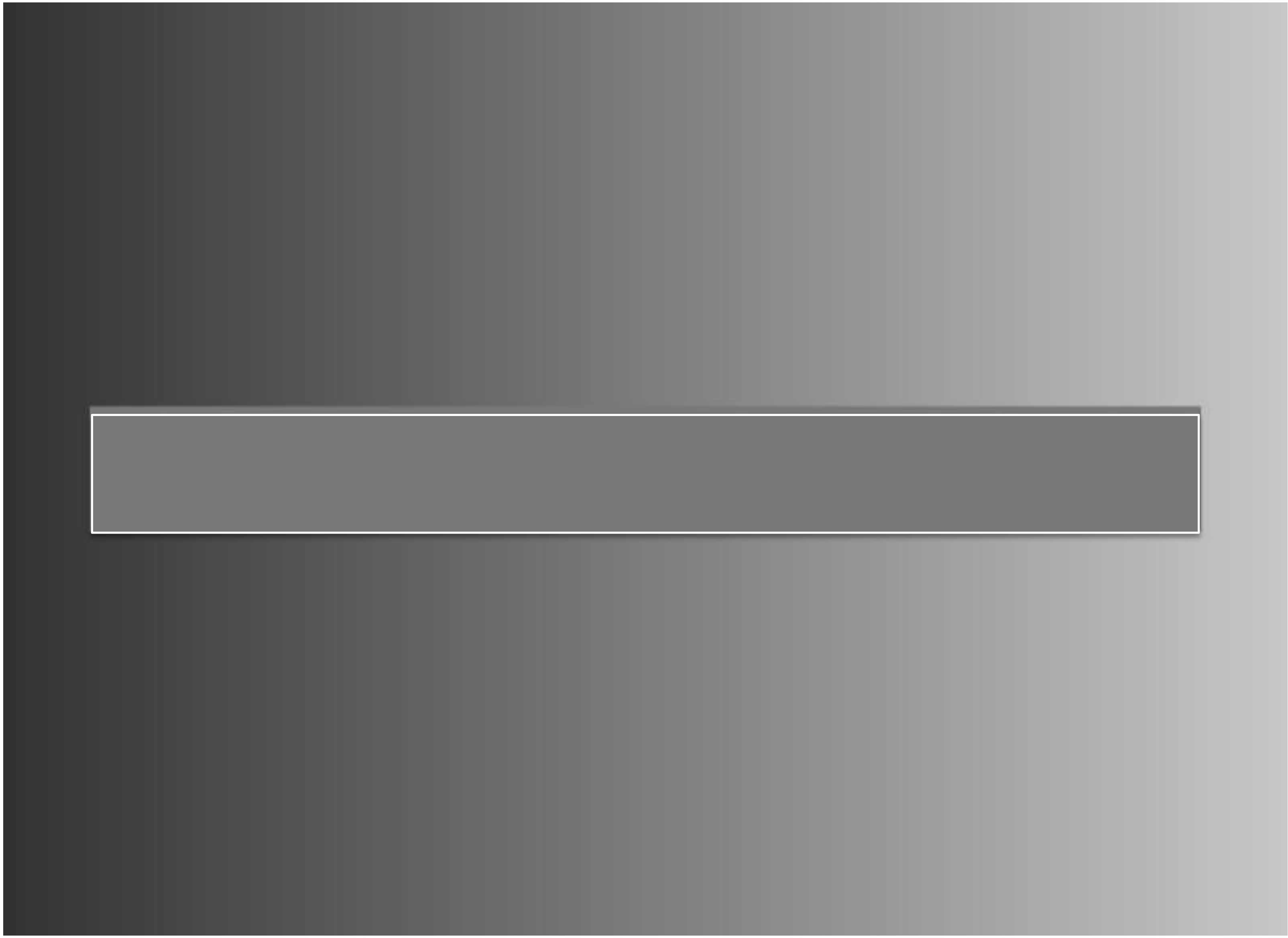
Limitations of Thresholding

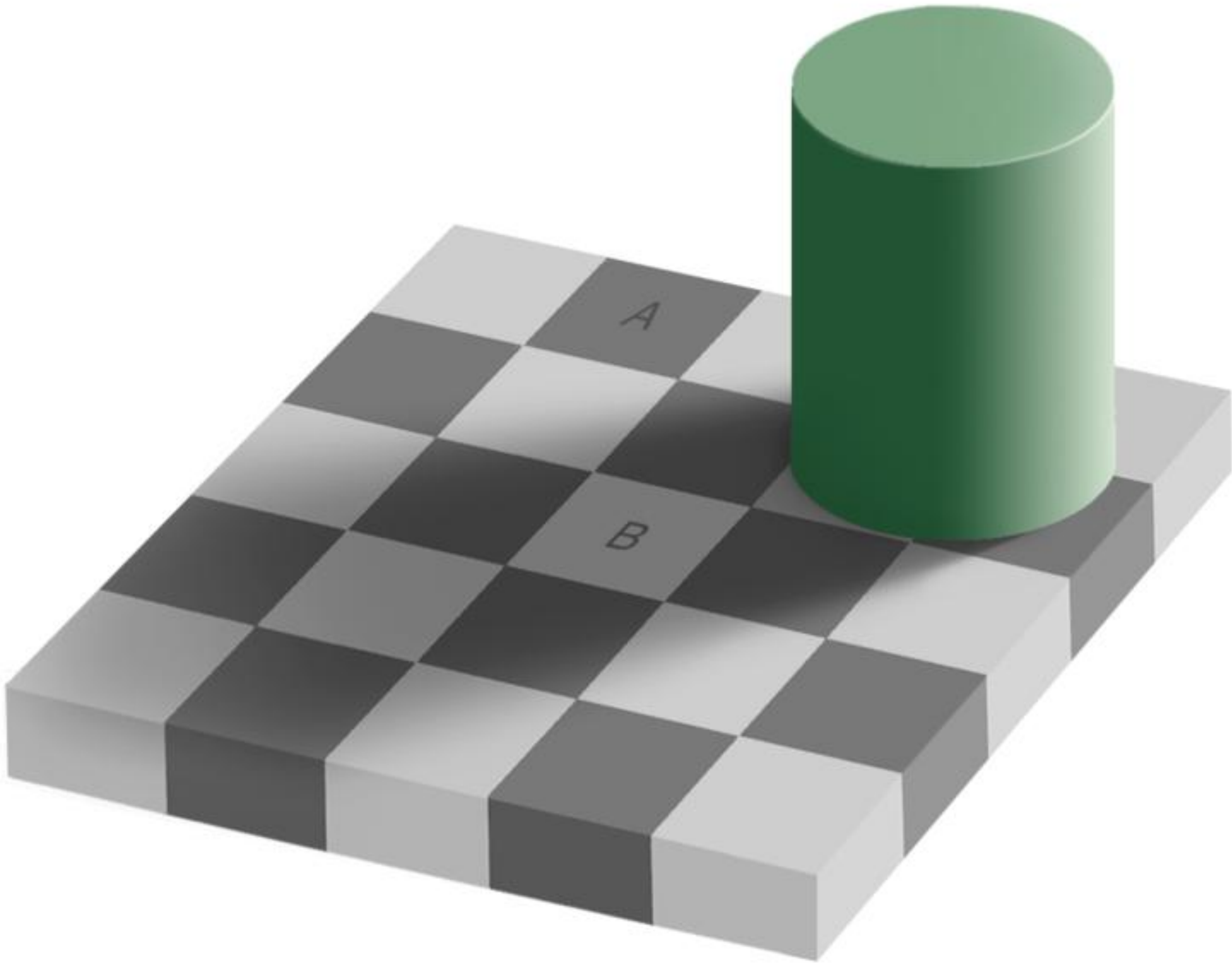
- Why can we segment images much better by eye than through thresholding processes?

- We might improve results by considering image **context:** Surface Coherence

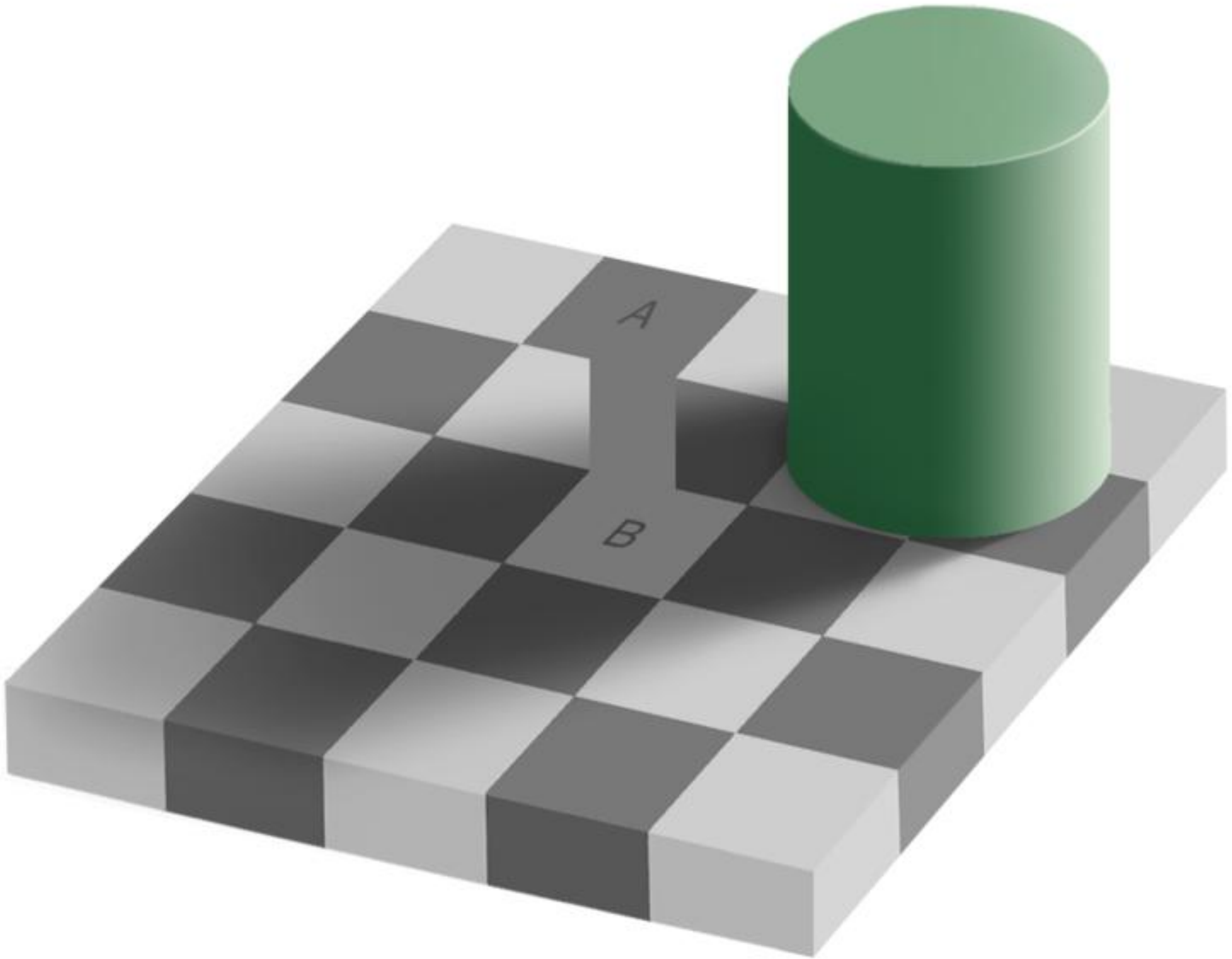


Gradient.illusion.arp.jpg

Aha! Humans are suckers for context!

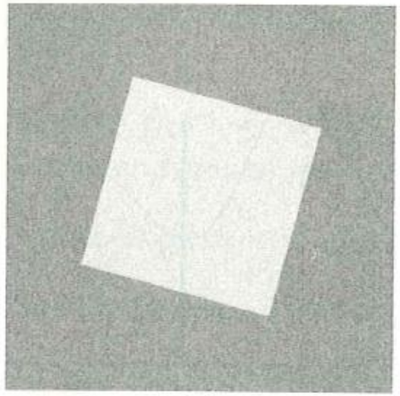


by Adrian Pingstone, based on the original created by Edward H. Adelson

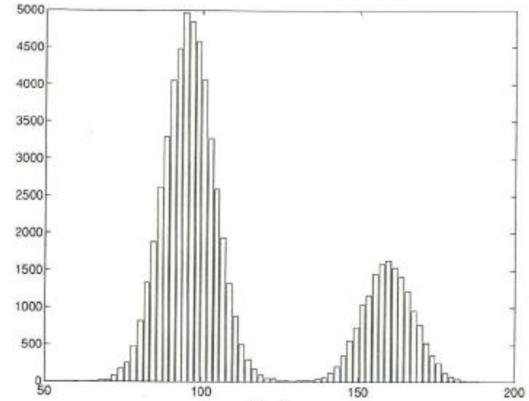


by Adrian Pingstone, based on the original created by Edward H. Adelson

Chapter 3 in Machine Vision by Jain et al.

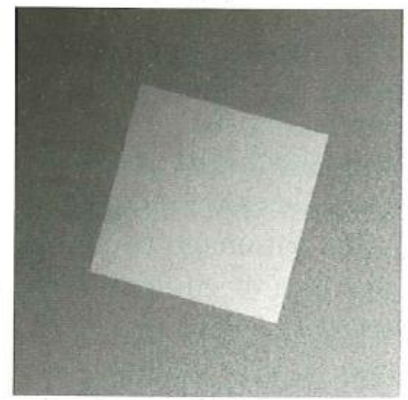


(a)

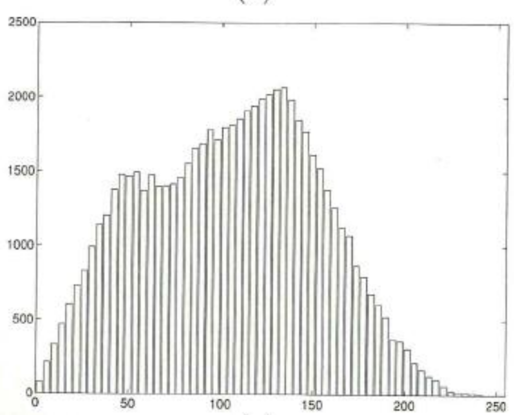


(b)

(c)

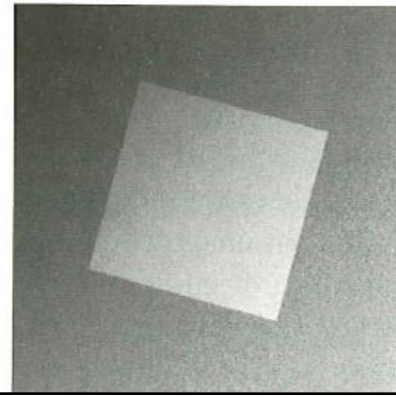


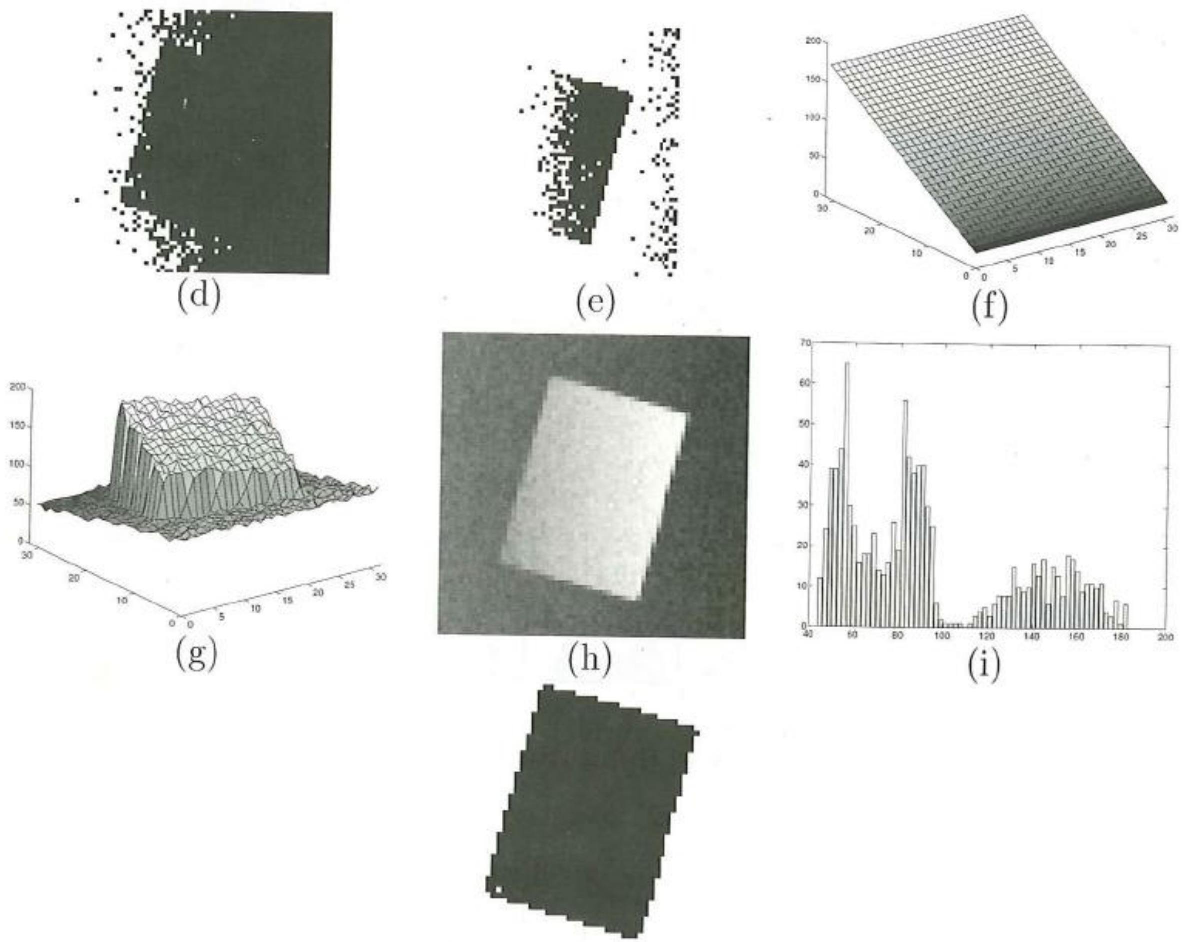
(d)



(e)

(f)





(d) (e) (f) (g) (h) (i) (j)

Note on Performance Assessment

- In real-life, we use two or even three separate sets of test data:
  1. A ***training set***, for tuning the algorithm

  2. A ***validation*** set for tuning the performance score

  3. An unseen ***test set*** to get a final performance score on the tuned algorithm

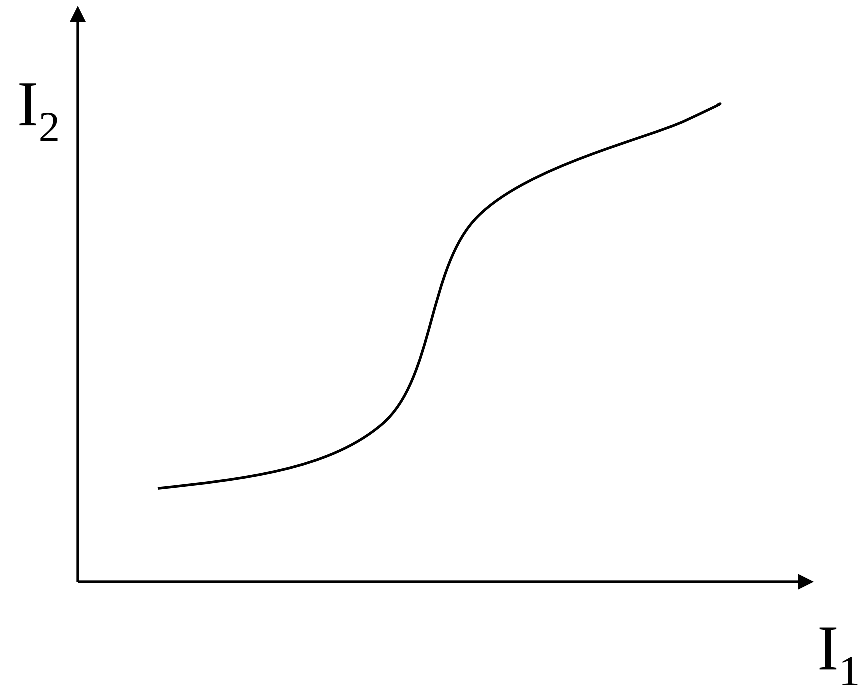
Image Transformations

Outline

- Grey-level transformations
  - Histogram equalization
- Geometric transformations
  - Affine transformations
  - Interpolation
  - Warping and morphing

Grey-level Transformations

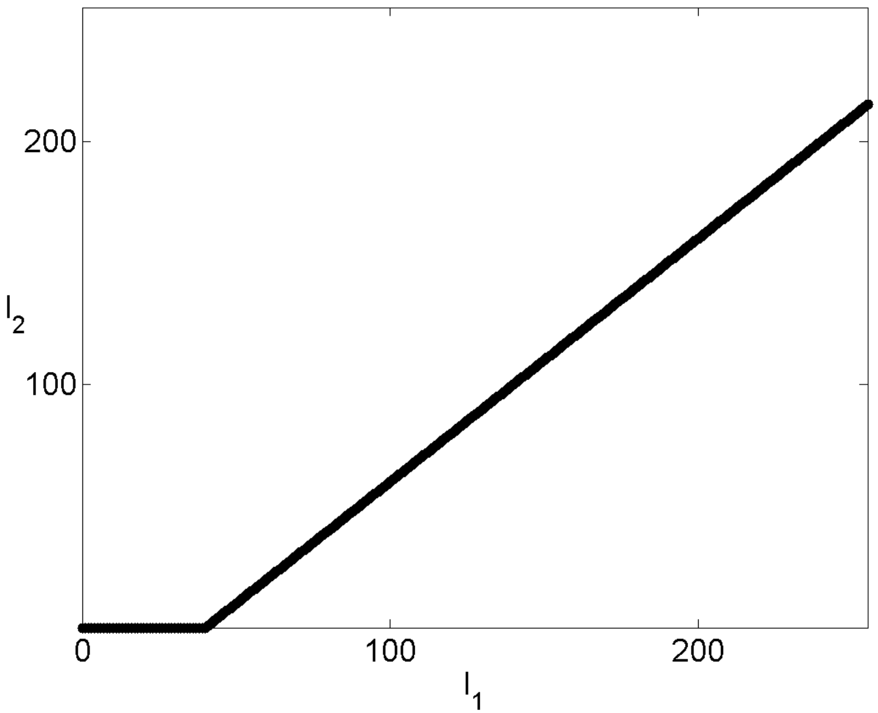
- Start with I_1, $I_1 : R^n \to Z_{256}$ or $I_1 : R^n \to R$
- Change the image grey level in each pixel by a fixed mapping f: $R \to R$:
- $I_2(x, y) = f(I_1(x,y))$



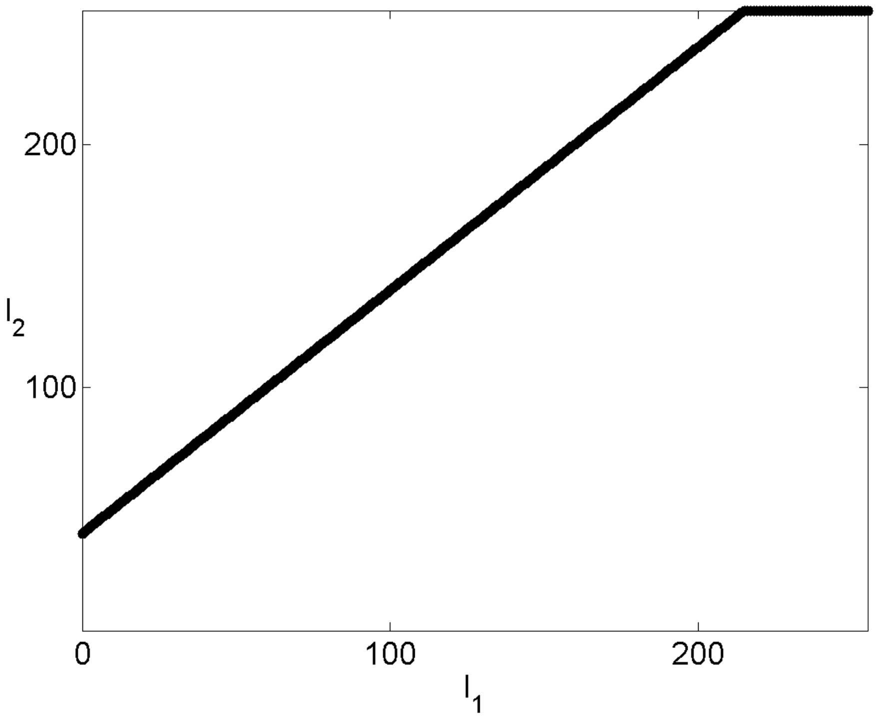
Linear – contrast stretching

- f is a linear function: $f(x) = \alpha x + \beta$
- We must preserve the range of grey level values as $\{0, 1, \ldots, 255\}$.

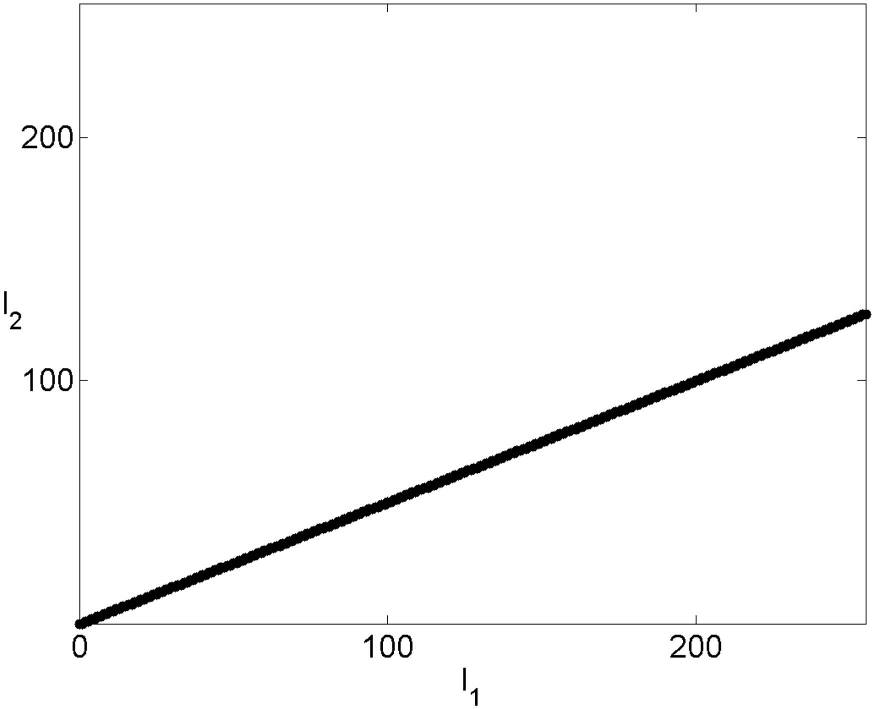
α=1.0, β=-40



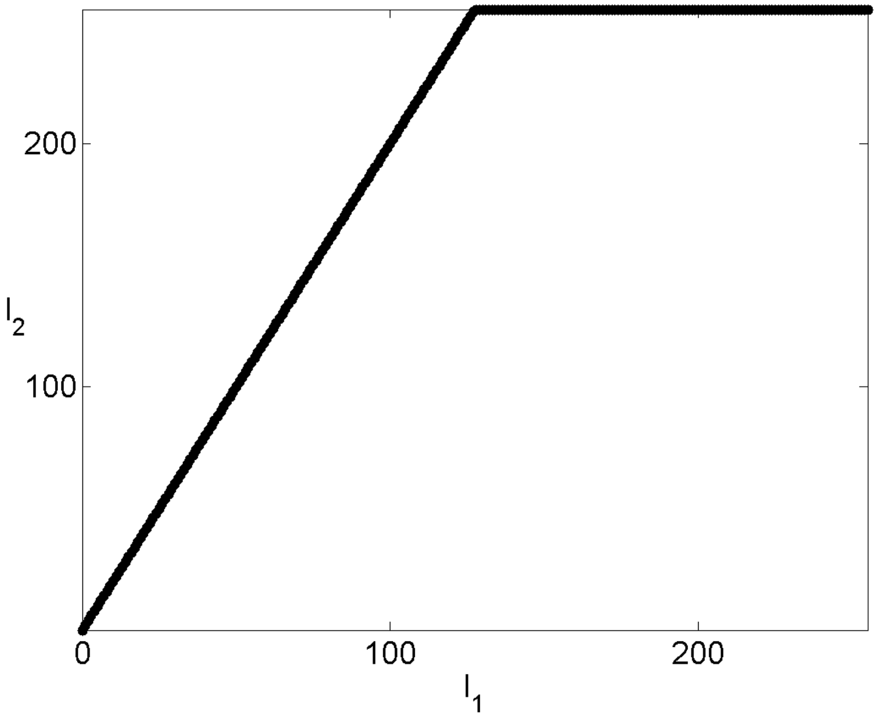
α=1.0, β=40



α=0.5, β=0



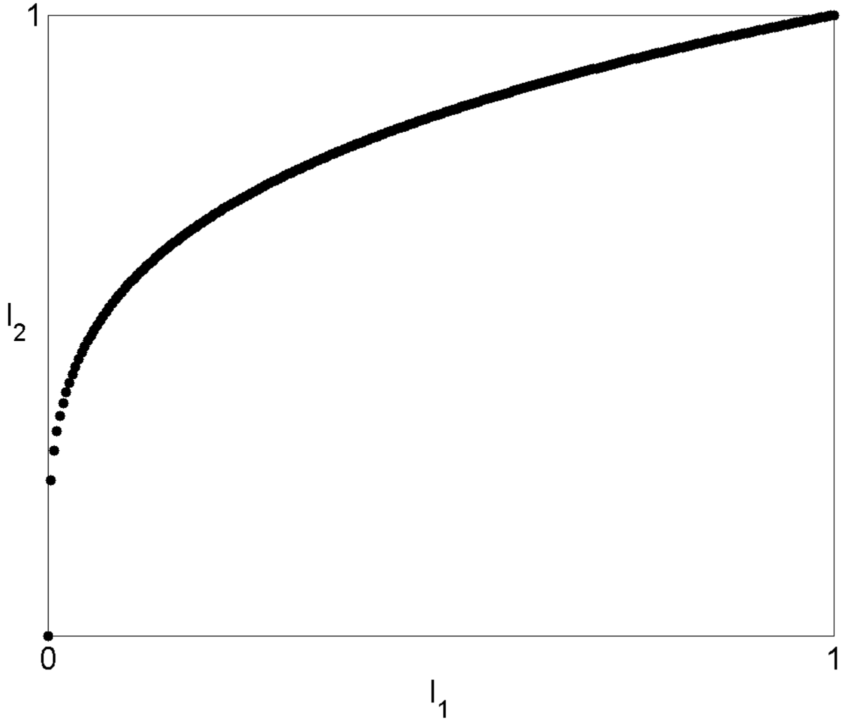
α=2.0, β=0



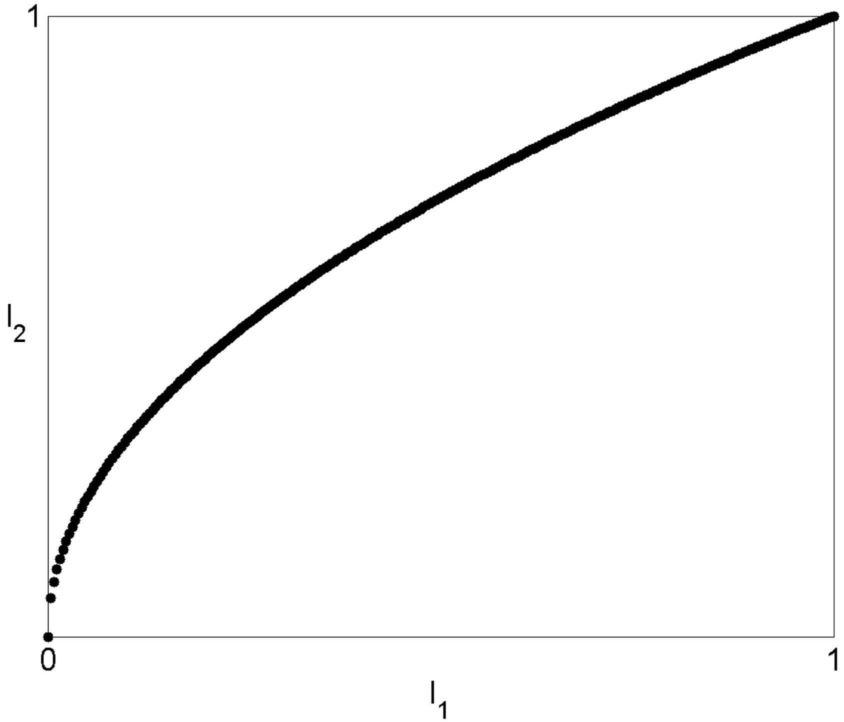
Non-linear – Gamma Correction

- Non-linear grey-level transformations are useful too
- Gamma correction adjusts for differences between camera sensitivity and the human eye
- $f(x) = Ax^{\gamma}$
- $A=255^{(1-\gamma)}$ ensures that the grey scale range is unchanged

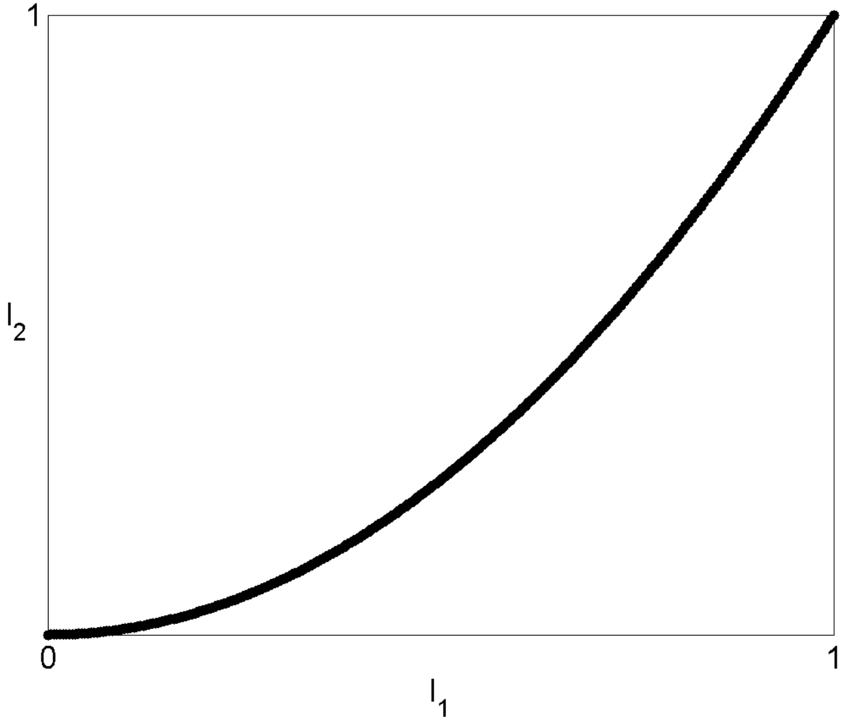
γ=0.25



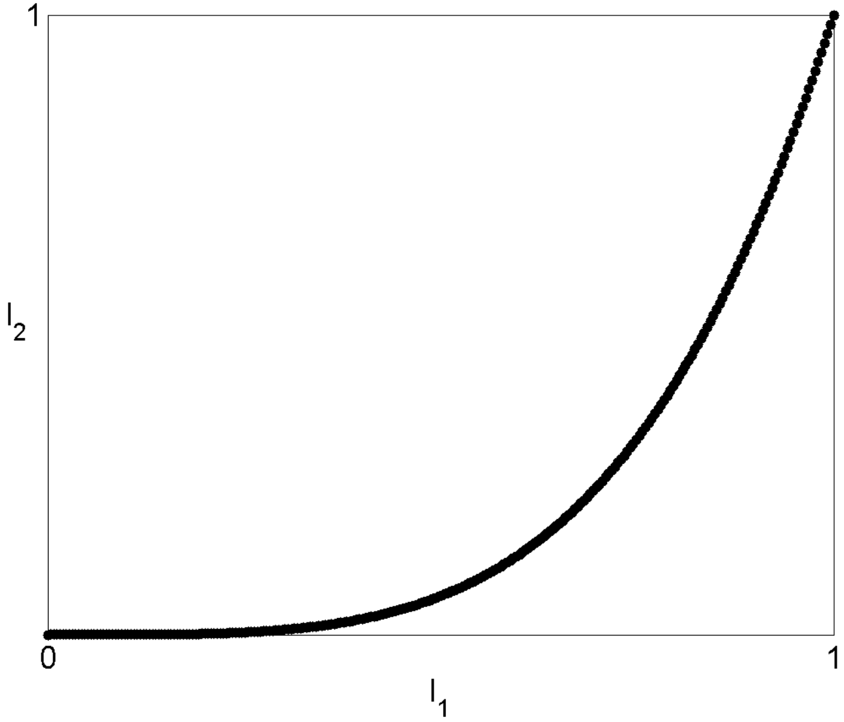
γ=0.5

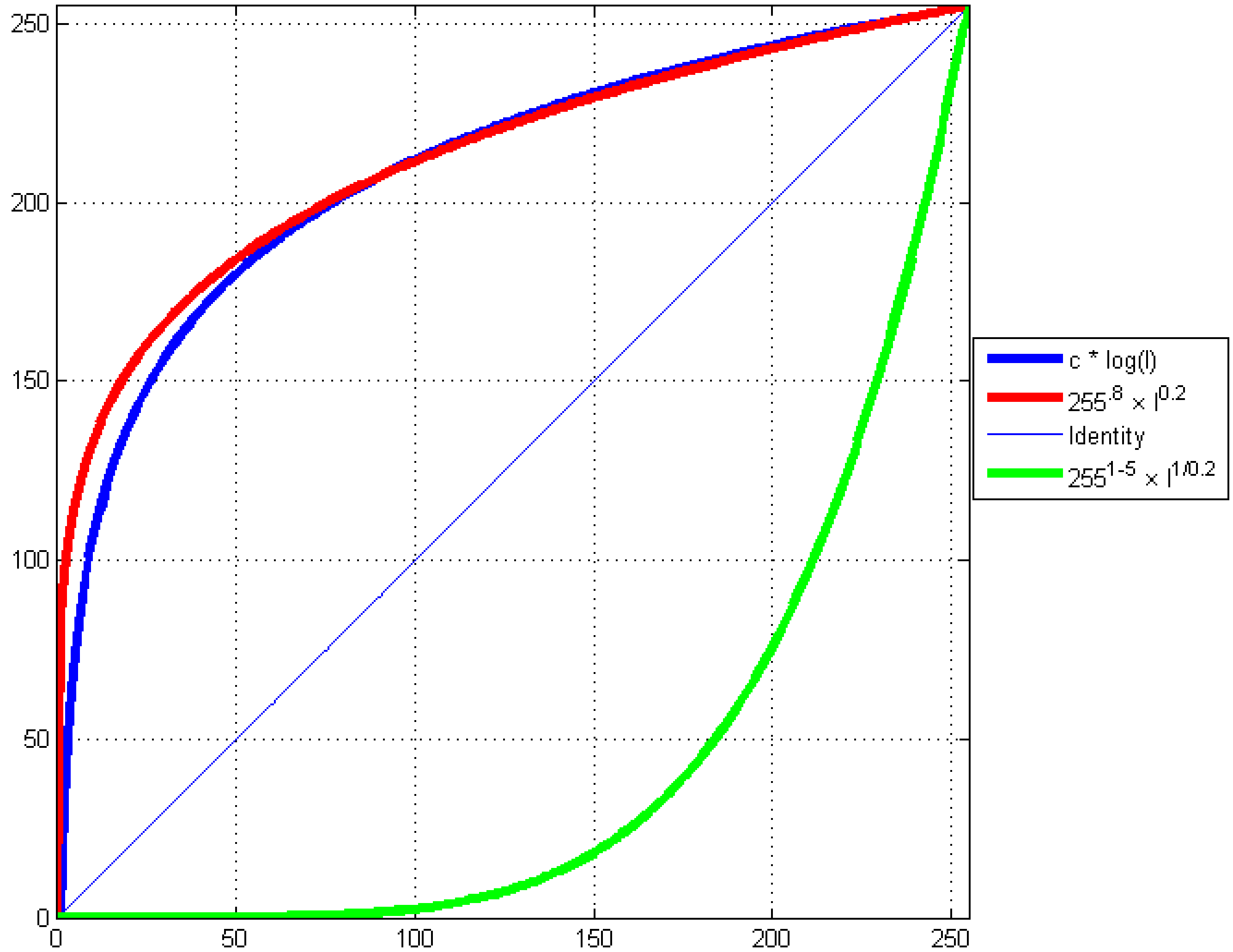


γ=2



γ=4





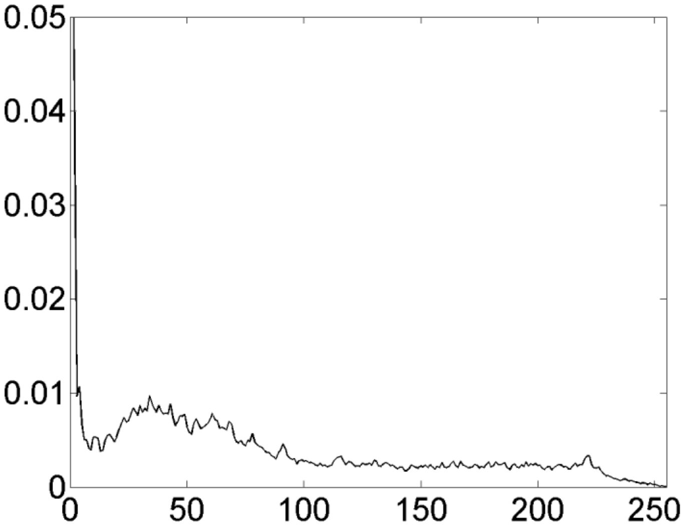
For example, CRT's would have gamma = 2.5, so pre-apply a gamma = 1/2.5.

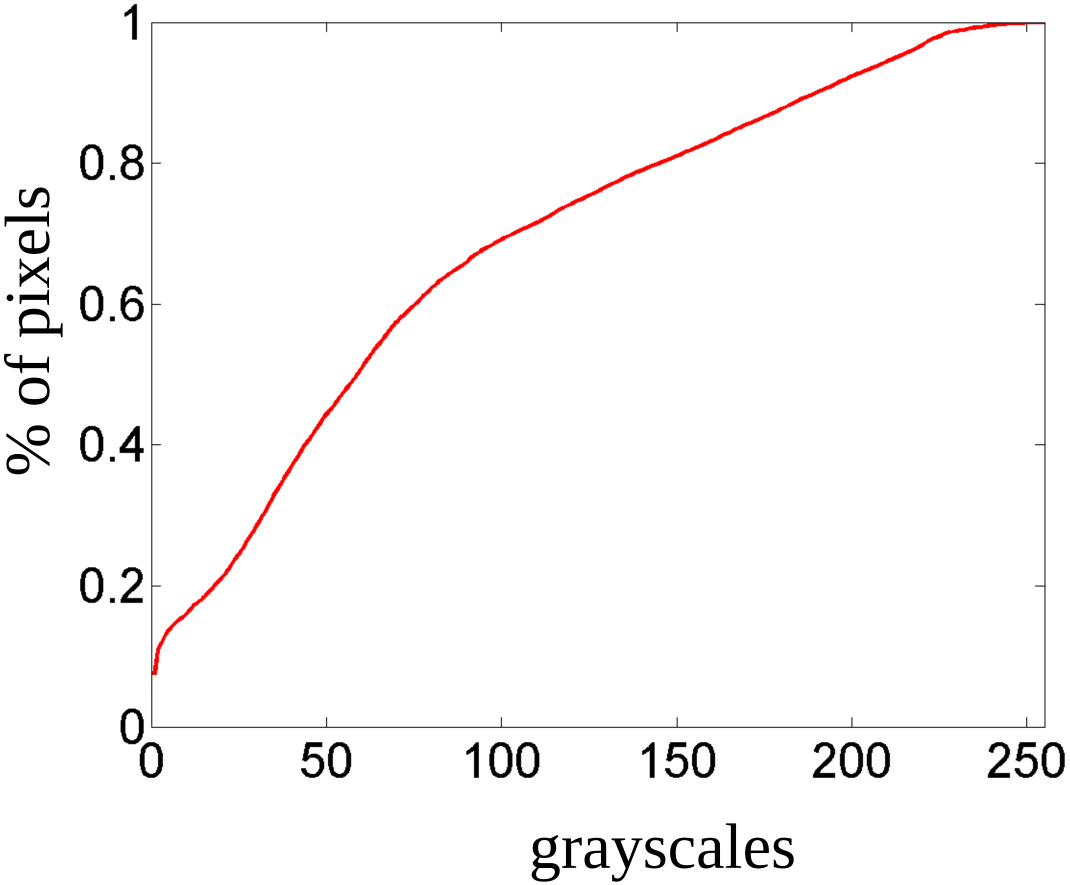
Histogram Equalization

- Tries to use all the grey levels equally often
- The grey level histogram is then flat

- Use the cumulative histogram for f

Cumulative histogram





Histogram Equalization

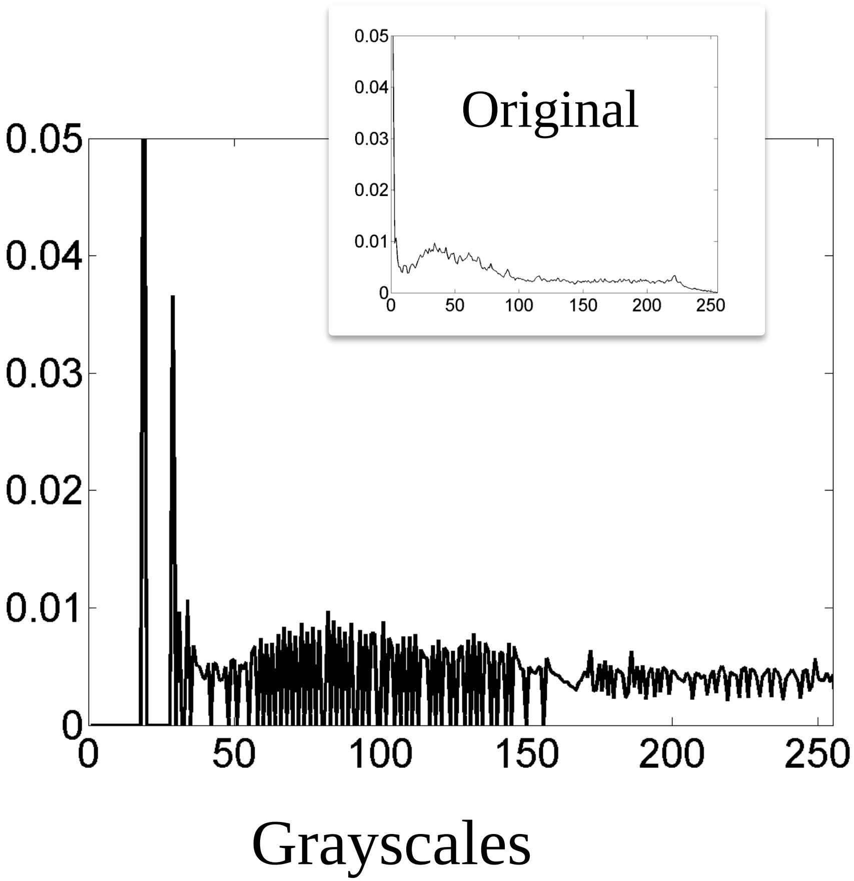
```
function eqIm = histEq(image)
[X,Y] = size(image);
% Construct the cumulative histogram
for i=0:255
    cumHist(i) = sum(sum(image <= i))/(X*Y);
end
% Use it to transform the grey level in each pixel.
eqIm = fix(255*cumHist(image));
```

Equalized Image

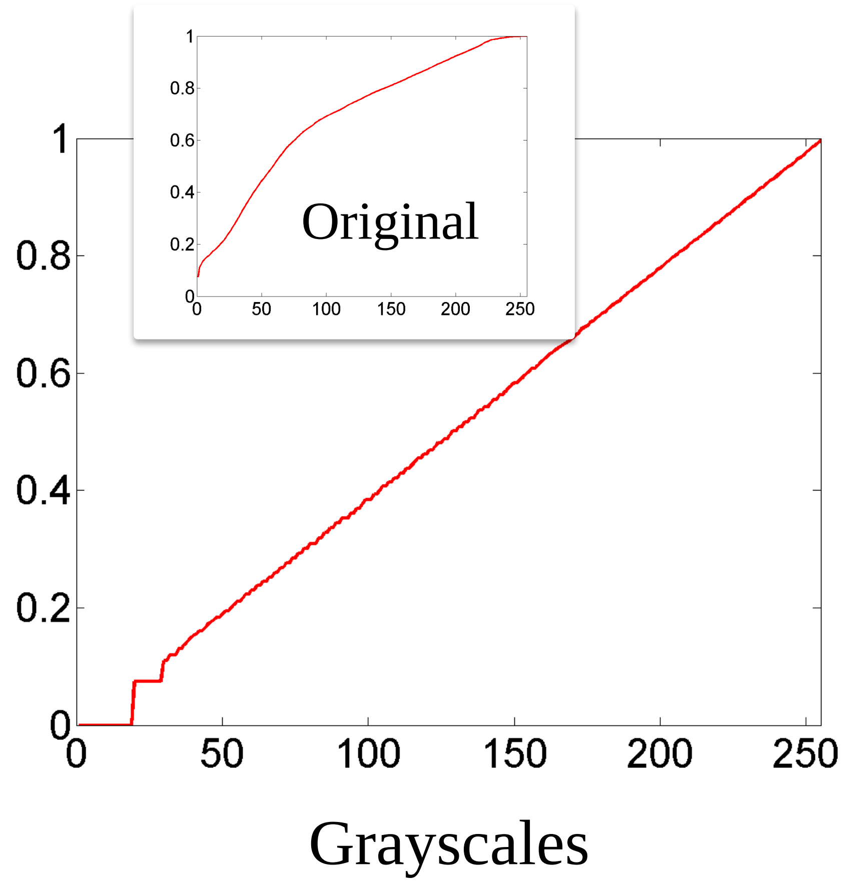
Original

Equalized

Equalized Histograms



Histogram after equalization



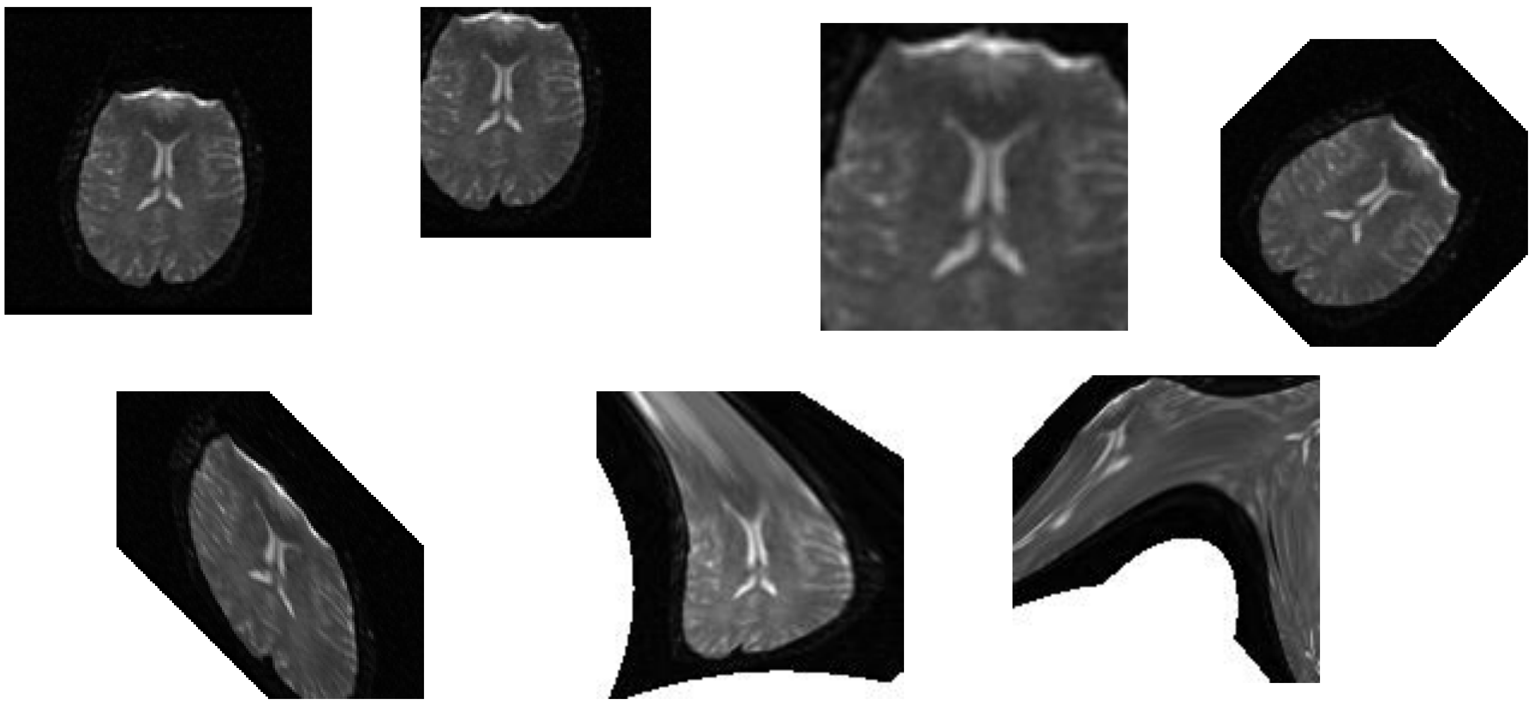
Cumulative histogram after equalization

Unassessed Exercise

- Write matlab functions for linear grey-level transformations and gamma correction. Try moderate and extreme settings on an image and observe the effects they have. Plot the grey-level histograms before and after the grey-level transformations.

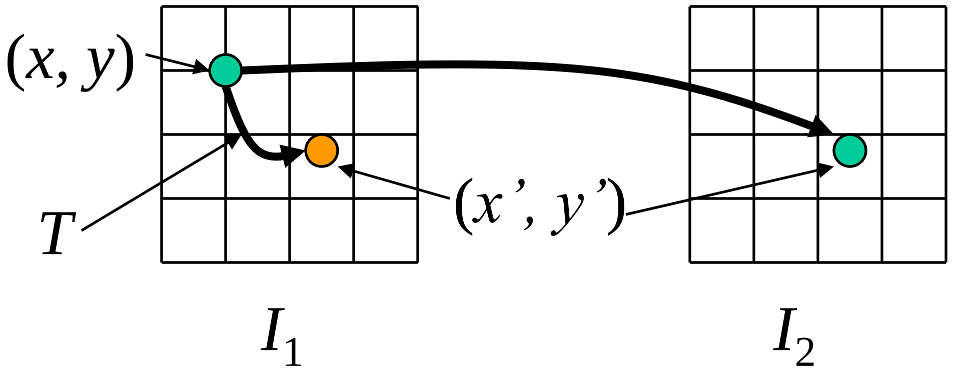
Geometric Transformations

- Change the location of image features



Geometric Transformations

- I_1 is the original image, I_2 is the warped image.
- $I_2(x', y') = I_1(x, y)$
- $(x', y') = T(x, y)$



Affine Transformations

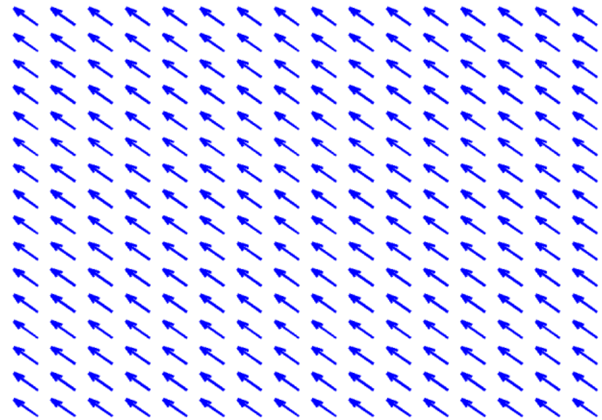
- $x' = ax + by + t_x$
- $y' = cx + dy + t_y$

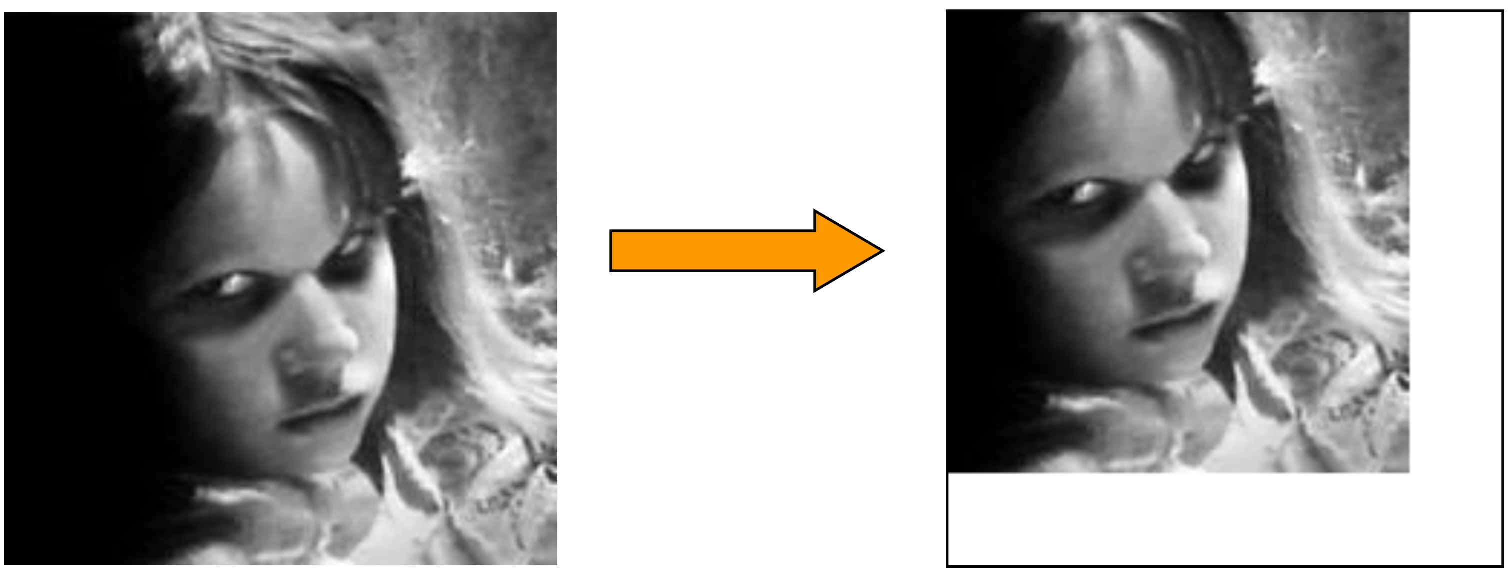
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

- Translation, scaling, rotation, shear.

Translation

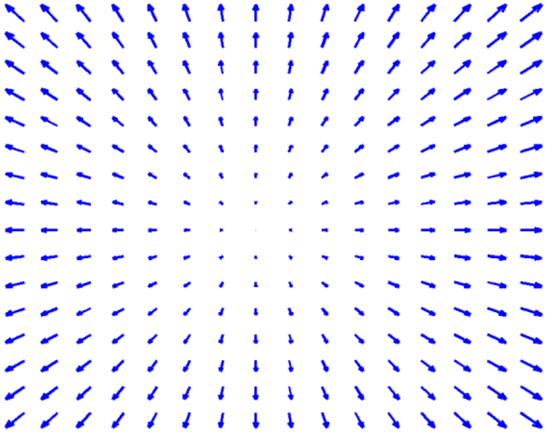
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} -W/4 \\ -H/4 \end{pmatrix}$$

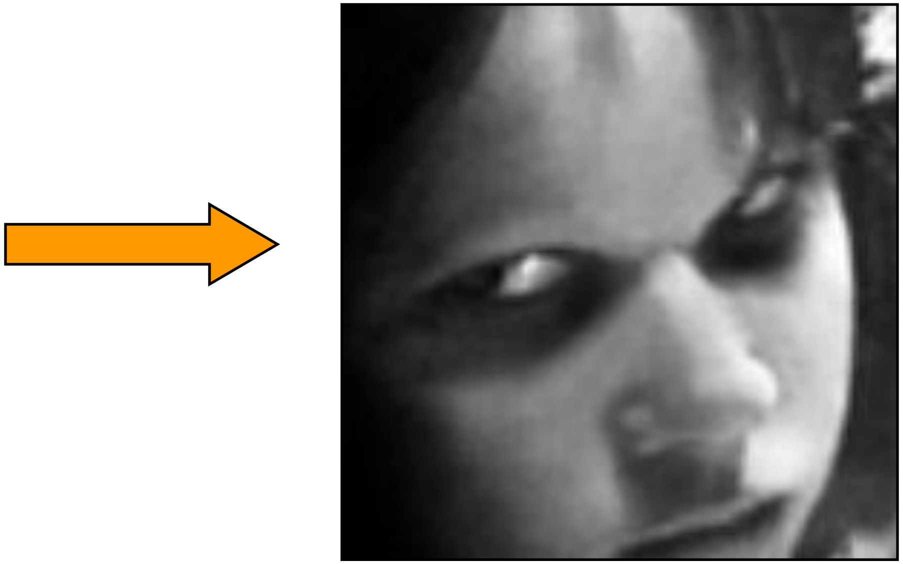




Scaling

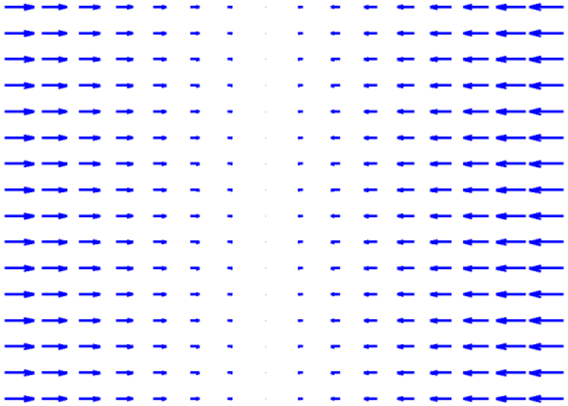
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \quad \begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} -W/2 \\ -H/2 \end{pmatrix}$$

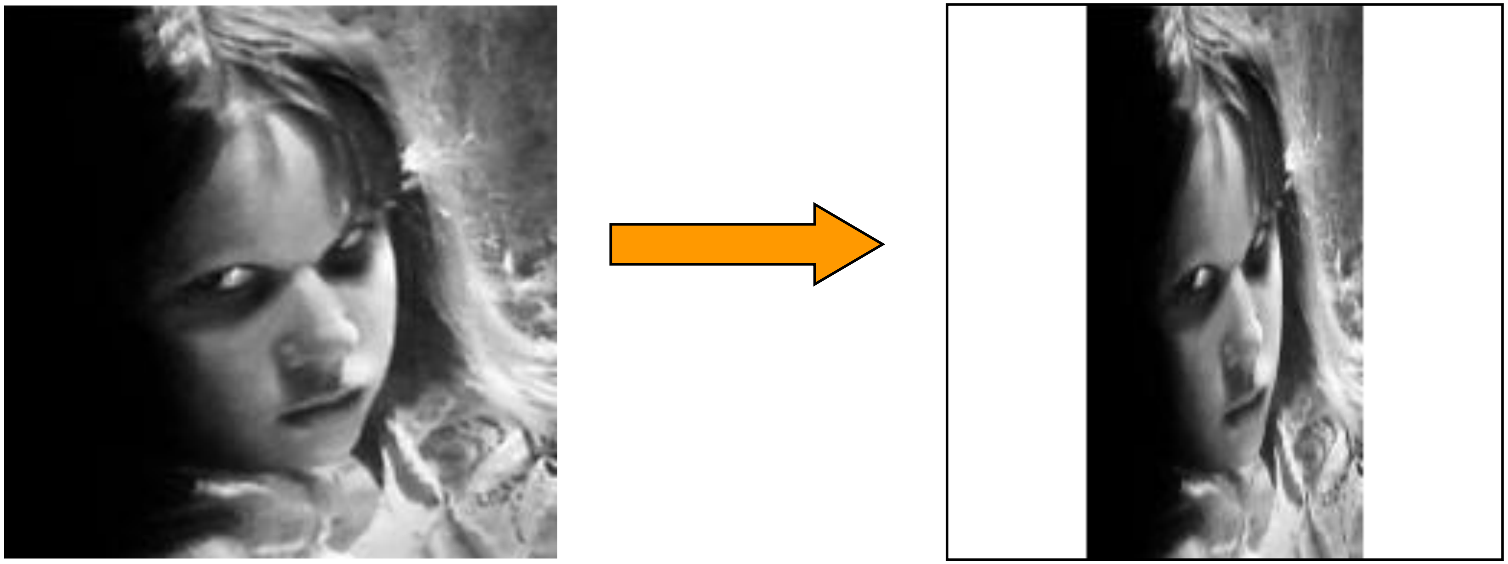




Stretch

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} W/4 \\ 0 \end{pmatrix}$$

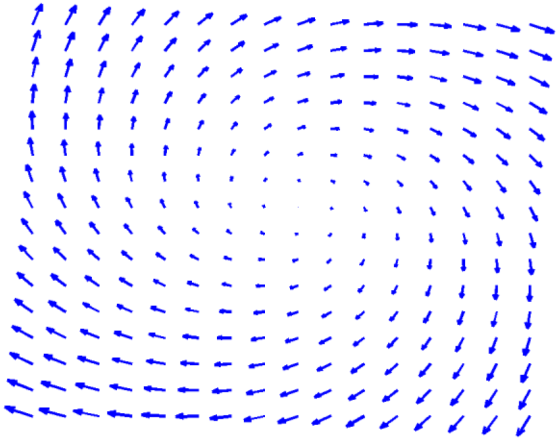




Rotation

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \theta = \pi / 4,$$

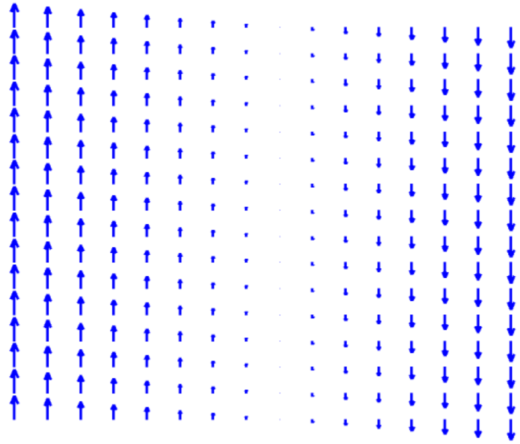
$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} -(W(\cos\theta - 1) + H\sin\theta)/2 \\ (W\sin\theta - H(\cos\theta - 1))/2 \end{pmatrix}$$



Shear (along y axis)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix}, s = 1$$

$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} 0 \\ -sH/2 \end{pmatrix}$$



Homogeneous Coordinates

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Remember:

$$x' = ax + by + t_x$$
$$y' = cx + dy + t_y$$

$$\Rightarrow$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

Homogeneous Coordinates

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

"Subsequent" operations are inserted here, by pre-multiplying

Implementation

- Always use the inverse transformation:

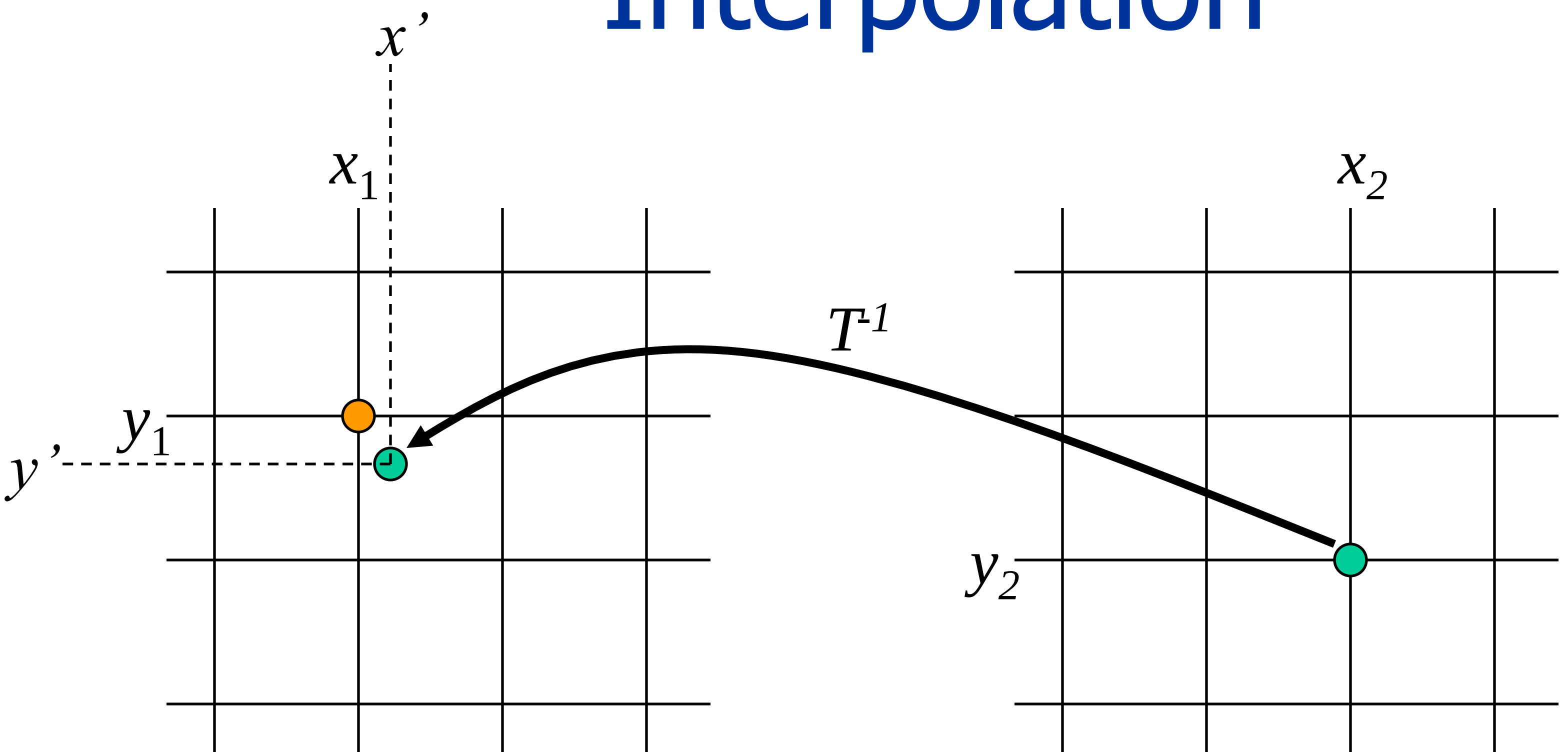
```
function warpedImage = transform(image, T)
[W,H] = size(image);
warpedImage = zeros(W,H);
invT = invert(T);
for x=1:W
    for y=1:H
        warpedImage(x,y)=image(invT(x,y));
    end
end
```

Interpolation

- Usually, $(x', y') = T^{-1}(x, y)$ are not integer coordinates.

- We estimate $I_1(x', y')$ by *interpolation* from surrounding positions.

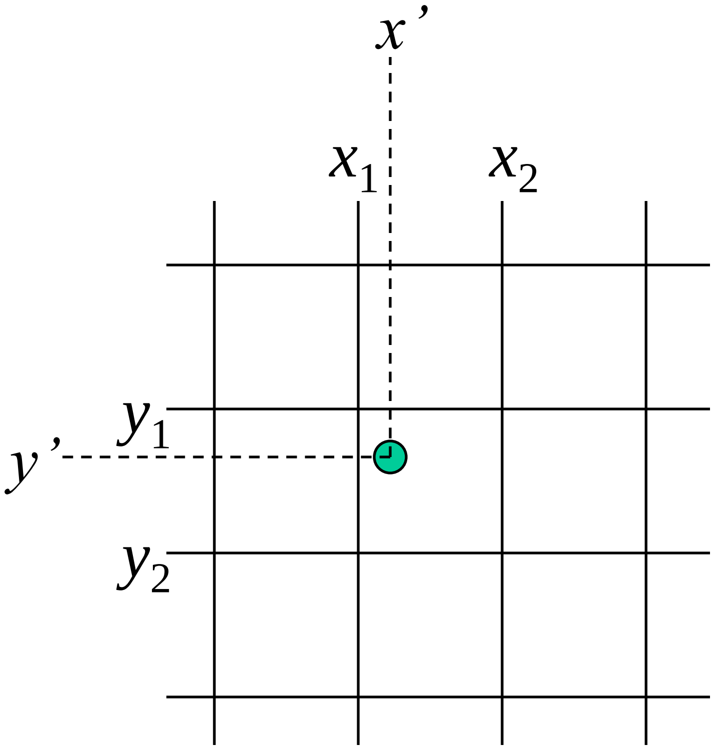
Nearest Neighbour Interpolation



Take the value at the nearest grid point:

$$I_1(x', y') = I_2(x_2, y_2)$$

Bilinear interpolation



Weighted average from neighbouring grid points:

$$
\begin{aligned}
I_1(x', y') = {} & \Delta x\, \Delta y\, \mathrm{I}_1(x_2, y_2) \\
& + \Delta x\, (1 - \Delta y)\, \mathrm{I}_1(x_2, y_1) \\
& + (1 - \Delta x)\, \Delta y\, \mathrm{I}_1(x_1, y_2) \\
& + (1 - \Delta x)\, (1 - \Delta y)\, \mathrm{I}_1(x_1, y_1)
\end{aligned}
$$

$\Delta x = x' - x_1, \quad \Delta y = y' - y_1.$

Higher order interpolation

- Quadratic interpolation fits a bi-quadratic function to a 3x3 neighbourhood of grid points

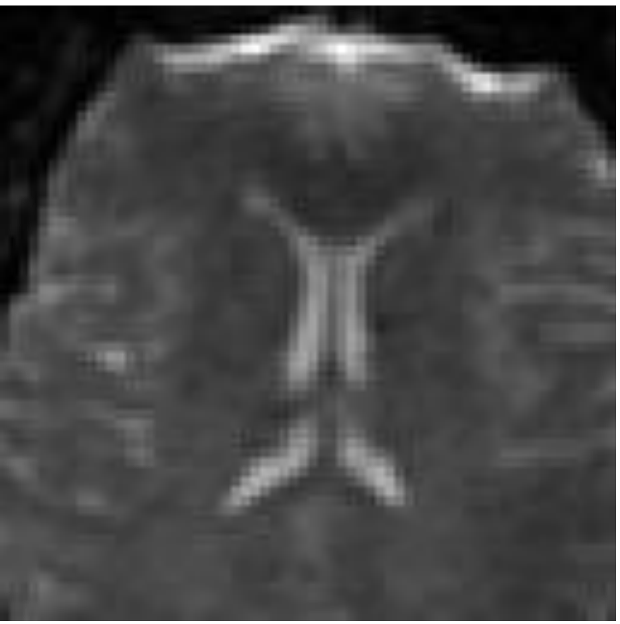
- Cubic interpolation fits a bi-cubic function to a 4x4 neighbourhood.

`interp2`

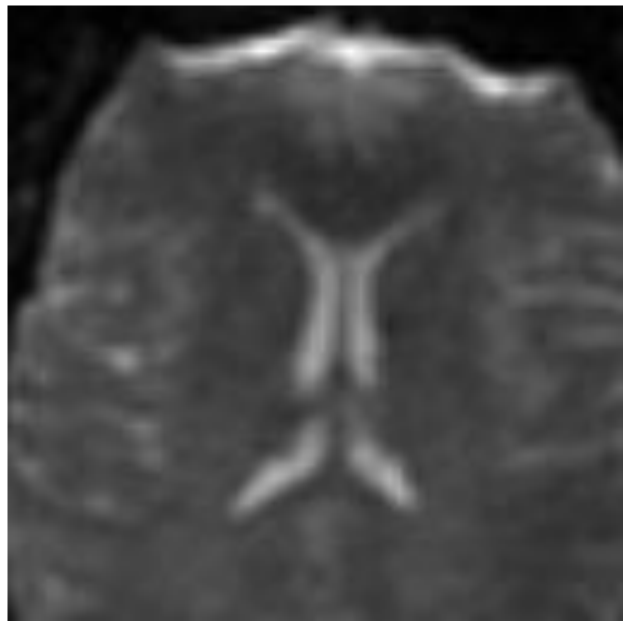
- The matlab function `interp2` does `nearest` neighbour, `linear` and `cubic` interpolation.

- Look it up and try it out!

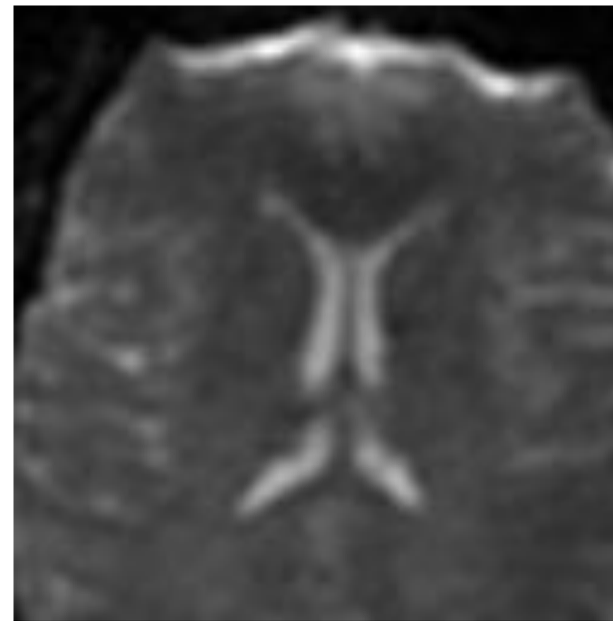
Interpolation comparison



Nearest neighbour



Bilinear



Cubic

Warps

- Polynomial transformations, e.g., quadratic transformations:
- $x' = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2$
- $y' = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2$

- Affine transformations map lines to lines.
- They are *first-order* polynomial transformations.
- Higher-order polynomials can bend lines.

Quadratic warp example

Control point warps

- Moves some pixels to specified locations.
- Interpolate the displacement at intermediate positions.
- Find a polynomial warp *P* that maps:

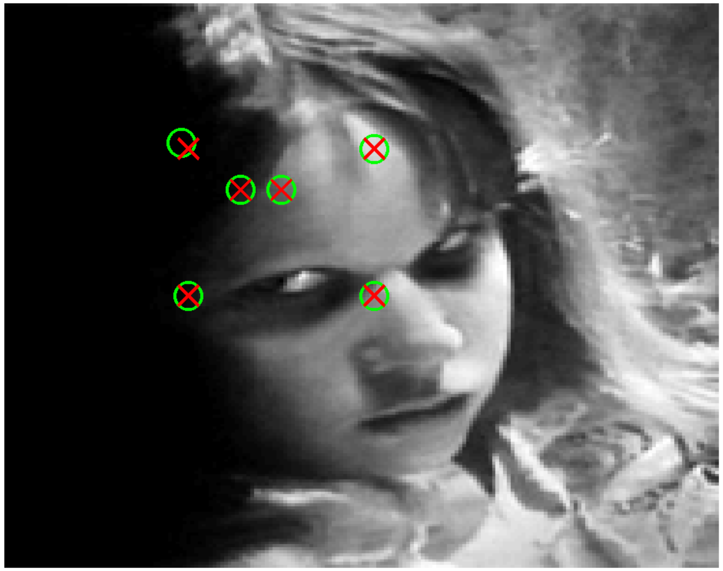
$$(x_1, y_1) \rightarrow (x_1', y_1')$$
$$(x_2, y_2) \rightarrow (x_2', y_2')$$
$$\vdots$$
$$(x_m, y_m) \rightarrow (x_m', y_m')$$

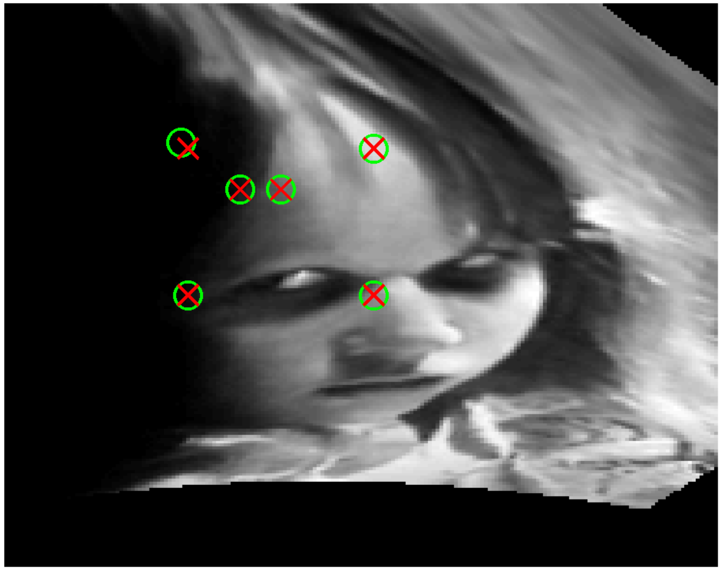
Control point warps

$$\begin{pmatrix} x_1' & y_1' \\ x_2' & y_2' \\ \vdots & \vdots \\ x_m' & y_m' \end{pmatrix} = \begin{pmatrix} 1 & x_1 & y_1 & x_1^2 & x_1 y_1 & y_1^2 \\ 1 & x_2 & y_2 & x_2^2 & x_2 y_2 & y_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_m & y_m & x_m^2 & x_m y_m & y_m^2 \end{pmatrix} \begin{pmatrix} a_0 & b_0 \\ a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \\ a_4 & b_4 \\ a_5 & b_5 \end{pmatrix}$$

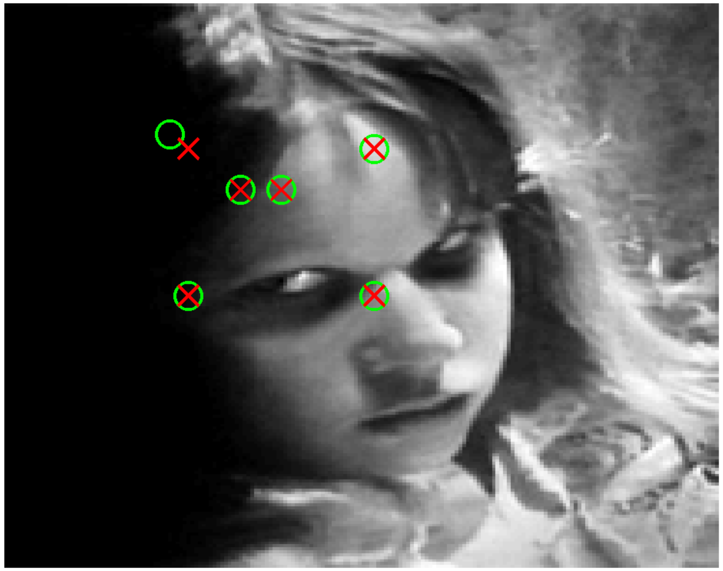
- $A = X\,P$
- Least squares estimate of P is ($m >= 6$):
- $P = (X^T X)^{-1} X^T A$.

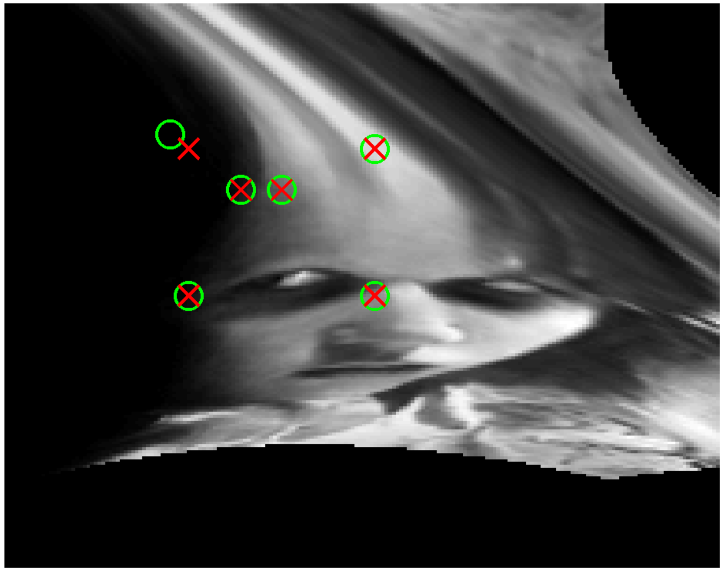
Example – Two pixel displacement



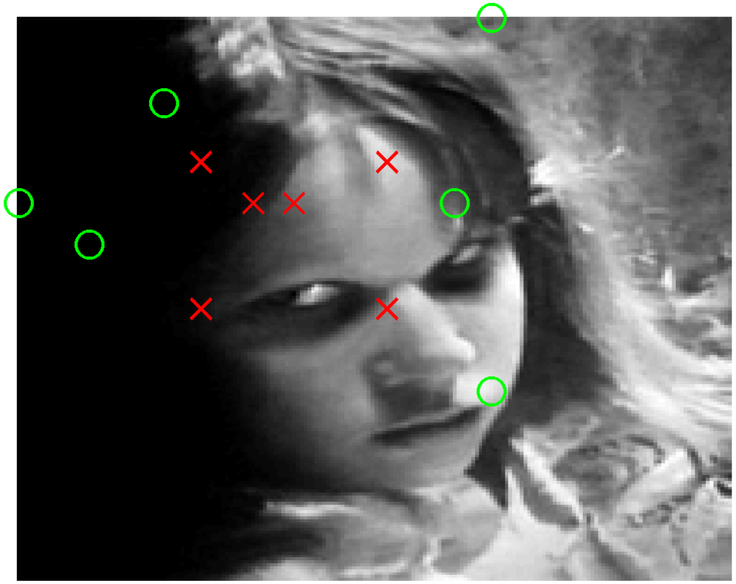


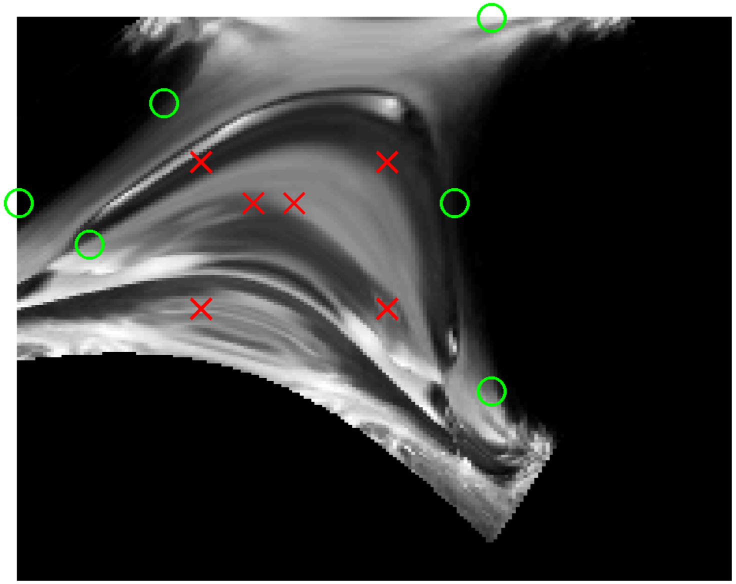
Five Pixel Displacement



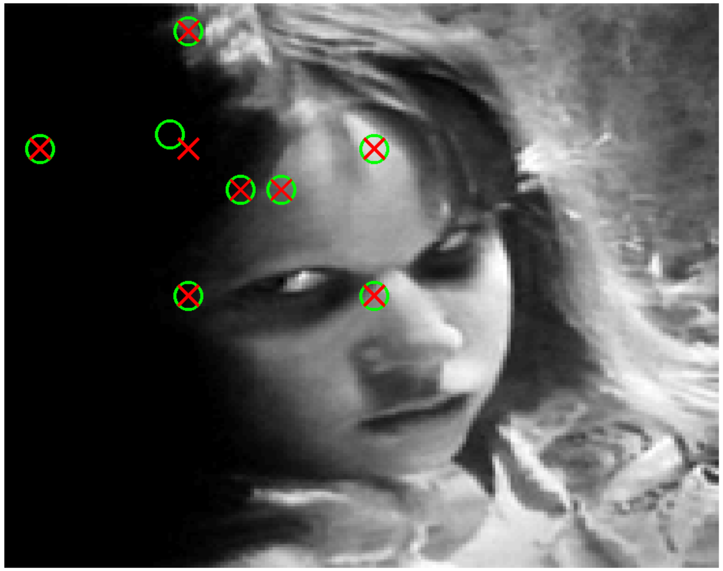


Extreme warp





Overdetermined



Applications

- Special effects
  - Film industry
  - Computer games

- Image registration
  - Medical imaging
  - Security

Image Morphing

How do we get the *i*-th image in the sequence?

Landmarks

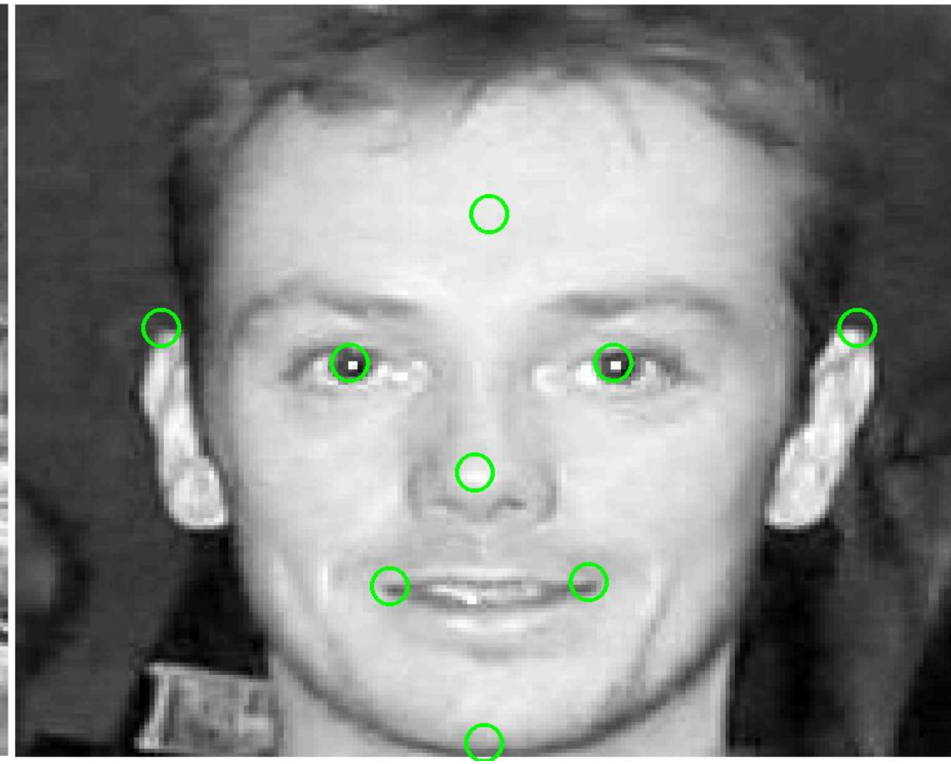


Image Registration

- Determines the transformation that aligns two similar images

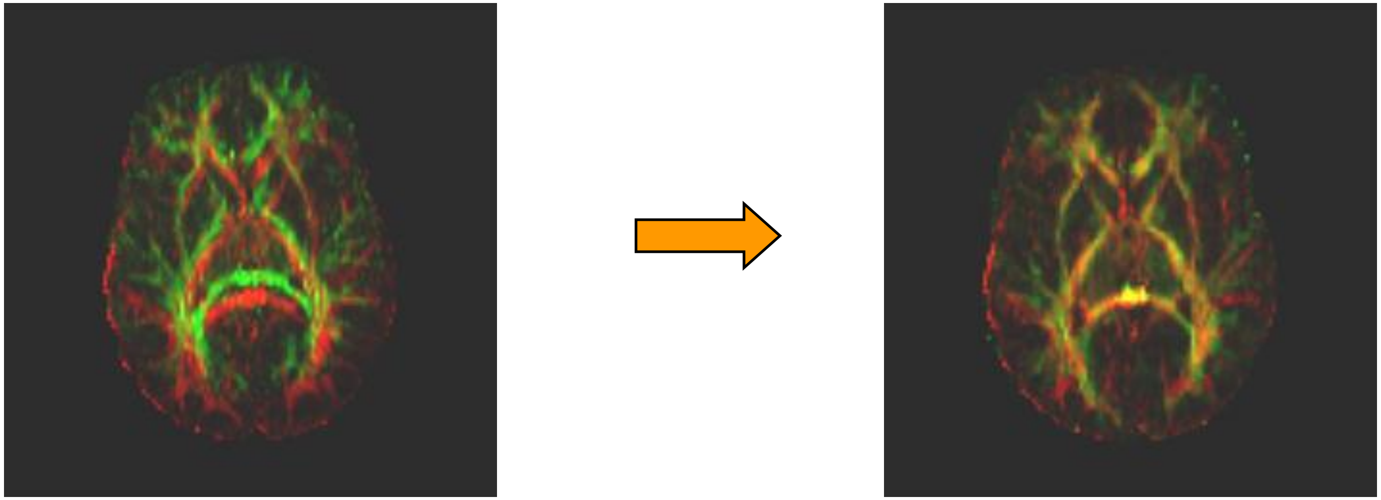


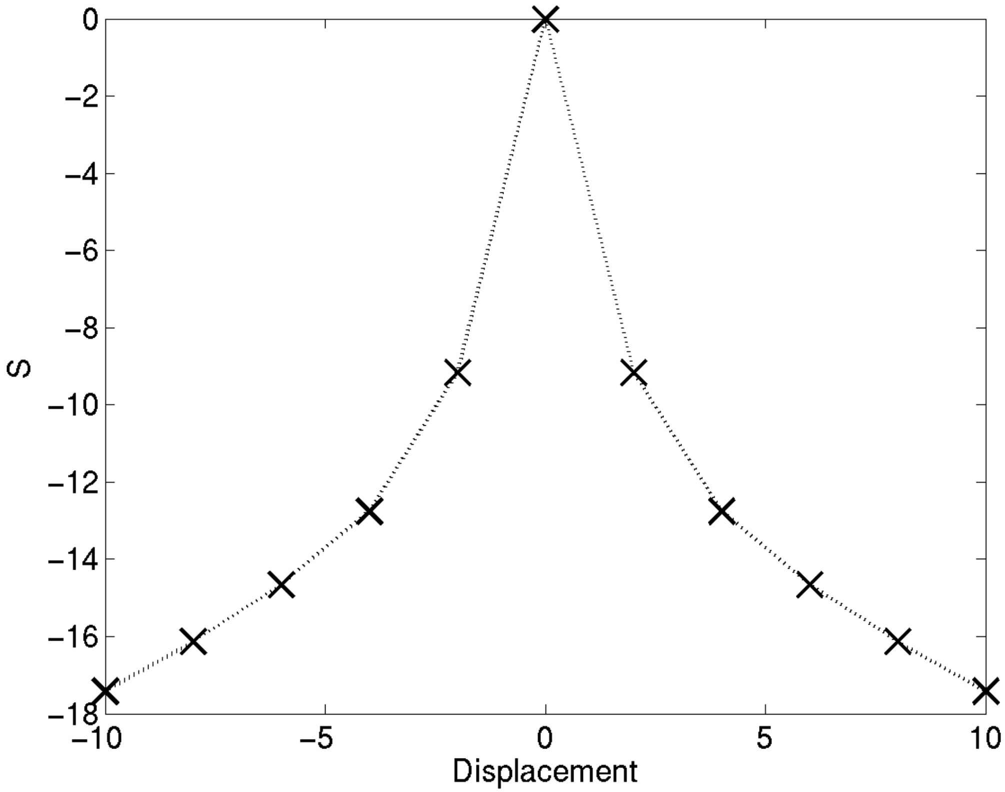
Image Similarity

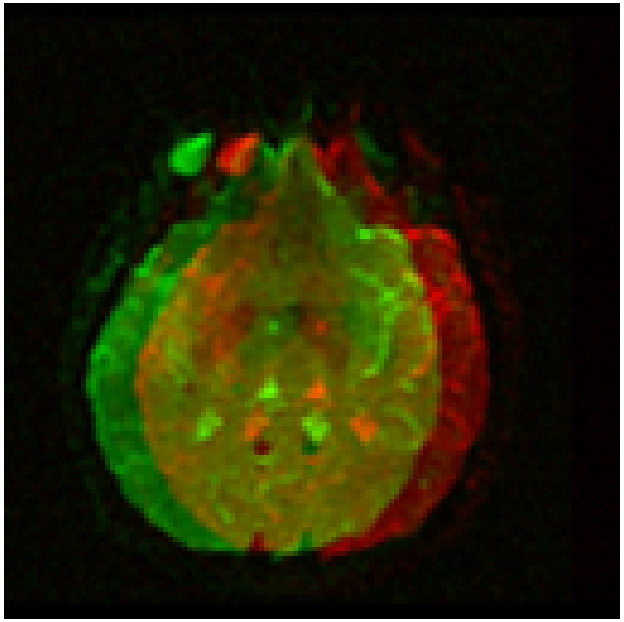
- We find the transformation that maximises the similarity of the images.
- A simple similarity measure is:

$$S(I_1, I_2) = -\left(\sum_{\underline{x} \in I_1} (I_1(\underline{x}) - I_2(\underline{x}))^2 \right)^{\frac{1}{2}}$$

- Many others are used including the *cross-correlation* and *mutual information*.

Registration example





Registration

- We have to search for the transformation the minimizes *S*.
- Simple in the 1D example.
- For more complex transformations, we use *numerical optimization* (see the matlab function `fminunc` for example).

Feature-Based Image Metamorphosis

Beier & Neely Siggraph'92

(Quick overview)

Explicit Correspondences

Individual face

Average face

$$\boldsymbol{u} = \frac{(\boldsymbol{X}-\boldsymbol{P}) \cdot (\boldsymbol{Q}-\boldsymbol{P})}{\|\boldsymbol{Q}-\boldsymbol{P}\|^2} \tag{1}$$

$$\boldsymbol{v} = \frac{(\boldsymbol{X}-\boldsymbol{P}) \cdot Perpendicular\,(\boldsymbol{Q}-\boldsymbol{P})}{\|\boldsymbol{Q}-\boldsymbol{P}\|} \tag{2}$$

$$\boldsymbol{X}' = \boldsymbol{P}' + \boldsymbol{u} \cdot (\boldsymbol{Q}'-\boldsymbol{P}') + \frac{\boldsymbol{v} \cdot Perpendicular\,(\boldsymbol{Q}'-\boldsymbol{P}')}{\|\boldsymbol{Q}'-\boldsymbol{P}'\|} \tag{3}$$