# **Towards a Quantum-Inspired Multi-Gene Linear Genetic Programming Model**

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#### **ABSTRACT**

This paper presents a new model for regression problems based on Multi-Gene and Quantum Inspired Linear Genetic Programming. We discuss theoretical aspects, operators, representation, and experimental results.

### **INTRODUCTION**

The development of new GP regression models is relevant to provide more accurate results through fewer evaluations, and two distinct approaches are possible: (i) modify the GP basic structure: finding new ways to codify a solution and thus providing new recombination operators. There is some work in this area, such as Linear GP [1, 2]; (ii) allowing more outputs per GP individual: a simple approach is to enable more functions per individual and to combine their outputs, like Multi-Gene (or Multi-Tree) GP (MGGP) [3].

Thus, new GP based models that can operate in both senses could generate better results within fewer evaluations. This paper proposes a Quantum-Inspired Multi-Gene Linear GP model (QIMuLGP) for regression tasks, a generalization of Quantum-Inspired Linear GP (QILGP) [2]. QIMuLGP modifies canonical GP structures, explores new recombination operators and enables several outputs per individual that can be combined applying the least squares method. We evaluated this approach on 11 datasets, comparing its results with GP, MGGP and QILGP.

#### **QUANTUM-INSPIRED MULTI-GENE GP** 2.

We propose a novel GP model in this paper: Quantum-Inspired Multi-Gene Linear GP (QIMuLGP). The main difference between the original QILGP [2] and this generalization is that each individual has more than one chromosome. Therefore the fitness of an individual results from a linear combination of each chromosome output, where the weights are adjusted using least squares method – like MGGP.

Figure 1 illustrates QIMuLGP structure and its basic operation. It has a quantum population with N quantum individuals (QIs) with M chromosomes each (e.g. N = 3 and M = 4 in Figure 1). QIMuLGP has two classical individuals (CIs): one to store an observed individual and other for the best individual found. Through CIs, x86 machine code programs are generated. Figure 1 also enumerates four basic steps that repeat N times to complete a generation: 1<sup>st</sup>. a QI is observed generating a CI (*Observed Individual*);  $2^{nd}$ . the *M* chromosomes of *Observed Individual* are linearly combined to calculate its fitness; 3<sup>rd</sup>. if its fitness is better, it is copied to *Best Classical Individual*;  $4^{\text{th}}$ . an operator P





is applied to the QI observed in step 1, taking as reference Best Classical Individual, increasing the probability that future observations of the QI results in CI more similar to the best found.

The observation of a QI comprises observing each of its chromosomes, which defines the chromosomes of the resulting CI. The Figure 2 shows the observation process. The evolution continues the same way as QILGP.

# **3. RESULTS AND DISCUSSIONS**

Tables 1 and 2 present the main results (RMSE) and standard deviation ( $\sigma$ ) for the test set as well as the time spent for performing an evaluation (milliseconds). We varied the number of evaluations according to the number of variables of each dataset (parenthesis in the first columns) multiplied by some default values (3,000, 5,000, 7,000, 11,000). In general, two patterns can be identified: (i) performing more evaluations can benefit almost all evolutionary algorithms; (ii) Multi-Gene approaches (MGGP and QIMuLGP) compare favorably with their canonical counterparts.

QIMuLGP enhanced the average RMSE of QILGP about 54%, with a dispersion reduction of 10%, but the computational cost was 24 times higher. The comparison with MGGP shows that QIMuLGP RMSE values were 19% higher on average. However, the proposed model had an speedup factor of 8.

Figure 1: Basic diagram of QIMuLGP model.



Figure 2: Creation of a gene by the observation of a quantum gene.

Fable 1:	Main	results	of	$\mathbf{GP}$	and	MGGP	for	$\mathbf{test}$	$\underline{\mathbf{set}}$

			GP		
Evals/vars	$3,\!000$	$5,\!000$	$7,\!000$	$11,\!000$	$\mathrm{ms/eval}$
ABA(8)	2.32489	2.34160	2.27052	2.25400	
$\sigma$	0.01600	0.02610	0.05027	0.03850	3.37462
MPG6(5)	5.85305	6.42550	4.90581	4.65530	
$\sigma$	0.34086	0.16460	0.27672	0.27380	0.23375
MPG8(7)	4.84849	4.93290	4.45391	4.30520	
$\sigma$	0.26502	0.09670	0.12933	0.16150	0.48711
FRD $(5)$	2.65968	2.66270	2.36383	2.54460	
$\sigma$	0.20180	0.28990	0.28181	0.30970	1.50323
LAS $(4)$	29.2293	29.8512	28.8769	25.6498	
$\sigma$	1.25435	0.97750	1.46215	2.12210	1.52929
CPU(6)	80.6985	94.9208	78.1299	74.3228	
$\sigma$	6.13706	7.82560	8.83915	6.10730	0.24385
PLA(2)	3.11145	3.59640	2.83740	2.66930	
$\sigma$	0.12580	0.41730	0.20408	0.10280	3.63541
QUA(3)	0.20042	0.19010	0.18941	0.18940	
$\sigma$	0.05219	0.00130	0.00015	0.00020	1.18020
ELE $(18)$	0.00505	0.00497	0.00486	0.00479	
$\sigma$	0.00007	0.00021	0.00013	0.00017	7.82954
TRE $(15)$	0.40564	0.38500	0.36876	0.39916	
$\sigma$	0.10046	0.01660	0.02252	0.17031	1.07788
BAS $(16)$	714.178	709.767	728.983	910.453	
$\sigma$	24.5729	16.2693	18.7587	55.0068	0.36862
	1				
			$\mathrm{MGGP}$		
Evals/vars	3000	5000	$\begin{array}{c} \mathrm{MGGP} \\ \mathrm{7000} \end{array}$	11000	ms/eval
Evals/vars ABA(8)	$\frac{3000}{2.09448}$	5000 2.09005	$\frac{\mathrm{MGGP}}{2.15677}$	$11000 \\ 2.09146$	ms/eval
$\frac{\text{Evals/vars}}{\text{ABA(8)}}$	$\frac{3000}{2.09448}\\0.01536$	5000 2.09005 0.00939	MGGP 7000 2.15677 0.27578	$\frac{11000}{2.09146}\\0.01209$	ms/eval 68.9318
$\frac{\text{Evals/vars}}{\text{ABA(8)}}$ $\frac{\sigma}{\text{MPG6 (5)}}$	$\begin{array}{r} 3000 \\ 2.09448 \\ 0.01536 \\ 2.82620 \end{array}$	$     5000 \\     2.09005 \\     0.00939 \\     2.79916 $	MGGP 7000 2.15677 0.27578 2.76916	$     11000 \\     2.09146 \\     0.01209 \\     2.78887 $	ms/eval 68.9318
$\frac{\text{Evals/vars}}{\text{ABA(8)}}$ $\frac{\sigma}{\text{MPG6 (5)}}$ $\sigma$	$\begin{array}{r} 3000 \\ 2.09448 \\ 0.01536 \\ 2.82620 \\ 0.03991 \end{array}$	$\begin{array}{r} 5000 \\ 2.09005 \\ 0.00939 \\ 2.79916 \\ 0.05446 \end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314	$\begin{array}{r} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218 \end{array}$	ms/eval 68.9318 19.0408
$\frac{\text{Evals/vars}}{\text{ABA(8)}}$ $\frac{\sigma}{\text{MPG6 (5)}}$ $\frac{\sigma}{\text{MPG8 (7)}}$	$\begin{array}{r} 3000 \\ 2.09448 \\ 0.01536 \\ 2.82620 \\ 0.03991 \\ 2.77983 \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314 2.74484	11000 2.09146 0.01209 2.78887 0.08218 2.79120	ms/eval 68.9318 19.0408
$\frac{\text{Evals/vars}}{\text{ABA(8)}}$ $\frac{\sigma}{\text{MPG6 (5)}}$ $\frac{\sigma}{\text{MPG8 (7)}}$ $\sigma$	$\begin{array}{r} 3000 \\ 2.09448 \\ 0.01536 \\ 2.82620 \\ 0.03991 \\ 2.77983 \\ 0.05648 \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314 2.74484 0.04656	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131 \end{array}$	ms/eval 68.9318 19.0408 21.7090
$\begin{array}{r} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ {\rm MPG6}\ (5)\\ \sigma\\ {\rm MPG8}\ (7)\\ \sigma\\ {\rm FRD}\ (5) \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314 2.74484 0.04656 1.16372	11000 2.09146 0.01209 2.78887 0.08218 2.79120 0.06131 1.11400	ms/eval 68.9318 19.0408 21.7090
$\frac{\text{Evals/vars}}{\text{ABA(8)}}$ $\frac{\sigma}{\text{MPG6 (5)}}$ $\frac{\sigma}{\text{MPG8 (7)}}$ $\frac{\sigma}{\text{FRD (5)}}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\end{array}$	$\begin{array}{c} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314 2.74484 0.04656 1.16372 0.07745	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523
$\begin{array}{r} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ {\rm MPG6~(5)}\\ \sigma\\ {\rm MPG8~(7)}\\ \sigma\\ {\rm FRD~(5)}\\ \sigma\\ {\rm LAS~(4)} \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401 \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544 \end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314 2.74484 0.04656 1.16372 0.07745 6.62436	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523
$\begin{array}{c} \text{Evals/vars} \\ \text{ABA(8)} \\ \sigma \\ \\ \text{MPG6 (5)} \\ \sigma \\ \\ \text{MPG8 (7)} \\ \sigma \\ \\ \text{FRD (5)} \\ \sigma \\ \\ \text{LAS (4)} \\ \sigma \\ \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\end{array}$	$\begin{array}{r} \mathrm{MGGP} \\ 7000 \\ 2.15677 \\ 0.27578 \\ 2.76916 \\ 0.04314 \\ 2.74484 \\ 0.04656 \\ 1.16372 \\ 0.07745 \\ 6.62436 \\ 0.37549 \end{array}$	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215
$\begin{array}{c} \text{Evals/vars} \\ \text{ABA(8)} \\ \sigma \\ \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732 \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594 \end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314 2.74484 0.04656 1.16372 0.07745 6.62436 0.37549 75.6617	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ 68.5437 \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ {\rm MPG6~(5)}\\ \sigma\\ {\rm MPG8~(7)}\\ \sigma\\ {\rm FRD~(5)}\\ \sigma\\ {\rm LAS~(4)}\\ \sigma\\ {\rm CPU~(6)}\\ \sigma\\ \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\end{array}$	$\begin{array}{r} \mathrm{MGGP} \\ 7000 \\ \hline 2.15677 \\ 0.27578 \\ \hline 2.76916 \\ 0.04314 \\ \hline 2.74484 \\ 0.04656 \\ \hline 1.16372 \\ 0.07745 \\ \hline 6.62436 \\ 0.37549 \\ \hline 75.6617 \\ \hline 6.05547 \\ \hline \end{array}$	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ 68.5437\\ 32.4190\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ {\rm MPG6}\ (5)\\ \sigma\\ {\rm MPG8}\ (7)\\ \sigma\\ {\rm FRD}\ (5)\\ \sigma\\ {\rm LAS}\ (4)\\ \sigma\\ {\rm CPU}\ (6)\\ \sigma\\ {\rm PLA}\ (2)\\ \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\\ 1.36655\end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\\ 1.32200\\ \end{array}$	$\begin{array}{r} \mathrm{MGGP} \\ 7000 \\ 2.15677 \\ 0.27578 \\ 2.76916 \\ 0.04314 \\ 2.74484 \\ 0.04656 \\ 1.16372 \\ 0.07745 \\ 6.62436 \\ 0.37549 \\ 75.6617 \\ 6.05547 \\ 1.30754 \end{array}$	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ 68.5437\\ 32.4190\\ 1.31548\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ {\rm MPG6}\ (5)\\ \sigma\\ {\rm MPG8}\ (7)\\ \sigma\\ {\rm FRD}\ (5)\\ \sigma\\ {\rm FRD}\ (5)\\ \sigma\\ {\rm LAS}\ (4)\\ \sigma\\ {\rm CPU}\ (6)\\ \sigma\\ {\rm PLA}\ (2)\\ \sigma\\ {\rm \sigma}\\ \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\\ 1.36655\\ 0.02762\\ \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\\ 1.32200\\ 0.01607\\ \end{array}$	$\begin{array}{r} \mathrm{MGGP} \\ 7000 \\ \hline 2.15677 \\ 0.27578 \\ \hline 2.76916 \\ 0.04314 \\ \hline 2.74484 \\ 0.04656 \\ \hline 1.16372 \\ 0.07745 \\ \hline 6.62436 \\ 0.37549 \\ \hline 75.6617 \\ \hline 6.05547 \\ \hline 1.30754 \\ 0.01867 \\ \hline \end{array}$	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ 68.5437\\ 32.4190\\ 1.31548\\ 0.01511\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149 179.986
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ {\rm MPG6}\ (5)\\ \sigma\\ {\rm MPG8}\ (7)\\ \sigma\\ {\rm FRD}\ (5)\\ \sigma\\ {\rm FRD}\ (5)\\ \sigma\\ {\rm LAS}\ (4)\\ \sigma\\ {\rm CPU}\ (6)\\ \sigma\\ {\rm PLA}\ (2)\\ \sigma\\ {\rm QUA}\ (3)\\ \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\\ 1.36655\\ 0.02762\\ 0.19185\\ \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\\ 1.32200\\ 0.01607\\ 0.19101\\ \end{array}$	$\begin{array}{r} {\rm MGGP} \\ 7000 \\ \hline 2.15677 \\ 0.27578 \\ \hline 2.76916 \\ 0.04314 \\ \hline 2.74484 \\ 0.04656 \\ \hline 1.16372 \\ 0.07745 \\ \hline 6.62436 \\ 0.37549 \\ \hline 75.6617 \\ \hline 6.05547 \\ \hline 1.30754 \\ 0.01867 \\ \hline 0.19196 \\ \hline \end{array}$	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ 68.5437\\ 32.4190\\ 1.31548\\ 0.01511\\ 0.19195\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149 179.986
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ \\ \sigma\\ {\rm MPG6 (5)}\\ \sigma\\ \\ {\rm MPG8 (7)}\\ \sigma\\ \\ {\rm FRD (5)}\\ \sigma\\ \\ {\rm FRD (5)}\\ \sigma\\ \\ {\rm FRD (5)}\\ \\ \sigma\\ \\ {\rm FRD (3)}\\ \\ \sigma\\ \\ \sigma\\ \\ \sigma\\ \\ {\rm FRD (3)}\\ \\ \sigma\\ \\ \sigma\\ \\ {\rm FRD (3)}\\ \\ \sigma\\ \\$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\\ 1.36655\\ 0.02762\\ 0.19185\\ 0.00143\\ \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\\ 1.32200\\ 0.01607\\ 0.19101\\ 0.00160\\ \end{array}$	$\begin{array}{r} {\rm MGGP} \\ 7000 \\ \hline 2.15677 \\ 0.27578 \\ \hline 2.76916 \\ 0.04314 \\ \hline 2.74484 \\ 0.04656 \\ \hline 1.16372 \\ 0.07745 \\ \hline 6.62436 \\ 0.37549 \\ \hline 75.6617 \\ \hline 6.05547 \\ \hline 1.30754 \\ 0.01867 \\ \hline 0.19196 \\ 0.00097 \\ \hline \end{array}$	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ 68.5437\\ 32.4190\\ 1.31548\\ 0.01511\\ 0.19195\\ 0.00222\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149 179.986 82.7172
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ {\rm MPG6}\ (5)\\ \sigma\\ {\rm MPG8}\ (7)\\ \sigma\\ {\rm FRD}\ (5)\\ \sigma\\ {\rm FRD}\ (5)\\ \sigma\\ {\rm LAS}\ (4)\\ \sigma\\ {\rm CPU}\ (6)\\ \sigma\\ {\rm PLA}\ (2)\\ \sigma\\ {\rm QUA}\ (3)\\ \sigma\\ {\rm ELE}\ (18)\\ \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\\ 1.36655\\ 0.02762\\ 0.19185\\ 0.00143\\ 0.00245\\ \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\\ 1.32200\\ 0.01607\\ 0.19101\\ 0.00160\\ 0.00235\\ \end{array}$	$\begin{array}{r} {\rm MGGP} \\ 7000 \\ \hline 2.15677 \\ 0.27578 \\ \hline 2.76916 \\ 0.04314 \\ \hline 2.74484 \\ 0.04656 \\ \hline 1.16372 \\ 0.07745 \\ \hline 6.62436 \\ 0.37549 \\ \hline 75.6617 \\ \hline 6.05547 \\ \hline 1.30754 \\ 0.01867 \\ \hline 0.19196 \\ 0.00097 \\ \hline 0.00685 \\ \hline \end{array}$	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ 68.5437\\ 32.4190\\ 1.31548\\ 0.01511\\ 0.19195\\ 0.00222\\ 0.00504\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149 179.986 82.7172
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ \\ \sigma\\ {\rm MPG6} (5)\\ \sigma\\ \\ {\rm MPG8} (7)\\ \sigma\\ \\ {\rm FRD} (5)\\ \sigma\\ \\ {\rm FRD} (5)\\ \\ \sigma\\ \\ \sigma\\ \\ {\rm FRD} (5)\\ \\ \sigma\\ \\ \sigma\\ \\ {\rm FRD} (5)\\ \\ \sigma\\ \\ \sigma\\ \\ \sigma\\ \\ {\rm FRD}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\\ 1.36655\\ 0.02762\\ 0.19185\\ 0.00143\\ 0.00245\\ 0.00021\\ \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\\ 1.32200\\ 0.01607\\ 0.19101\\ 0.00160\\ 0.00235\\ 0.00008\\ \end{array}$	$\begin{array}{r} {\rm MGGP} \\ 7000 \\ \hline 2.15677 \\ 0.27578 \\ \hline 2.76916 \\ 0.04314 \\ \hline 2.74484 \\ 0.04656 \\ \hline 1.16372 \\ 0.07745 \\ \hline 6.62436 \\ 0.37549 \\ \hline 75.6617 \\ \hline 6.05547 \\ \hline 1.30754 \\ 0.01867 \\ \hline 0.19196 \\ 0.00097 \\ \hline 0.00685 \\ 0.01827 \\ \hline \end{array}$	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ 68.5437\\ 32.4190\\ 1.31548\\ 0.01511\\ 0.19195\\ 0.00222\\ 0.00504\\ 0.00676\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149 179.986 82.7172 107.327
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ \\ \\ \sigma\\ {\rm MPG6 (5)}\\ \\ \sigma\\ \\ {\rm MPG8 (7)}\\ \\ \sigma\\ \\ \\ {\rm FRD (5)}\\ \\ \\ \sigma\\ \\ \\ {\rm FRD (5)}\\ \\ \\ \sigma\\ \\ \\ {\rm FRD (5)}\\ \\ \\ \\ \sigma\\ \\ \\ {\rm FRD (5)}\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\\ 1.36655\\ 0.02762\\ 0.19185\\ 0.00245\\ 0.00143\\ 0.00245\\ 0.00021\\ 0.21615\\ \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\\ 1.32200\\ 0.01607\\ 0.19101\\ 0.00160\\ 0.00235\\ 0.00008\\ 0.21601\\ \end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314 2.74484 0.04656 1.16372 0.07745 6.62436 0.37549 75.6617 6.05547 1.30754 0.01867 0.19196 0.00097 0.00685 0.01827 0.23694	$\begin{array}{c} 11000\\ 2.09146\\ 0.01209\\ 2.78887\\ 0.08218\\ 2.79120\\ 0.06131\\ 1.11400\\ 0.03803\\ 6.91258\\ 0.36189\\ 68.5437\\ 32.4190\\ 1.31548\\ 0.01511\\ 0.19195\\ 0.00222\\ 0.00504\\ 0.00676\\ 0.21140\\ \end{array}$	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149 179.986 82.7172 107.327
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ \\ \sigma\\ {\rm MPG6}\ (5)\\ \sigma\\ \\ {\rm \sigma}\\ {\rm MPG8}\ (7)\\ \sigma\\ \\ {\rm FRD}\ (5)\\ \sigma\\ \\ {\rm FRD}\ (5)\\ \sigma\\ \\ {\rm CPU}\ (6)\\ \sigma\\ \\ {\rm CPU}\ (6)\\ \sigma\\ \\ {\rm OPLA}\ (2)\\ \\ \sigma\\ \\ {\rm PLA}\ (2)\\ \\ \sigma\\ \\ {\rm FRD}\ (3)\\ \\ \sigma\\ \\ {\rm FRD}\ (15)\\ \\ \sigma\\ \\ {\rm TRE}\ (15)\\ \\ \sigma\\ \\ \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\\ 1.36655\\ 0.02762\\ 0.19185\\ 0.00245\\ 0.00245\\ 0.00245\\ 0.00021\\ 0.21615\\ 0.00837\\ \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\\ 1.32200\\ 0.01607\\ 0.19101\\ 0.00160\\ 0.00235\\ 0.00008\\ 0.21601\\ 0.00309\\ \end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314 2.74484 0.04656 1.16372 0.07745 6.62436 0.37549 75.6617 6.05547 1.30754 0.01867 0.19196 0.00097 0.00685 0.01827 0.23694 0.09619	11000 2.09146 0.01209 2.78887 0.08218 2.79120 0.06131 1.11400 0.03803 6.91258 0.36189 68.5437 32.4190 1.31548 0.01511 0.19195 0.00222 0.00504 0.00676 0.21140 0.00361	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149 179.986 82.7172 107.327 100.430
$\begin{array}{c} {\rm Evals/vars}\\ {\rm ABA(8)}\\ \sigma\\ \\ \sigma\\ {\rm MPG6}\left(5\right)\\ \sigma\\ \\ \sigma\\ {\rm MPG8}\left(7\right)\\ \sigma\\ \\ {\rm FRD}\left(5\right)\\ \\ \sigma\\ \\ {\rm FRD}\left(1\right)\\ \\ \sigma\\ \\ {\rm FRD}\left(15\right)\\ \\ \sigma\\ \\ {\rm FRE}\left(15\right)\\ \\ \sigma\\ \\ {\rm BAS}\left(16\right)\\ \end{array}$	$\begin{array}{r} 3000\\ 2.09448\\ 0.01536\\ 2.82620\\ 0.03991\\ 2.77983\\ 0.05648\\ 1.19303\\ 0.08375\\ 7.18401\\ 0.43145\\ 78.3732\\ 50.6581\\ 1.36655\\ 0.02762\\ 0.19185\\ 0.00143\\ 0.00245\\ 0.00143\\ 0.00245\\ 0.00021\\ 0.21615\\ 0.00837\\ 1437.11\\ \end{array}$	$\begin{array}{r} 5000\\ 2.09005\\ 0.00939\\ 2.79916\\ 0.05446\\ 2.79353\\ 0.06879\\ 1.10039\\ 0.03939\\ 7.27544\\ 0.31578\\ 81.5594\\ 41.5579\\ 1.32200\\ 0.01607\\ 0.19101\\ 0.00160\\ 0.00235\\ 0.00008\\ 0.21601\\ 0.00309\\ 941.210\\ \end{array}$	MGGP 7000 2.15677 0.27578 2.76916 0.04314 2.74484 0.04656 1.16372 0.07745 6.62436 0.37549 75.6617 6.05547 1.30754 0.3754 0.01867 0.19196 0.00097 0.00685 0.01827 0.23694 0.09619 887.302	11000 2.09146 0.01209 2.78887 0.08218 2.79120 0.06131 1.11400 0.03803 6.91258 0.36189 68.5437 32.4190 1.31548 0.01511 0.19195 0.00222 0.00504 0.00504 0.00676 0.21140 0.00361 962.363	ms/eval 68.9318 19.0408 21.7090 56.9523 49.4215 42.4149 179.986 82.7172 107.327 100.430

### 4. CONCLUSIONS

We applied QILMuGP in set of 11 regression benchmarks, and it was found that QILMuGP greatly improve the accuracy when comparing to its simplified version (QILGP);

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Table 2: Main results of QILGP and QIMuLGP for tost sot

T-1/	2000	<b>F</b> 000	QILGP	11000	l / 1
Evals/vars	3000	5000	7000	11000	ms/eval
ABA(8)	2.49880	2.42167	2.33994	2.30975	0.05410
$\sigma$	0.10909	0.07016	0.06592	0.08731	0.97413
MPG6 (5)	5.65584	4.84559	4.30004	3.82211	
σ	0.66805	0.35753	0.54516	0.60763	0.12580
MPG8(7)	5.02799	4.48504	4.19458	3.72854	
$\sigma$	0.52746	0.58040	0.64397	0.71112	0.13271
FRD(5)	3.83546	3.40657	3.20121	2.80673	
$\sigma$	0.16641	0.18097	0.13782	0.08024	0.30340
LAS $(4)$	28.9468	28.1398	26.0149	22.9493	
$\sigma$	1.91894	2.34428	2.53016	2.14484	0.24450
CPU(6)	74.1313	71.0203	69.7652	57.5957	
$\sigma$	15.5803	14.0342	16.3019	8.33990	0.08650
PLA(2)	3.17037	2.82697	2.68247	2.39313	
$\sigma$	0.16257	0.20336	0.10046	0.12619	0.33500
QUA(3)	0.18977	0.18946	0.18948	0.18952	
$\sigma$	0.00643	0.00635	0.00635	0.00631	0.46000
ELE (18)	0.58902	1.16181	1.16172	2.31833	
$\sigma$	1.30531	2.58649	2.58656	5.17306	3.54139
TRE (15)	0.33768	0.31326	0.30514	0.29105	
$\sigma$	0.03757	0.04039	0.04268	0.03810	0.32467
BAS (16)	844.192	757.278	748.243	732.011	
$\sigma$	26.4487	68.6481	35.7884	86.1586	0.13319
			QIMuLGP	,	1
Evals/vars	3000	5000	7000	11000	$\mid$ ms/eval
ABA(8)	2.15181	2.12817	2.14707	2.13025	/
$\sigma$	0.08167	0.08105	0.10566	0.08106	11.6134
	0.07040	0.01450	0 77700		
MPG6(5)	2.87843	2.81456	2.77730	2.74637	
$\begin{array}{c} \text{MPG6} (5) \\ \sigma \end{array}$	$2.87843 \\ 0.24769$	$2.81456 \\ 0.20421$	$2.77730 \\ 0.22262$	$2.74637 \\ 0.25929$	3.04560
$\frac{\text{MPG6 (5)}}{\sigma}$ $\frac{\sigma}{\text{MPG8 (7)}}$	$\begin{array}{r} 2.87843 \\ 0.24769 \\ \hline 2.88234 \end{array}$	$\frac{2.81456}{0.20421}$ $\frac{2.81248}{2.81248}$	$     \begin{array}{r}       2.77730 \\       0.22262 \\       \hline       2.81635     \end{array} $	$\frac{2.74637}{0.25929}$ $\frac{2.76679}{0.25929}$	3.04560
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \text{MPG8 (7)} \\ \sigma \end{array}$	$\begin{array}{r} 2.87843 \\ 0.24769 \\ \hline 2.88234 \\ 0.38581 \end{array}$	$\begin{array}{r} 2.81456 \\ 0.20421 \\ \hline 2.81248 \\ 0.36920 \end{array}$	$\begin{array}{r} 2.77730 \\ 0.22262 \\ \hline 2.81635 \\ 0.35363 \end{array}$	$\begin{array}{r} 2.74637 \\ 0.25929 \\ \hline 2.76679 \\ 0.35807 \end{array}$	3.04560 3.85343
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \text{MPG8 (7)} \\ \sigma \\ \hline \text{FRD (5)} \end{array}$	$\begin{array}{r} 2.87843 \\ 0.24769 \\ \hline 2.88234 \\ 0.38581 \\ \hline 2.06512 \end{array}$	$\begin{array}{r} 2.81456 \\ 0.20421 \\ \hline 2.81248 \\ 0.36920 \\ \hline 1.88007 \end{array}$	$\begin{array}{r} 2.77730\\ \hline 0.22262\\ \hline 2.81635\\ \hline 0.35363\\ \hline 1.70563\end{array}$	$\begin{array}{r} 2.74637 \\ 0.25929 \\ \hline 2.76679 \\ 0.35807 \\ \hline 1.52325 \end{array}$	3.04560 3.85343
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \hline \text{MPG8 (7)} \\ \sigma \\ \hline \text{FRD (5)} \\ \sigma \\ \end{array}$	$\begin{array}{r} 2.87843 \\ 0.24769 \\ \hline 2.88234 \\ 0.38581 \\ \hline 2.06512 \\ 0.13197 \end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \end{array}$	$\begin{array}{r} 2.77730\\ \hline 0.22262\\ \hline 2.81635\\ \hline 0.35363\\ \hline 1.70563\\ \hline 0.10826\end{array}$	$\begin{array}{r} 2.74637 \\ 0.25929 \\ \hline 2.76679 \\ 0.35807 \\ \hline 1.52325 \\ 0.09174 \end{array}$	$\begin{array}{r} 3.04560 \\ 3.85343 \\ 6.93420 \end{array}$
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \text{MPG8 (7)} \\ \sigma \\ \hline \text{FRD (5)} \\ \sigma \\ \hline \text{LAS (4)} \end{array}$	$\begin{array}{r} 2.87843 \\ 0.24769 \\ 2.88234 \\ 0.38581 \\ 2.06512 \\ 0.13197 \\ 10.5812 \end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ \hline 2.81635\\ 0.35363\\ \hline 1.70563\\ 0.10826\\ \hline 9.17668\end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\end{array}$	$\begin{array}{r} 3.04560 \\ 3.85343 \\ 6.93420 \end{array}$
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \hline \text{MPG8 (7)} \\ \sigma \\ \hline \text{FRD (5)} \\ \sigma \\ \hline \text{LAS (4)} \\ \sigma \end{array}$	$\begin{array}{r} 2.87843 \\ 0.24769 \\ 2.88234 \\ 0.38581 \\ 2.06512 \\ 0.13197 \\ 10.5812 \\ 2.01984 \end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191 \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ \hline 2.81635\\ 0.35363\\ \hline 1.70563\\ 0.10826\\ \hline 9.17668\\ \hline 1.90272 \end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ 1.60768\end{array}$	$\begin{array}{r} 3.04560 \\ 3.85343 \\ 6.93420 \\ 6.38325 \end{array}$
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \text{MPG8 (7)} \\ \sigma \\ \text{FRD (5)} \\ \sigma \\ \text{LAS (4)} \\ \sigma \\ \text{CPU (6)} \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191\\ \hline 61.6998 \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ \hline 2.81635\\ 0.35363\\ \hline 1.70563\\ 0.10826\\ \hline 9.17668\\ \hline 1.90272\\ \hline 178.479\end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \end{array}$	$\begin{array}{r} 3.04560 \\ 3.85343 \\ 6.93420 \\ 6.38325 \end{array}$
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \hline \text{MPG8 (7)} \\ \sigma \\ \hline \text{FRD (5)} \\ \sigma \\ \hline \text{LAS (4)} \\ \sigma \\ \hline \text{CPU (6)} \\ \sigma \\ \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ \hline 2.06512\\ 0.13197\\ \hline 10.5812\\ 2.01984\\ \hline 94.6812\\ 107.974\\ \end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191\\ \hline 61.6998\\ 33.4788\end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ \hline 2.81635\\ 0.35363\\ \hline 1.70563\\ 0.10826\\ \hline 9.17668\\ \hline 1.90272\\ \hline 178.479\\ \hline 248.655\end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \hline 218.108\\ \end{array}$	$\begin{array}{r} 3.04560 \\ \hline 3.85343 \\ \hline 6.93420 \\ \hline 6.38325 \\ \hline 2.90633 \end{array}$
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \text{MPG8 (7)} \\ \sigma \\ \text{FRD (5)} \\ \sigma \\ \text{LAS (4)} \\ \sigma \\ \text{CPU (6)} \\ \sigma \\ \text{PLA (2)} \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ 1.51029\end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191\\ \hline 61.6998\\ 33.4788\\ \hline 1.50002 \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ \hline 2.81635\\ 0.35363\\ \hline 1.70563\\ 0.10826\\ \hline 9.17668\\ \hline 1.90272\\ \hline 178.479\\ \hline 248.655\\ \hline 1.49180\\ \end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \hline 218.108\\ \hline 1.48186\\ \hline \end{array}$	3.04560 3.85343 6.93420 6.38325 2.90633
$\begin{array}{c} \mathrm{MPG6} (5) \\ \sigma \\ \mathrm{MPG8} (7) \\ \sigma \\ \mathrm{FRD} (5) \\ \sigma \\ \mathrm{LAS} (4) \\ \sigma \\ \mathrm{CPU} (6) \\ \sigma \\ \mathrm{PLA} (2) \\ \sigma \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ 1.51029\\ 0.04283\end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ \hline 1.62191\\ \hline 61.6998\\ \hline 33.4788\\ \hline 1.50002\\ 0.04069\\ \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ \hline 2.81635\\ 0.35363\\ \hline 1.70563\\ 0.10826\\ \hline 9.17668\\ \hline 1.90272\\ \hline 178.479\\ \hline 248.655\\ \hline 1.49180\\ 0.04147\\ \end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \hline 218.108\\ \hline 1.48186\\ 0.05259\\ \end{array}$	$\begin{array}{r} 3.04560 \\ \hline 3.85343 \\ \hline 6.93420 \\ \hline 6.38325 \\ \hline 2.90633 \\ \hline 8.90900 \end{array}$
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \hline \\ \text{MPG8 (7)} \\ \sigma \\ \hline \\ \text{FRD (5)} \\ \sigma \\ \hline \\ \text{LAS (4)} \\ \sigma \\ \hline \\ \text{CPU (6)} \\ \sigma \\ \hline \\ \text{PLA (2)} \\ \sigma \\ \hline \\ \\ \text{QUA (3)} \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ 1.51029\\ 0.04283\\ 0.19564\end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191\\ \hline 61.6998\\ 33.4788\\ \hline 1.50002\\ 0.04069\\ \hline 0.19032\\ \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ \hline 2.81635\\ 0.35363\\ \hline 1.70563\\ 0.10826\\ \hline 9.17668\\ \hline 1.90272\\ \hline 178.479\\ 248.655\\ \hline 1.49180\\ 0.04147\\ \hline 0.35561\\ \end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \hline 218.108\\ \hline 1.48186\\ 0.05259\\ \hline 0.19337\\ \end{array}$	3.04560         3.85343         6.93420         6.38325         2.90633         8.90900
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \hline \\ \text{MPG8 (7)} \\ \sigma \\ \hline \\ \text{FRD (5)} \\ \sigma \\ \hline \\ \text{LAS (4)} \\ \sigma \\ \hline \\ \text{CPU (6)} \\ \sigma \\ \hline \\ \text{PLA (2)} \\ \sigma \\ \hline \\ \text{QUA (3)} \\ \sigma \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ \hline 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ \hline 1.51029\\ 0.04283\\ \hline 0.19564\\ 0.01749\\ \end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191\\ \hline 61.6998\\ 33.4788\\ \hline 1.50002\\ 0.04069\\ \hline 0.19032\\ 0.00785\\ \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ \hline 2.81635\\ 0.35363\\ \hline 1.70563\\ 0.10826\\ \hline 9.17668\\ 1.90272\\ \hline 178.479\\ 248.655\\ \hline 1.49180\\ 0.04147\\ \hline 0.35561\\ 0.37354\\ \end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \hline 218.108\\ \hline 1.48186\\ 0.05259\\ \hline 0.19337\\ 0.00917\\ \end{array}$	$\begin{array}{r} 3.04560 \\ \hline 3.85343 \\ \hline 6.93420 \\ \hline 6.38325 \\ \hline 2.90633 \\ \hline 8.90900 \\ \hline 8.25233 \end{array}$
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{MPG8 (7)} \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{FRD (5)} \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{LAS (4)} \\ \sigma \\ \hline \sigma \\ \\ \hline \text{CPU (6)} \\ \sigma \\ \hline \sigma \\ \\ \hline \text{PLA (2)} \\ \sigma \\ \hline \sigma \\ \\ \hline \text{QUA (3)} \\ \sigma \\ \hline \sigma \\ \\ \hline \text{ELE (18)} \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ 1.51029\\ 0.04283\\ 0.19564\\ 0.01749\\ 0.00695\end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191\\ \hline 61.6998\\ 33.4788\\ \hline 1.50002\\ 0.04069\\ \hline 0.19032\\ 0.00785\\ \hline 0.00266\end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ 2.81635\\ 0.35363\\ 1.70563\\ 0.10826\\ 9.17668\\ 1.90272\\ 178.479\\ 248.655\\ 1.49180\\ 0.04147\\ 0.35561\\ 0.37354\\ 0.00259\end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \hline 218.108\\ \hline 1.48186\\ 0.05259\\ \hline 0.19337\\ \hline 0.00917\\ \hline 0.00390\\ \end{array}$	3.04560         3.85343         6.93420         6.38325         2.90633         8.90900         8.25233
$\begin{array}{c} \mathrm{MPG6} \ (5) \\ \sigma \\ \mathrm{MPG8} \ (7) \\ \sigma \\ \mathrm{FRD} \ (5) \\ \sigma \\ \mathrm{LAS} \ (4) \\ \sigma \\ \mathrm{CPU} \ (6) \\ \sigma \\ \mathrm{PLA} \ (2) \\ \sigma \\ \mathrm{QUA} \ (3) \\ \sigma \\ \mathrm{ELE} \ (18) \\ \sigma \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ 1.51029\\ 0.04283\\ 0.19564\\ 0.01749\\ 0.00695\\ 0.00923\\ \end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191\\ \hline 61.6998\\ 33.4788\\ \hline 1.50002\\ 0.04069\\ \hline 0.19032\\ 0.00785\\ \hline 0.00266\\ 0.00009\\ \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ 2.81635\\ 0.35363\\ 1.70563\\ 0.10826\\ 9.17668\\ 1.90272\\ 178.479\\ 248.655\\ 1.49180\\ 0.04147\\ 0.35561\\ 0.37354\\ 0.00259\\ 0.00006\\ \end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \hline 218.108\\ \hline 1.48186\\ 0.05259\\ \hline 0.19337\\ 0.00917\\ \hline 0.00390\\ 0.00302\\ \end{array}$	3.04560 3.85343 6.93420 6.38325 2.90633 8.90900 8.25233 63.2082
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{MPG8 (7)} \\ \sigma \\ \hline \sigma \\ \hline \text{FRD (5)} \\ \sigma \\ \hline \sigma \\ \hline \text{LAS (4)} \\ \sigma \\ \hline \sigma \\ \hline \text{CPU (6)} \\ \sigma \\ \hline \sigma \\ \hline \text{PLA (2)} \\ \sigma \\ \hline \sigma \\ \hline \text{QUA (3)} \\ \sigma \\ \hline \text{ELE (18)} \\ \sigma \\ \hline \sigma \\ \hline \text{TRE (15)} \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ \hline 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ \hline 1.51029\\ 0.04283\\ \hline 0.19564\\ 0.01749\\ \hline 0.00695\\ 0.00923\\ \hline 0.23948\\ \end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ 2.81248\\ 0.36920\\ 1.88007\\ 0.18702\\ 9.48600\\ 1.62191\\ 61.6998\\ 33.4788\\ 1.50002\\ 0.04069\\ 0.19032\\ 0.00785\\ 0.00266\\ 0.00009\\ 0.23853\end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ 2.81635\\ 0.35363\\ 1.70563\\ 0.10826\\ 9.17668\\ 1.90272\\ 178.479\\ 248.655\\ 1.49180\\ 0.04147\\ 0.35561\\ 0.37354\\ 0.00259\\ 0.00006\\ 0.23182\end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \hline 218.108\\ \hline 1.48186\\ 0.05259\\ \hline 0.19337\\ 0.00917\\ \hline 0.00390\\ \hline 0.00302\\ \hline 0.22692\\ \end{array}$	3.04560         3.85343         6.93420         6.38325         2.90633         8.90900         8.25233         63.2082
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \\ \sigma \\ \text{MPG8 (7)} \\ \sigma \\ \\ \text{FRD (5)} \\ \sigma \\ \\ \text{CPU (5)} \\ \sigma \\ \\ \text{CPU (6)} \\ \sigma \\ \\ \\ \sigma \\ \\ \text{PLA (2)} \\ \sigma \\ \\ \\ \sigma \\ \\ \\ \text{QUA (3)} \\ \sigma \\ \\ \\ \\ \\ \sigma \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ 1.51029\\ 0.04283\\ 0.19564\\ 0.01749\\ 0.00695\\ 0.00923\\ 0.23948\\ 0.04567\end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191\\ \hline 61.6998\\ 33.4788\\ \hline 1.50002\\ 0.04069\\ \hline 0.19032\\ 0.00785\\ \hline 0.00266\\ 0.00009\\ \hline 0.23853\\ 0.04297\end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ 2.81635\\ 0.35363\\ \hline 1.70563\\ 0.10826\\ \hline 9.17668\\ 1.90272\\ \hline 178.479\\ 248.655\\ \hline 1.49180\\ 0.04147\\ \hline 0.35561\\ 0.37354\\ \hline 0.00259\\ 0.00006\\ \hline 0.23182\\ 0.04221\\ \end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ \hline 2.76679\\ 0.35807\\ \hline 1.52325\\ 0.09174\\ \hline 7.82286\\ \hline 1.60768\\ \hline 148.720\\ \hline 218.108\\ \hline 1.48186\\ 0.05259\\ \hline 0.19337\\ 0.00917\\ \hline 0.00390\\ 0.00302\\ \hline 0.22692\\ 0.04040\\ \end{array}$	$\begin{array}{r} 3.04560\\ \hline 3.85343\\ \hline 6.93420\\ \hline 6.38325\\ \hline 2.90633\\ \hline 8.90900\\ \hline 8.25233\\ \hline 63.2082\\ \hline 52.4000\\ \end{array}$
$\begin{array}{c} \text{MPG6 (5)} \\ \sigma \\ \\ \sigma \\ \\ \hline \text{MPG8 (7)} \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{FRD (5)} \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{LAS (4)} \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{CPU (6)} \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{PLA (2)} \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{QUA (3)} \\ \sigma \\ \\ \hline \hline \text{ELE (18)} \\ \sigma \\ \\ \hline \hline \text{TRE (15)} \\ \\ \sigma \\ \\ \hline \sigma \\ \\ \hline \text{BAS (16)} \\ \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ 1.51029\\ 0.04283\\ 0.19564\\ 0.01749\\ 0.00695\\ 0.00923\\ 0.23948\\ 0.04567\\ 737 169\end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ \hline 2.81248\\ 0.36920\\ \hline 1.88007\\ 0.18702\\ \hline 9.48600\\ 1.62191\\ \hline 61.6998\\ 33.4788\\ \hline 1.50002\\ 0.04069\\ \hline 0.19032\\ 0.00785\\ \hline 0.00266\\ 0.00009\\ \hline 0.23853\\ 0.04297\\ \hline 747.094 \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ 2.81635\\ 0.35363\\ 1.70563\\ 0.10826\\ 9.17668\\ 1.90272\\ 178.479\\ 248.655\\ 1.49180\\ 0.04147\\ 0.35561\\ 0.37354\\ 0.00259\\ 0.00006\\ 0.23182\\ 0.04221\\ 984 207\end{array}$	$\begin{array}{r} 2.74637\\ 0.25929\\ 2.76679\\ 0.35807\\ 1.52325\\ 0.09174\\ 7.82286\\ 1.60768\\ 148.720\\ 218.108\\ 1.48186\\ 0.05259\\ 0.19337\\ 0.00917\\ 0.00390\\ 0.00302\\ 0.22692\\ 0.04040\\ 780.099\end{array}$	3.04560         3.85343         6.93420         6.38325         2.90633         8.90900         8.25233         63.2082         52.4000
$\begin{array}{c} \mathrm{MPG6} \ (5) \\ \sigma \\ \mathrm{MPG8} \ (7) \\ \sigma \\ \mathrm{FRD} \ (5) \\ \sigma \\ \mathrm{LAS} \ (4) \\ \sigma \\ \mathrm{CPU} \ (6) \\ \sigma \\ \mathrm{PLA} \ (2) \\ \sigma \\ \mathrm{PLA} \ (2) \\ \sigma \\ \mathrm{QUA} \ (3) \\ \sigma \\ \mathrm{ELE} \ (18) \\ \sigma \\ \mathrm{TRE} \ (15) \\ \sigma \\ \mathrm{BAS} \ (16) \\ \end{array}$	$\begin{array}{r} 2.87843\\ 0.24769\\ 2.88234\\ 0.38581\\ \hline 2.06512\\ 0.13197\\ 10.5812\\ 2.01984\\ 94.6812\\ 107.974\\ \hline 1.51029\\ 0.04283\\ \hline 0.19564\\ 0.01749\\ \hline 0.00695\\ 0.00923\\ \hline 0.23948\\ 0.04567\\ \hline 737.169\\ 87.4307\\ \end{array}$	$\begin{array}{r} 2.81456\\ 0.20421\\ 2.81248\\ 0.36920\\ 1.88007\\ 0.18702\\ 9.48600\\ 1.62191\\ 61.6998\\ 33.4788\\ 1.50002\\ 0.04069\\ 0.19032\\ 0.00785\\ 0.00266\\ 0.00009\\ 0.23853\\ 0.04297\\ 747.094\\ 39.7709\\ \end{array}$	$\begin{array}{r} 2.77730\\ 0.22262\\ 2.81635\\ 0.35363\\ 1.70563\\ 0.10826\\ 9.17668\\ 1.90272\\ 178.479\\ 248.655\\ 1.49180\\ 0.04147\\ 0.35561\\ 0.37354\\ 0.00259\\ 0.00006\\ 0.23182\\ 0.04221\\ 984.207\\ 543.770\\ \end{array}$	$\begin{array}{c} 2.74637\\ 0.25929\\ 2.76679\\ 0.35807\\ 1.52325\\ 0.09174\\ 7.82286\\ 1.60768\\ 148.720\\ 218.108\\ 1.48186\\ 0.05259\\ 0.19337\\ 0.00917\\ 0.00390\\ 0.00302\\ 0.22692\\ 0.04040\\ 780.099\\ 79.8790\\ \end{array}$	3.04560         3.85343         6.93420         6.38325         2.90633         8.90900         8.25233         63.2082         52.4000         1.85350

and MGGP obtained slightly better results than QILMuGP, however using twice the computational effort.

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