the structuring element are gray level arrays.

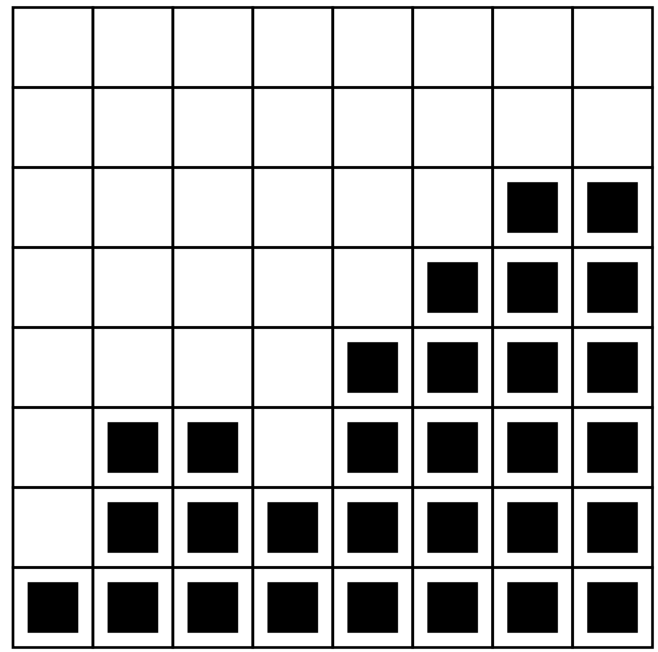
- Used to remove speckle noise.
- Also for smoothing, edge detection and segmentation.

Umbra

$$U(f) = \{(x, y) : y \leq f(x)\}$$



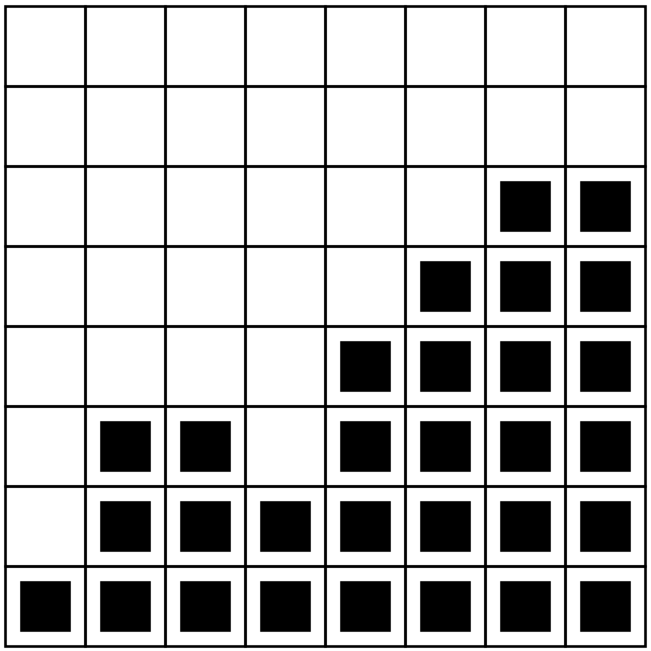
f



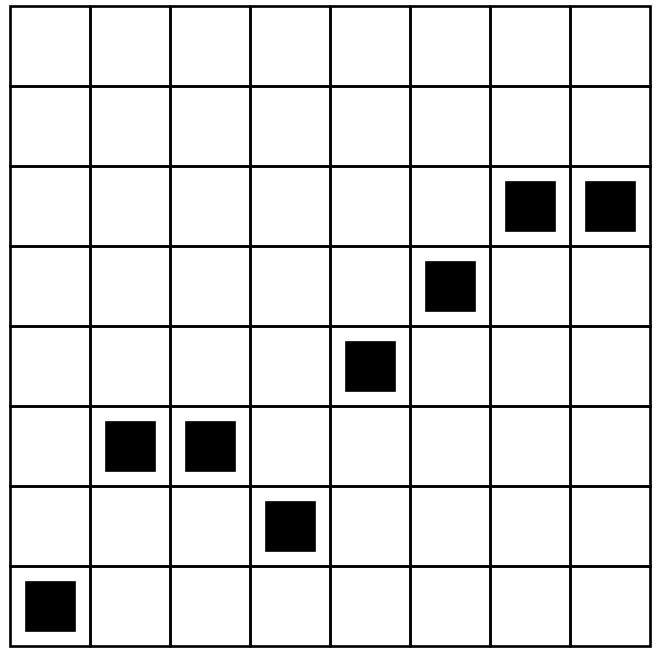
$U(f)$

Top surface

$$T(R) = \{(x, y) : y \geq z \text{ for all } (x, z) \in R\}$$



R



$T(R)$

Gray level erode and dilate

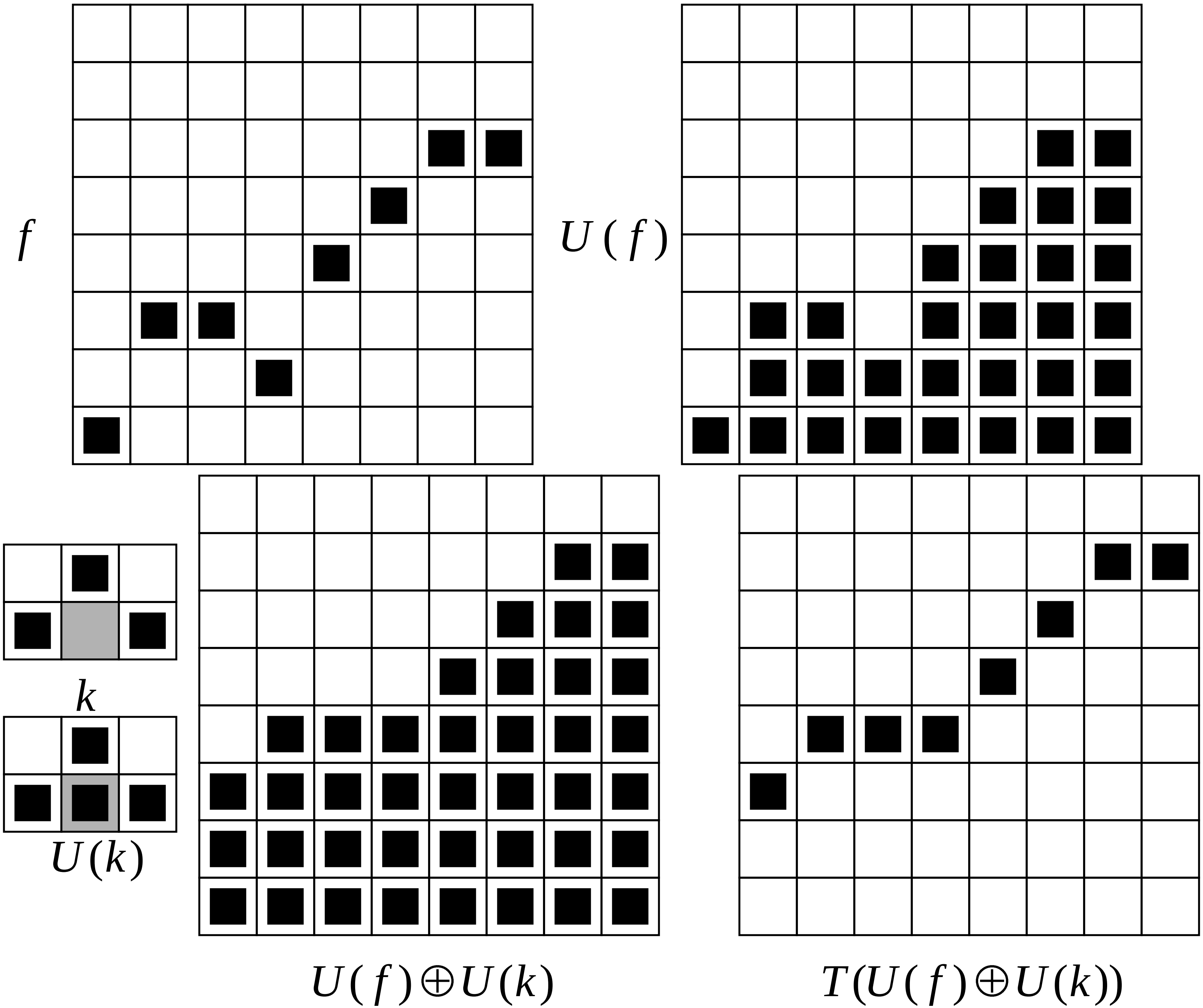
- Erosion:

$$f \ominus k = T(U(f) \ominus U(k))$$

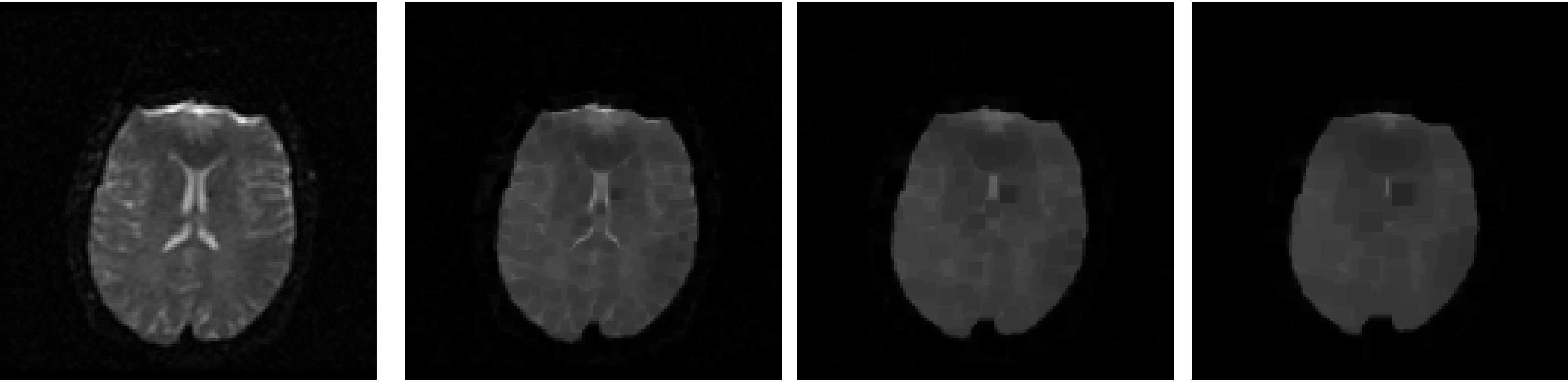
- Dilation:

$$f \oplus k = T(U(f) \oplus U(k))$$

- Open and close defined as before.

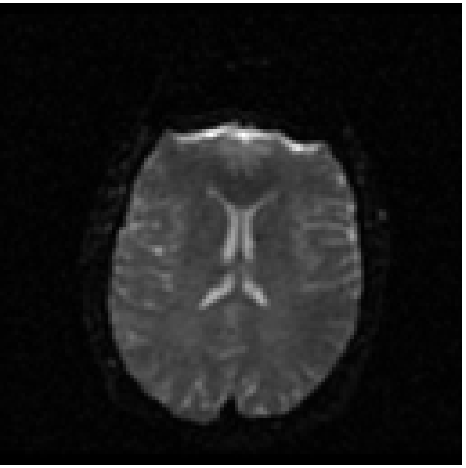

f
$U(f)$
k
$U(k)$
$U(f) \oplus U(k)$
$T(U(f) \oplus U(k))$

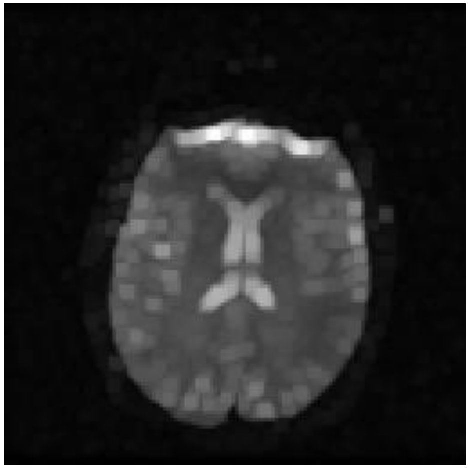
Erosion



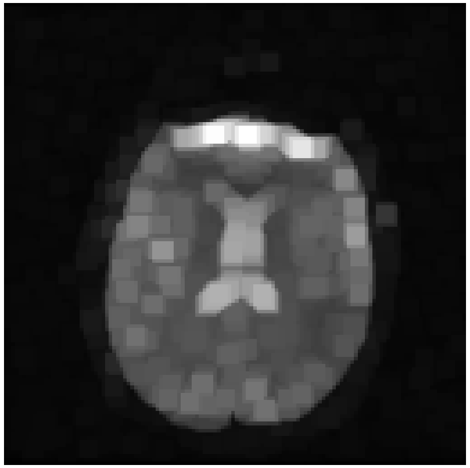
S=ones(3,3) S=ones(5,5) S=ones(7,7)

Dilation

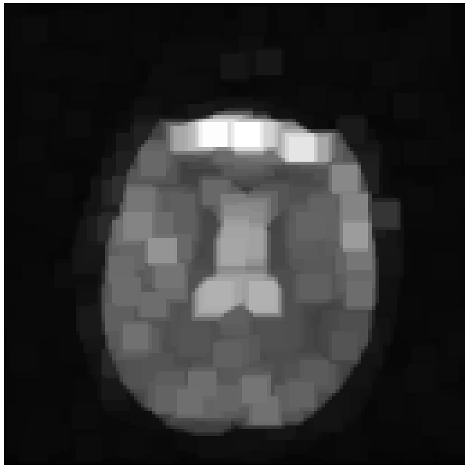




S=ones(3,3)

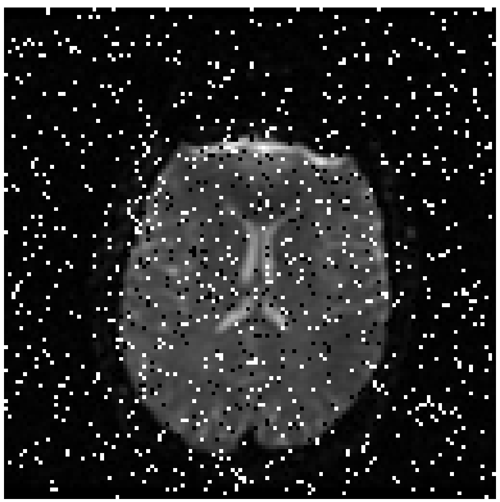


S=ones(5,5)



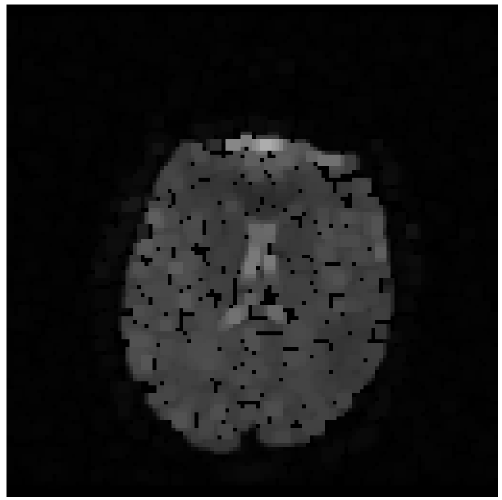
S=ones(7,7)

Speckle removal

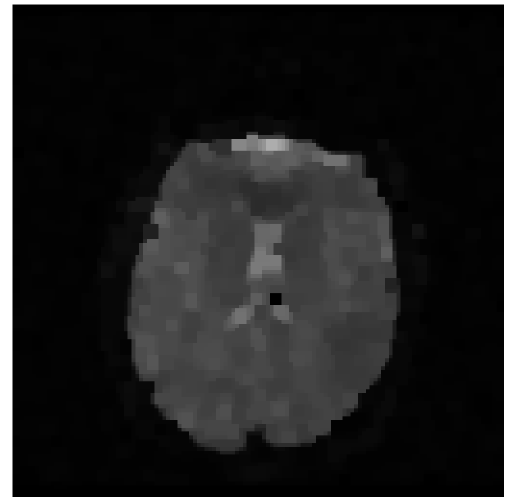


Salt and pepper noise

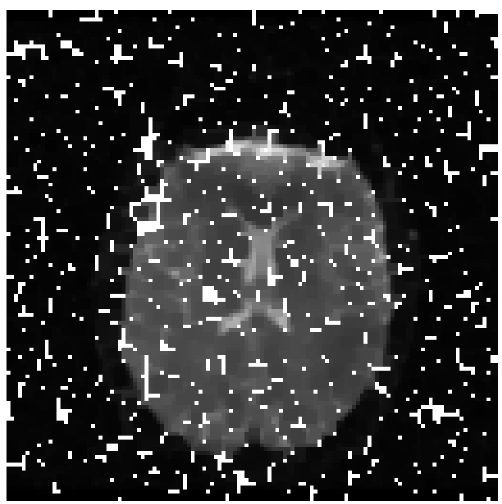
Erode



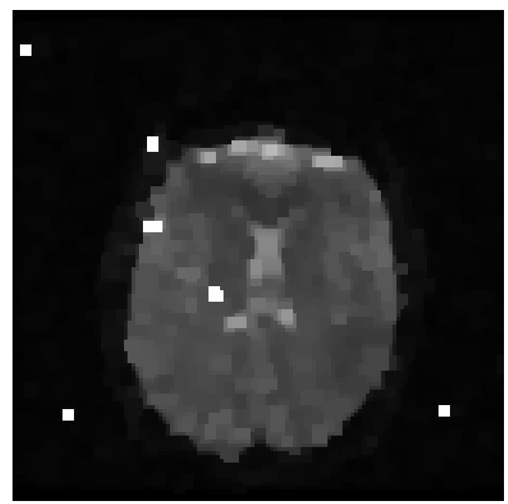
Open



Dilate



Close



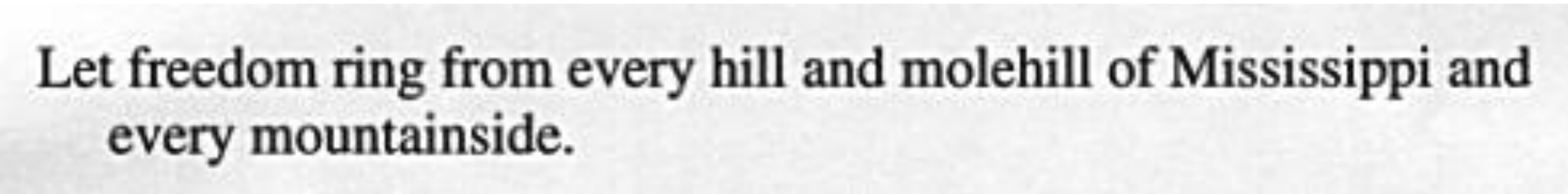
Favorites

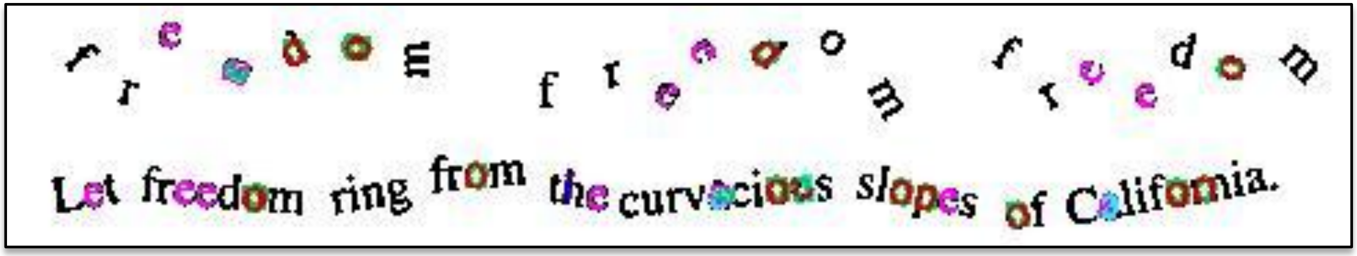
- E = medfilt2()
- [D,L] = bwdist()

Summary

- Simple morphological operations
  - Erode and dilate
  - Open and close
- Applications:
  - Granulometry
  - Thinning and thickening
  - Skeletons and the medial axis transform
- Gray level morphology

Find the Letter ‘e’





Perception: Change Detection?

- Examples from Daniel Simons's many projects

- UBC examples of change blindness

Background Subtraction

- Chromakeying is often impractical
- Camera pose is sometimes locked – opportunity!

Distance Measures

- Chromakey distance – remember?

$$\mathbf{I}_\alpha = \left|\mathbf{I} - \mathbf{g}\right| > T$$

T = ~20

$\mathbf{g}$ = (0 255 0) (for example)

- Plain Background-subtraction metric:

$$\mathbf{I}_\alpha = \left|\mathbf{I} - \mathbf{I}_{bg}\right| > \mathbf{T}$$

$\mathbf{T}$ = [20 20 10] (for example)

$\mathbf{I_{bg}}$ = Background Image