Generating Massive Amount of High-Quality Random Numbers using GPU

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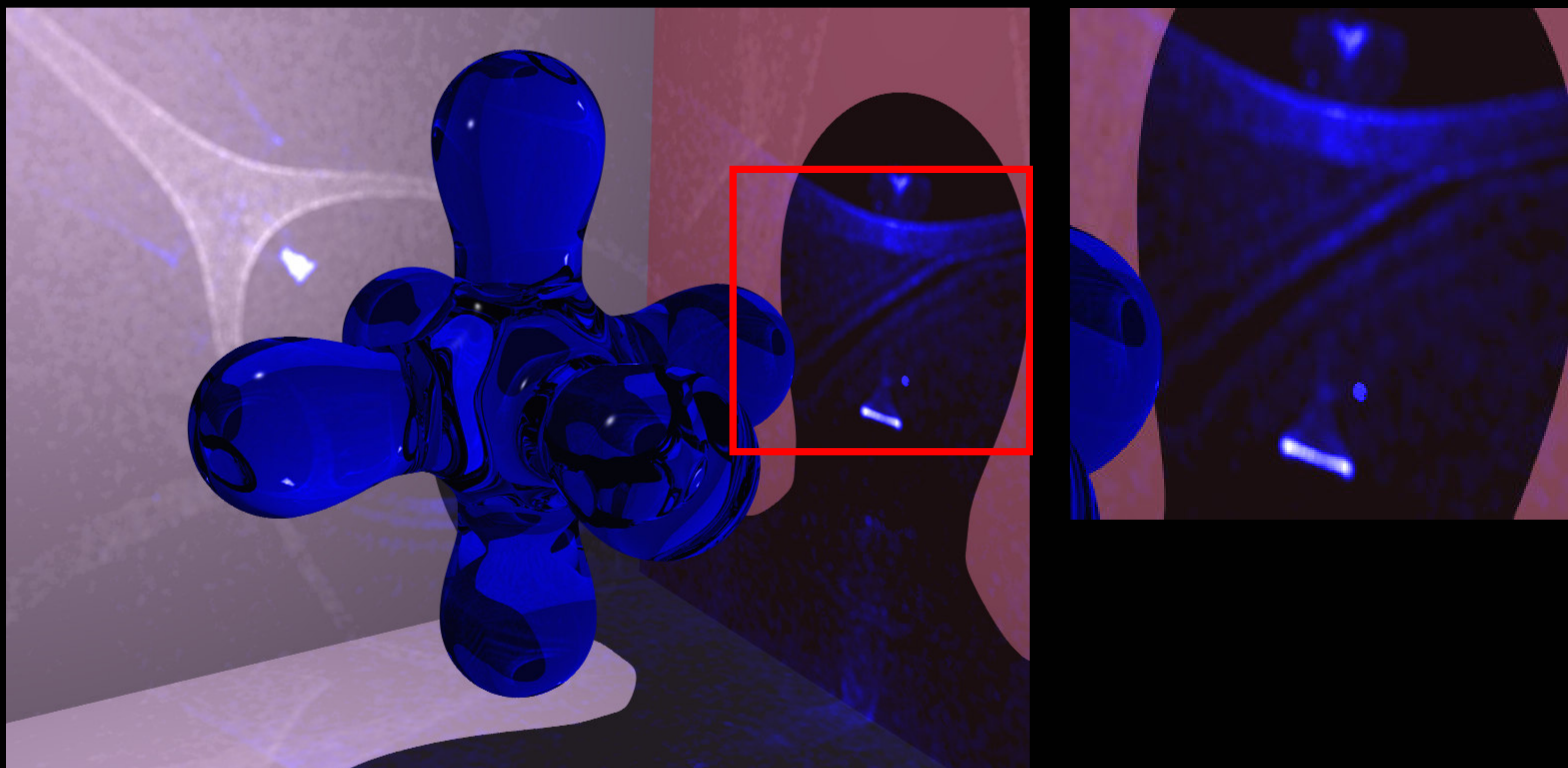
Pseudo-random number generator (PRNG)

- Provide uniform random numbers
- Example : rand() in C
- Important for stochastic algorithms
  - Evolutionary Computing
  - Photon-mapping rendering
- Huge Amount
- Speed
- Quality
  - Poor randomness → slow convergence

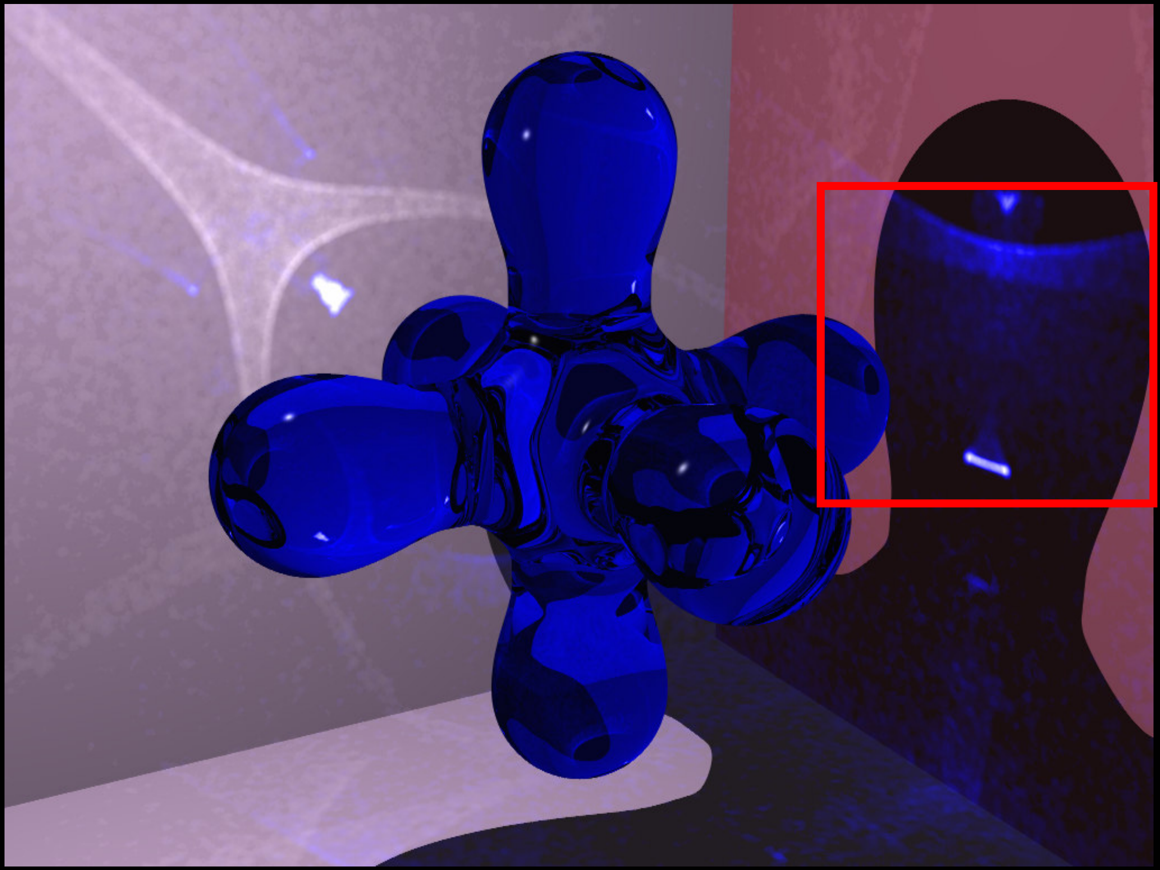
PRNG for Stochastic Rendering

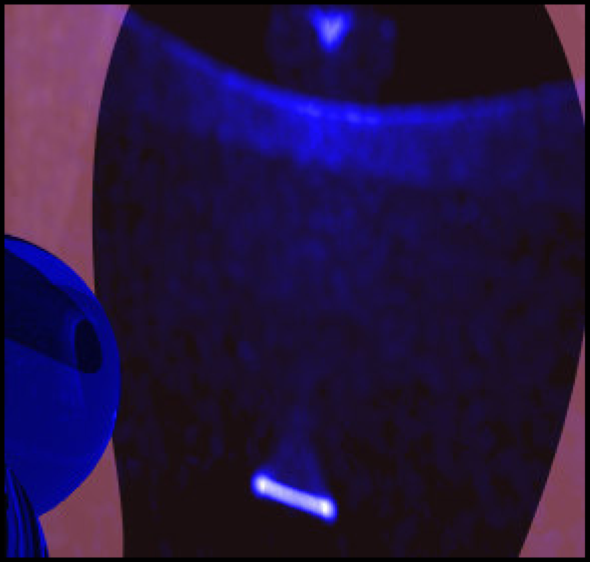
- Artifact for poor quality PRNG



PRNG for Stochastic Rendering

- From High quality PRNG





Some common PRNG

- linear congruential generator (LCG)
  - $R_{n+1} = aR_n + b \pmod{m}$
- lagged Fibonacci generator
  - $R_n = R_{n-j} \# R_{n+k} \pmod{m}$ (where # is a binary operator)
- High precision integer arithmetic
- Cannot fit in all GPU

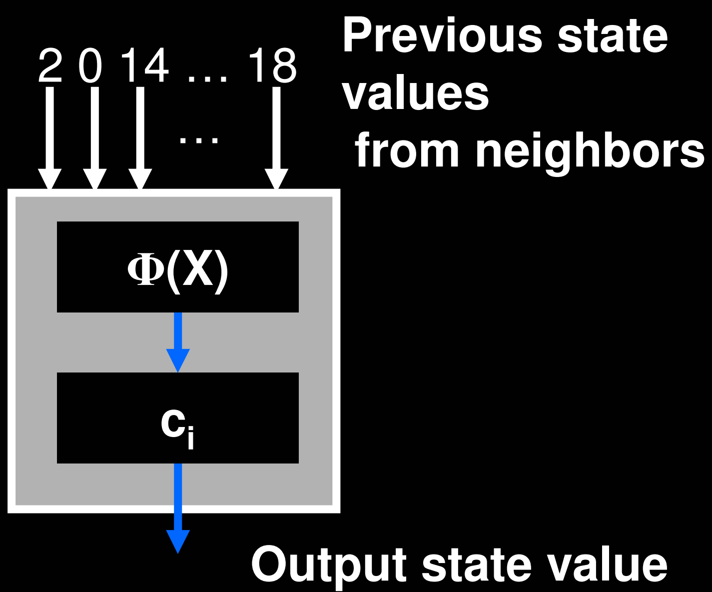
PRNG on GPU

- Cellular Automata-based PRNG [Wolfram]
- No high precision integer arithmetics
- Homogeneous cell operation and connectivity
- Quality
  - Configure to produce high quality random sequence

CA-based PRNG

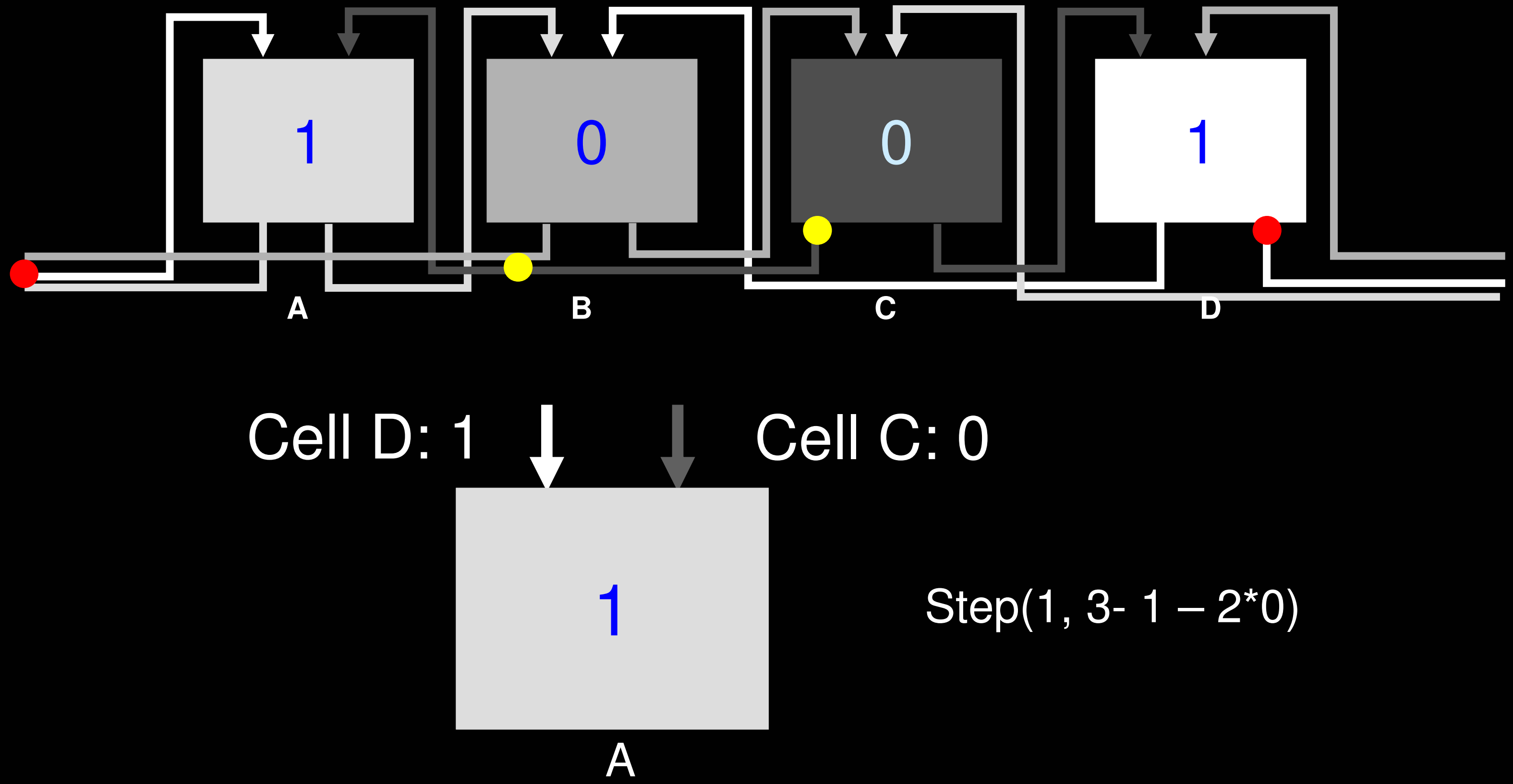
- Array of connected *cells* with homogeneous behavior
- Each Cell have a state and a common cell equation
- Cell Equation :

$$c_i^g = \phi(c_{i+n_0}^{g-1}, c_{i+n_1}^{g-1}, \ldots, c_{i+n_j}^{g-1})$$

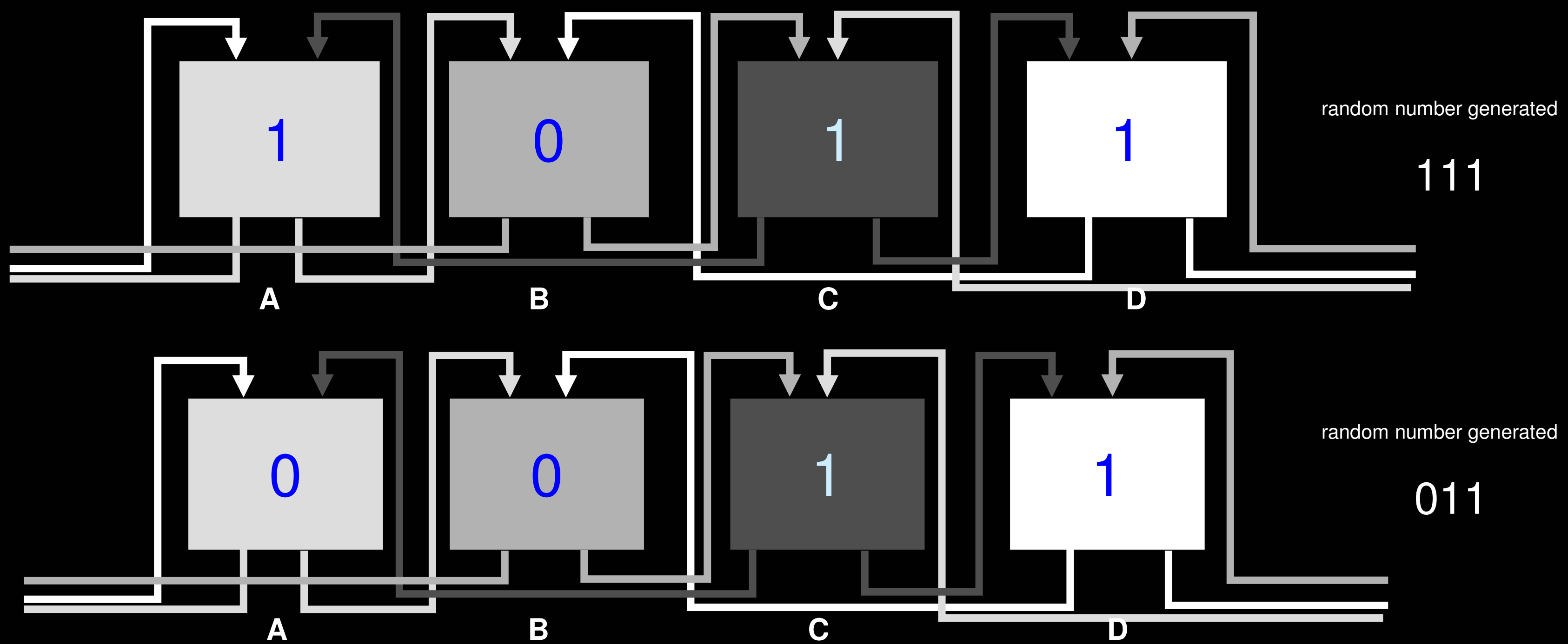


Mechanism

- 4 Cell, Connectivity (-1,2)
- Cell Equation : step(1, 3- c1- 2*c2)



Mechanism (cont')



GPU Implementation Issue

- Cell resembles texel in GPU
- 64 cells and 4 connected CA PRNG for 32-bits random number
- Cell equation evaluation
  - Fast table lookup
  - 4 connectivities = 4 input, 2^4 = 16 possible output
- Reorganize bits
  - Bits in a random number is scattered among texels
  - Output floating point value *f*

$$f = \left(\left(\left(r_0 / 2\right) + r_1\right) / 2 + \ldots\ldots + r_{31}\right) / 2$$

  - r_i is the *i*-th bit in the random number

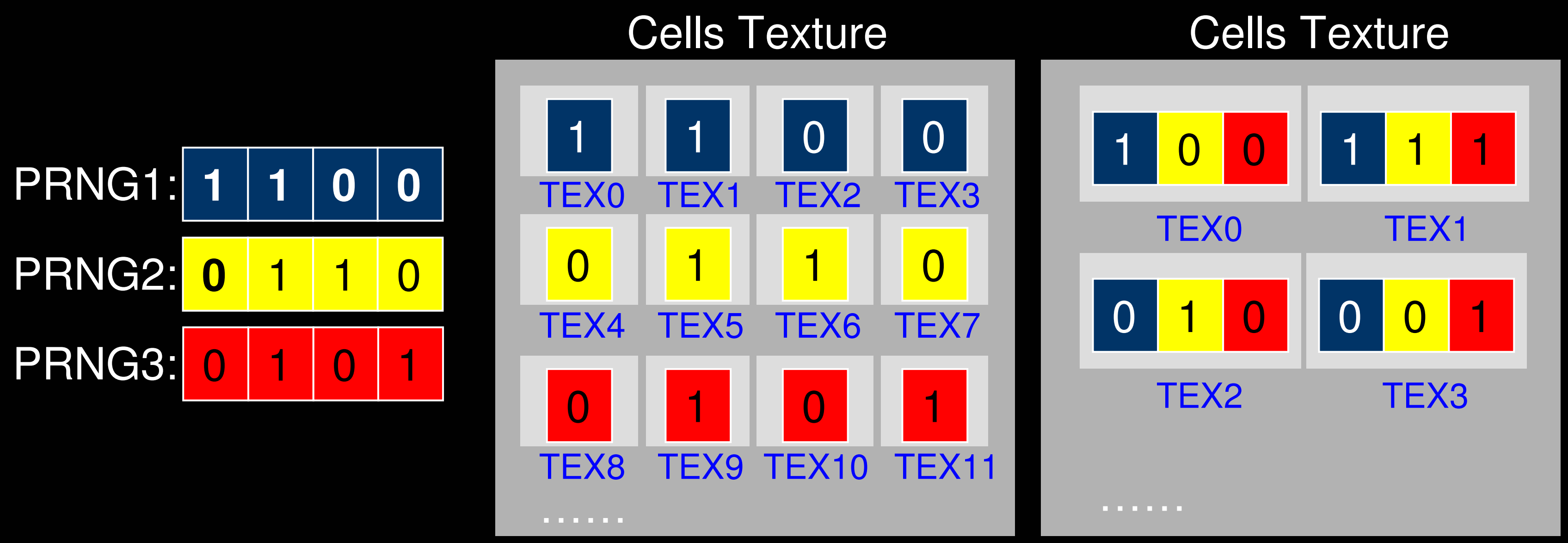
Shader Code

```
float4 caprng( in half2 coords: TEX0,in const uniform samplerRECT cells): COLOR0
{
   float2 Connector; float4 newState; float4 neigborStates[4]; int i;
   for (i = 0 ; i < 4; i++)
   {
              Connector.x = fmod(coords.x -connectivity(i),CA SIZE);
              Connector.y = coords.y;
              neigborStates[i] = round(texRECT(cells,Connector));
   } // cell equation evaluation
   newState.x = celleqn(neigborStates);
   return newState;
}
```

```
float4 pack(in half2 index : TEX0, in const uniform samplerRECT cells): COLOR0
{  int i; float4 outbits; float4 states; float2 texindex; outbits = 0;
   // packing all 32 bits
   for (i = 0 ; i < 32 ; i++)
   {
              texindex.x = i*2+1;
              texindex.y = index.y;
              states = texRECT(cells, texindex);
              outbits += states;
              outbits /= 2;
   }return outbits;
}
```

Parallelized PRNG

- Fully utilize 4096 ×4096 texels (7800GTX)
- Each cell occupies single bit in texel
- Why not pack more inside each texel ?
  - Fully utilize the mantissa part of the texel
- 23 × 4 random sequences simultaneously.
- Combine 2 schemes : 64×4096×92 PRNGs



Optimize for Quality

- Genetic Algorithm
  - CA base PRNG configuration with best quality
- Initialize candidates
  - Encoded cell equation and connectivities
  - $2^n + n$ bits
- Evaluate candidates by objective function
- Generate next generation
  - Crossover
  - Mutation
- Repeat until excess certain threshold

Objective Function

- Objective function

  $objective = w_0 \times e + w_1 \times \varphi$
- w_i is the weighting
- e is the n-bit entropy

$$e = \frac{-\sum_{i=0}^{2^n-1} p_i \log p_i}{n}$$

- *φ is the result of Diehard test*

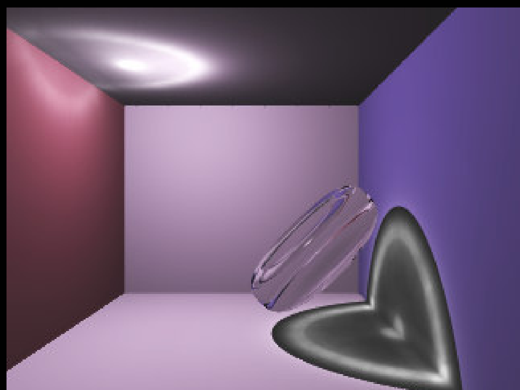
Objective Function (cont')

- Diehard test
  - 14 tests (e.g. birthday spacing, GDC test, etc.)
  - Chi-square

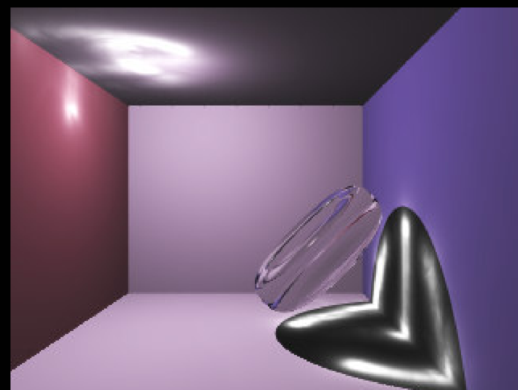
$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

  - Overall p-value
    - Chi-square test on all p-values with Gaussian distribution
- Best 4 connected, 64 Cells CA PRNG
  - Connectivity (56,2,21,49)
  - Cell equation in tightly packed format (1001100110100101)

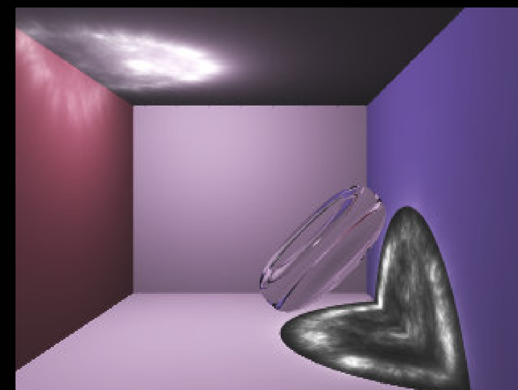
Convergence



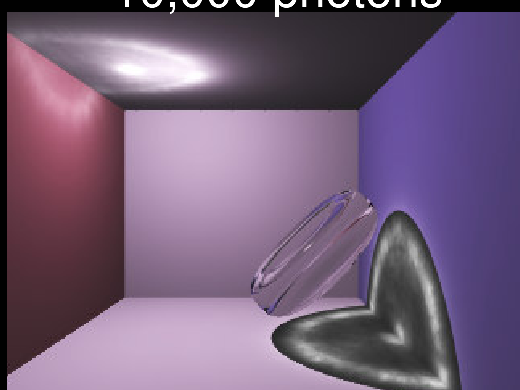
Control
10,000 photons



Generation 1
e=0.2673 φ =0.0



Generation 2
e=0.5852 φ =0.0



Generation 4
e=0.5944 φ =0.0

Generation 8
e=0.9464 φ =0.143

Generation 11
e=0.9514 φ =0.3513

Performance

- Performance compare with CPU
- Single PRNG
- 1,000 Parallel PRNG

Random numbers generated	GPU CA-PRNG	Software CA-PRNG
10,000 1,000	0.064s 0.004s	0.043s 0.004s
100,000 10,000	0.031s 0.942s	0.425s 0.042s
1,000,000 100,000	0.31s 10.081s	4.274s 0.391s
10,000,000 1,000,000	3.098s 100.082s	43.003s 4.163s
100,000,000	31.875s	430s

Conclusion

- CA architecture PRNG is highly suitable for GPU
- Parallel PRNG on GPU
- Optimization for quality
- A high quality and high performance gain
- Future works
  - Support of variable precision random sequence
  - Experiment with Evolution Computing applications

End

Thanks for your attention

- Reference :
  - W. M. Pang, T. T. Wong and P. A. Heng, Shader X5: Advanced Rendering Techniques, Edited by W. Engel, Charles River Media, 2007, pp. 579-590.