William B. Langdon and Oliver Krauss

Genetic Improvement of Data for Maths Functions

Online 10-14. July 2021

Genetic Improvement of data

- Optimize constants, i.e. data
- Maintain software
- Evolve new or better functionality
- Different type of Genetic Improvement

Why is this relevant?

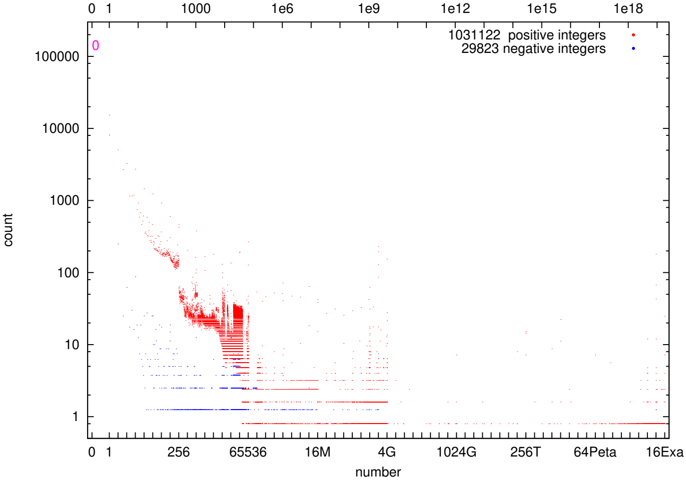


Figure: 1,202,711 integer constants in GNU C library

Evolved functions

Table: Accuracy and total time time (seconds) for CMA-ES

Start	Evolved		accuracy	secs
sqrt $\rightarrow$	cbrt()	$\sqrt[3]{x}$	dp i.e. $\leq 6.7\ 10^{-16}$	270
sqrt $\rightarrow$	log2()	$\log_2 x$	dp i.e. $\leq 2.2\ 10^{-16}$	6
sqrt $\rightarrow$	invsqrt()	$x^{-1/2}$	dp i.e. $\leq 2.2\ 10^{-16}$	6
sqrt $\rightarrow$	reciprocol	x^{-1}	dp i.e. $\leq 2.2\ 10^{-16}$	6

*dp = double precision accuracy

How math functions work I

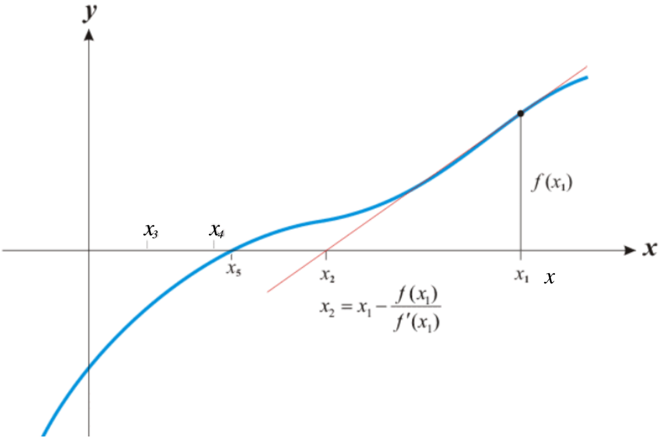


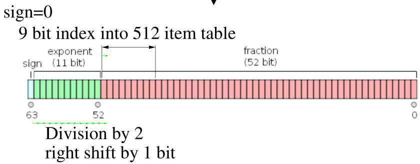
Figure: Newton Raphson Approximation

How math functions work II

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

load double precision value

select initial guess from lookup table




```
const float __t_sqrt[1024] = {
0.7078,0.7064, 0.7092,0.7050, 0.7106,0.7037, 0.7119,0.7023, 0.7133,0.7010,
0.7147,0.6996, 0.7160,0.6983, 0.7174,0.6970, 0.7187,0.6957, 0.7201,0.6943,
0.7215,0.6930, 0.7228,0.6917, 0.7242,0.6905, 0.7255,0.6892, 0.7269,0.6879,
0.7282,0.6866, 0.7295,0.6854, 0.7309,0.6841, 0.7322,0.6829, 0.7335,0.6816,
0.7349,0.6804, 0.7362,0.6792, 0.7375,0.6779, 0.7388,0.6767, 0.7402,0.6755,
0.7415,0.6743, 0.7428,0.6731, 0.7441,0.6719, 0.7454,0.6708, 0.7467,0.6696,
0.7480,0.6684, 0.7493,0.6672, 0.7507,0.6661, 0.7520,0.6649, 0.7532,0.6638,
0.7545,0.6627, 0.7558,0.6615, 0.7571,0.6604, 0.7584,0.6593, 0.7597,0.6582,
0.7610,0.6570, 0.7623,0.6559, 0.7635,0.6548, 0.7648,0.6537, 0.7661,0.6527,
0.7674,0.6516, 0.7686,0.6505, 0.7699,0.6494, 0.7712,0.6484, 0.7725,0.6473,
0.7737,0.6462, 0.7750,0.6452, 0.7762,0.6441, 0.7775,0.6431, 0.7787,0.6421,
0.7800,0.6410, 0.7812,0.6400, 0.7825,0.6390, 0.7837,0.6380, 0.7850,0.6370,
0.7862,0.6359, 0.7875,0.6349, 0.7887,0.6339, 0.7900,0.6330, 0.7912,0.6320,
0.7924,0.6310, 0.7937,0.6300, 0.7949,0.6290, 0.7961,0.6281, 0.7973,0.6271,
0.7986,0.6261, 0.7998,0.6252, 0.8010,0.6242, 0.8022,0.6233, 0.8034,0.6223,
0.8046,0.6214, 0.8059,0.6205, 0.8071,0.6195, 0.8083,0.6186, 0.8095,0.6177,
0.8107,0.6168, 0.8119,0.6158, 0.8131,0.6149, 0.8143,0.6140, 0.8155,0.6131,
```

Figure: Newton Raphson with Lookup Table

Evolving cube root from square root I

- Manual modification of glibc sqrt
- Covariance matrix adaption evolution strategy (CMA-ES)
  - For each *bin* in the lookup table
  - Fitness is *result cubed*
  - Random tests of several thousand double precision numbers

Evolving cube root from square root II

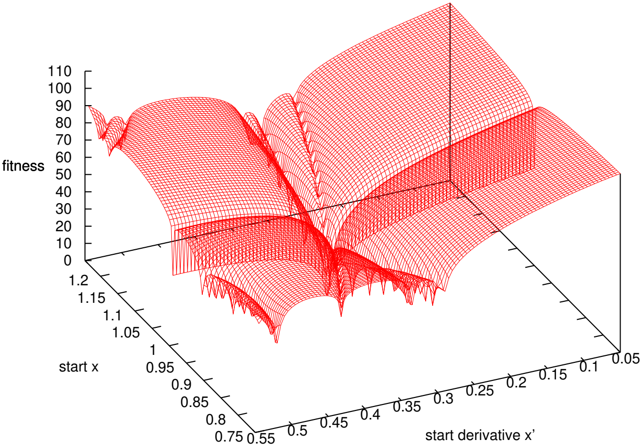


Figure: Fitness Landscape for cube root in GI (smaller is better)

Results

Table: Accuracy and total time time (seconds) for CMA-ES

Start	Evolved		accuracy	secs
sqrt →	cbrt()	$\sqrt[3]{x}$	dp i.e. $\leq$6.7 10^{-16}*	270
sqrt →	log2()	$\log_2 x$	dp i.e. $\leq$2.2 10^{-16}	6
sqrt →	invsqrt()	$x^{-1/2}$	dp i.e. $\leq$2.2 10^{-16}	6
sqrt →	reciprocol	x^{-1}	dp i.e. $\leq$2.2 10^{-16}	6

*Accuracy better than C++ and Java implementations. Runtime faster than Java implementation [1]

Conclusion

- Software can be maintained via GI
- Low effort
  - Takes just a few seconds
  - Source code and test case already exist
- Small changes
  - modifications comprehensible
  - higher acceptance by developers?
- **Try it yourself!**

Further information available at upcoming ACM TELO publication http://www0.cs.ucl.ac.uk/staff/W.Langdon/ftp/papers/Langdon_TELO.pdf [2]. Replication package on GitHub https://github.com/oliver-krauss/Replication_GI_Division_Free_Division

Contact

Bill Langdon
w.langdon@cs.ucl.ac.uk

Oliver Krauss
oliver.krauss@fh-hagenberg.at

Bibliography I

[1] O. Krauss and W. B. Langdon, "Automatically Evolving Lookup Tables for Function Approximation", en, in *Genetic Programming*, T. Hu, N. Lourenço, E. Medvet, and F. Divina, Eds., ser. Lecture Notes in Computer Science, Cham: Springer International Publishing, 2020, pp. 84–100.

[2] W. B. Langdon and O. Krauss, "Genetic improvement of data for maths functions", *ACM Transactions on Evolutionary Learning and Optimization*, vol. 1, no. 1, 2021. [Online]. Available: http://www0.cs.ucl.ac.uk/staff/W.Langdon/ftp/papers/Langdon_TELO.pdf.